UNIVERSITY OF CALIFORNIA PUBLICATIONS
ASTRONOMY

LICK OBSERVATORY BULLETINS

VOLUME XII
NUMBERS 359 TO 384
1924-1927

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CONTENTS

PAGE
NUMBER 359.—Corrected Elements and Ephemeris of Minor Planet 1924 TD (Baade). H. Thiele 1
Preliminary Elements and Ephemeris of Minor Planet Y. O. 22. A. D. Maxwell and T. D. Shea 2
Preliminary Elements and Ephemeris of Minor Planet Y. O. 23. J. D. Shea, A. D. Maxwell, and A. P. Alexeievsky 3
NUMBER 360.—Observations of the Satellites of *Mars*. H. M. Jeffers 4
Measures of Comets and of Baade's Asteroid. H. M. Jeffers 8
NUMBER 361.—The Cluster Messier 11. R. J. Trumpler 10
NUMBER 362.—On the Number of Solutions in Leuschner's Direct Method for Determining the Orbits of Disturbed Bodies. R. H. Sciobereti 17
NUMBER 363.—Elements and Ephemeris of Comet *a* 1925 (Shajn–Comas Solá). H. B. Kaster and Katherine Prescott 31
Elements and Ephemeris of Comet *b* 1925 (Reid). A. D. Maxwell 33
Elements and Ephemeris of Comet *c* 1925 (Orkisz). J. D. Shea and A. P. Alexeievsky 35
Second Elements of Comet *c* 1919 (Metcalf–Borrelly). W. F. Meyer 37
NUMBER 364.—On the Determination of the Systematic Error in the Boss Proper Motions in Declination. P. van de Kamp 38
NUMBER 365.—The Orbit of Krueger 60. R. G. Aitken 45
The Orbit of β 581. R. G. Aitken 47
NUMBER 366.—Photographs of *Mars* Made with Light of Different Colors. W. H. Wright 48
NUMBER 367.—Measures of Comets and of Planet 1924 TD (Baade). H. M. Jeffers 62
NUMBER 368.—Azimuth Corrections for Sunset and Sunrise Transits. R. H. Tucker 65
NUMBER 369.—The Radial Velocities of Twenty Southern Variable Stars of Class Me. Leah B. Allen 71
A Study of the Changes in the Spectrum of T *Centauri*. Leah B. Allen 73
NUMBER 370.—Preliminary Elements and Ephemeris of Comet *j* 1925 (Van Biesbroeck). A. D. Maxwell and L. C. Damsgard. 76
Preliminary Elements and Ephemeris of Brooks' Periodic Comet *g* 1925. N. W. Storer and H. S. Mendenhall. 77
Second Elements of Y. O. 22 (1924 UA). A. D. Maxwell 78
NUMBER 371.—Second Elements and Ephemeris of Comet *j* 1925 (Van Biesbroeck). A. D. Maxwell and L. C. Damsgard. 79
NUMBER 372.—The Spectrum of α *Cygni* Between Wave-lengths 3020A and 3300A. W. H. Wright and Dorothy Applegate. 81
NUMBER 373.—Preliminary Elements and Ephemeris of Comet *b* 1926 (Blathwayt). A. D. Maxwell and N. W. Storer 87
NUMBER 374.—A Determination of the Sun's Velocity with Respect to Stars of Magnitude 9 to 10. P. van de Kamp 88
NUMBER 375.—On the Continuous Hydrogen Absorption in Spectra of Class A Stars. C. S. Yü 104
NUMBER 376.—Third Elements and Ephemeris of Comet *j* 1925 (Van Biesbroeck). A. D. Maxwell and L. C. Damsgard 122
NUMBER 377.—Measures of Comets. H. M. Jeffers 124
NUMBER 378.—A Photoelectric Study of *u Herculis*. R. H. Baker 130
NUMBER 379.—A Re-determination of the Radial Velocity Curves of Certain Cepheid Variable Stars. T. S. Jacobsen 138
NUMBER 380.—A Proposed Spectroscopic Method for Determining the Absolute Luminosities of Class A Stars. C. S. Yü. 155
NUMBER 381.—The Orbit of the Spectroscopic Binary 112 *Herculis*. W. F. Meyer 165
NUMBER 382.—Elements and Ephemeris of Comet *f* 1926 (Comas Solá). C. H. Smiley and Margaret K. Holbrook 168
Preliminary Parabolic Elements of Comet *f* 1926 (Comas Solá). L. A. Brigham and Katherine Prescott. 169
NUMBER 383.—Eighty-five New Double Stars, Twenty-fifth List. R. G. Aitken 170
Measures of Double Stars in the Years 1923–1926. R. G. Aitken 173
NUMBER 384.—A Photometric Study of the Flash Spectrum. E. F. Carpenter 183

PLATES

PLATE I.—FIGS. 1–4.—*Mars* and San Jose as Photographed from Mount Hamilton.
FIGS. 5–7.—Photographs of *Mars* by Ultra-violet, Violet and Infra-red Light 55
PLATE II.—FIGS. 1–8.—Photographs of *Mars* made by Light of Different Colors.
FIG. 9.—Photographs of *Venus* taken by Violet and by Infra-red Light 56
PLATE III.—The Spectrum of α *Cygni* 84
PLATE IV.—FIG. 1.—Spectrum of θ *Leonis* A0.
FIG. 2.—Spectrum of β *Urs. Majoris* A0.
FIG. 3.—Spectrum of θ *Leonis* (Enlarged).
FIG. 4.—Spectrum of β *Urs. Majoris* (Enlarged) 113
PLATE V.—FIG. 6.—Apparent Intensity Graphs (Slit Observations) 113
PLATE VI.—FIG. 7.—Stellar Energy Curves (Reduced from Fig. 6) 113
PLATE VII.—FIG. 8.—Apparent Intensity Graphs 114
PLATE VIII.—FIG. 9.—Stellar Energy Curves 114
PLATE IX.—Flash Spectrum (Enlarged Section) 185
PLATE X.—Flash Spectrum (Enlarged Section) 185

ERRATA

Volume XII.

Page 81, Title, *for* 3000A *read* 3300A
Page 91, Table 3, Star 4102, *for* H *read* Hδ
Page 98, Star H.D. 200274, declination, insert +
Page 99, 1, Col. 1, line 16, *for* —K *read* +K
Page 106, Table I, *for ζ Tauri read ξ Tauri*
Page 106, Footnote 14, *for* 50 *read* 52
Page 107, Table I, *for* 107 *Virginis read* 109 *Virginis*
Page 119, Table IV, *for δ Leonis read θ Leonis.*
Page 119, Table IV, *for* 107 *Virginis read* 109 *Virginis*
Page 140, Col. 2, line 21 from bottom, *for* declination circles *read* hour circles
Page 142, Col. 2, line 3 from bottom, *for* six hours *read* twelve hours.
Page 142, Col. 2, last line, *for* $f_0 \cos \phi$ *read* $2 f_0 \cos \phi$
Page 146, Col. 1, line 19, *for* increasingly *read* increasing
Page 146, Col. 1, line 30, *for* $0\overset{d}{.}00000088$, *read* $0\overset{d}{.}00000286$
Page 146, Col. 1, line 32, *for* $0\overset{d}{.}00000286$, *read* $0\overset{d}{.}00000484$
Page 152, Col. 2, footnote 28, for *65* read *85*
Page 161, Table I, H.R. 3569, *for* ν U Ma *read* ι U Ma
Page 162, Table I, H.R. 7420, *for* ν Cyg *read* ι Cyg.

Volume II.

Page 181, Col. 2, Hu 617 *for* 138.0, 141.0, 139.5 *read* 222.0, 221.0, 221.5
Page 182, Col. 1, Hu 620, *for* D M. $+0\overset{\circ}{.}1913$ *read* $+0\overset{\circ}{.}1903$
for 0.85 *read* 0.95 and *for* 0.86 (mean) *read* 0.90

Volume III.

Page 104, Col. 2, Hu 941, for 322.0, 324.2, 323.1 *read* 142.0, 144.2, 143.1

Volume VII.

Page 126, line 14 from bottom, star τ *Cassiopeiae, for* +20.3 *read* —20.3

UNIVERSITY OF CALIFORNIA PUBLICATIONS
ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 359

CORRECTED ELEMENTS AND EPHEMERIS OF MINOR PLANET 1924 T D (BAADE)

BY

H. Thiele

From Lick Observatory observations between October 27 and December 2 by Jeffers the following elements and ephemeris were deduced by variation of the ratio and the product of the distances on October 27 and December 2 corresponding to the preliminary elements published in *Lick Obs. Bull.* **11**, 357.

Elements

Epoch 1924 Oct. 27.5 Gr. M. T.

M	6°	34′	18″.41	
ω	130	37	10.29	1924.0
Ω	216	25	57.01	
i	26	09	26.25	
φ	32	40	01.41	
μ		813″.314		

Constants for the Equator 1924.0

$$x = r\,[9.9845820]\sin(\;74^\circ\;08'\;44''.83 + v)$$
$$y = r\,[9.9997803]\sin(343\;\;39\;\;04.91 + v)$$
$$z = r\,[9.4211527]\sin(\;66\;\;58\;\;11.26 + v)$$

Constants for the Equator 1925.0

$$x = r\,[9.9845714]\sin(\;74^\circ\;09'\;33''.22 + v)$$
$$y = r\,[9.9997795]\sin(343\;\;39\;\;49.39 + v)$$
$$z = r\,[9.4213059]\sin(\;66\;\;58\;\;20.43 + v)$$

These elements represent an observation by Jeffers at the Lick Observatory, January 4, Gr. C. T. as follows:

$$\text{O—C},\quad \Delta\alpha = +0^s.56,\quad \Delta\delta = +3''.9,$$

if the position of the comparison star B. D. $-5^\circ.284$ is based on a recent meridian circle observation by Tucker. The residuals become:

$$\text{O—C},\quad \Delta\alpha = +1^s.05,\quad \Delta\delta = +4''.5$$

if the position of the comparison star in the *San Fernando Photographic Catalogue* is adopted.

Ephemeris for Greenwich Civil Midnight

1925		True α	True δ	log Δ	log r	m
Feb.	1.0	$2^h31^m25^s.6$	$-2^\circ22'05''$	0.2113	0.2556	12.5
	2.0	33 30.9	15 39	0.2156		
	3.0	35 35.8	09 10	0.2198		12.6
	4.0	37 40.2	−2 02 40	0.2240		
	5.0	39 44.2	−1 56 08	0.2282		12.6
	6.0	41 47.9	49 35	0.2323		
	7.0	43 51.1	43 00	0.2364		12.7
	8.0	45 54.0	36 24	0.2405		
	9.0	47 56.4	29 47	0.2445		12.8
	10.0	49 58.5	23 10	0.2485		
	11.0	52 00.3	16 32	0.2525		12.8
	12.0	54 01.7	09 54	0.2564		
	13.0	56 02.8	−1 03 15	0.2603	0.2747	12.9
	14.0	2 58 03.6	−0 56 36	0.2642		
	15.0	3 00 04.0	49 58	0.2680		13.0
	16.0	02 04.2	43 20	0.2718		
	17.0	04 04.1	36 43	0.2756		13.0
	18.0	06 03.8	30 06	0.2794		
	19.0	08 03.1	23 31	0.2831		13.1
	20.0	10 02.2	16 56	0.2868		
	21.0	12 01.0	10 22	0.2904		13.1
	22.0	13 58.7	−0 03 50	0.2941		
	23.0	15 58.1	+0 02 41	0.2977		13.2
	24.0	17 56.2	09 10	0.3013		
	25.0	19 54.2	15 38	0.3048	0.2930	13.2
	26.0	21 51.9	22 03	0.3083		
	27.0	23 49.4	28 27	0.3118		13.3
	28.0	25 46.7	34 48	0.3152		
Mar.	1.0	3 27 43.8	+0 41 07	0.3186		13.3

The earlier part of the ephemeris was transmitted to the Harvard College Observatory for distribution. The computed magnitudes are based on those given in *Beobachtungs-Zirkular der A. N.* 37.

Berkeley Astronomical Department,
December 15, 1924.

PRELIMINARY ELEMENTS AND EPHEMERIS OF MINOR PLANET Y. O. 22

BY

A. D. MAXWELL and J. D. SHEA

The following observations of Y. O. 22 by the discoverer, Dr. Van Biesbroeck of the Yerkes Observatory, were received by mail:

	1924 Gr.M.T.	α (1925.0)	δ (1925.0)
1. Dec.	1.86105	$5^h\ 27^m\ 56^s.02$	$+34°\ 26'\ 06''.5$
2.	15.59397	13 44.58	32 33.5
3.	20.81802	08 23.45	28 10.2
4. Dec.	24.67366	5 04 38.09	+34 22 36.2

An orbit was first based upon the last three of these observations, and later extended to include the first, using the observation of December 15 as middle date.

ELEMENTS

Epoch	1924 Dec. 15.58337 Gr.M.T.	
M	$14°\ 03'\ 25''.5$	
ω	54 29 22.8	
☊	11 19 36.3	1925.0
i	7 56 54.8	
φ	3 33 19.6	
μ	$687''.83$	
P	5.158 years.	
$\log a$	0.475018	

The residuals are:

O—C	Dec. 1	Dec. 20	Dec. 24
$\cos\delta\,\Delta\alpha$	$+1''.2$	$+0''.2$	$-1''.6$
$\Delta\delta$	-0.1	$+2.8$	-0.9

These residuals are so small that it was not deemed advisable to remove them by a differential correction.

CONSTANTS FOR THE EQUATOR 1925.0

$$x=r\,[9.999840]\sin(155°\ 42'\ 37''.1+v)$$
$$y=r\,[9.932005]\sin(\ 66\ \ 39\ \ 15.4+v)$$
$$z=r\,[9.715346]\sin(\ 63\ \ 08\ \ 49.7+v)$$

EPHEMERIS FOR GREENWICH CIVIL MIDNIGHT

1925	True α	True δ	$\log\Delta$
Jan. 30.0	$4^h\ 46^m\ 55^s$	$+32°\ 47'.0$	
Feb. 3.0	47 18	36.1	0.340
7.0	48 10	25.8	
11.0	49 28	16.1	0.359
15.0	51 13	+32 07.1	
19.0	53 22	+31 58.7	0.378
23.0	55 55	51.0	
Feb. 27.0	4 58 50	43.8	0.397
Mar. 3.0	5 02 06	37.2	
7.0	05 42	31.6	0.416
11.0	09 36	25.3	
15.0	13 47	19.9	0.434
19.0	18 14	14.6	
23.0	22 55	09.4	0.451
Mar. 27.0	5 27 51	+31 04.2	

The magnitude of the planet in December was estimated to be 14. The computed magnitudes at the beginning and end of the ephemeris are 14 and 15, respectively.

BERKELEY ASTRONOMICAL DEPARTMENT,
January 29, 1925.

PRELIMINARY ELEMENTS AND EPHEMERIS OF MINOR PLANET Y. O. 23

BY

J. D. Shea, A. D. Maxwell and A. P. Alexeievsky

The preliminary orbit of Y. O. 23 was derived by differential correction of the orbit of Y. O. 22 from the first, third, and fourth of the following observations all of which were made by the discoverer, Dr. Van Biesbroeck of the Yerkes Observatory:

	1924 Gr.M.T.	α (1925.0)	δ (1925.0)
1.	Dec. 1.86105	5^h 31^m $25^s.12$	$+35°$ $10'$ $20''.0$
2.	19.69736	14 11.08	+34 26 02.3
3.	20.81802	13 07.60	22 10.6
4.	Dec. 24.67366	5 09 36.53	+34 08 07.1

ELEMENTS

Epoch	1924 Dec. 20.80262 Gr.M.T.	
M	$125°$ $09'$ $22''.1$	
ω	356 44 27.8	1925.0
Ω	302 08 50.8	
i	12 54 27.0	
φ	13 00 51.4	
μ	$639''.35$	
P	5.5497 years.	
$\log a$	0.496180	

The residuals are:

O—C	Dec. 1	Dec. 19	Dec. 24
$\cos\delta\,\Delta\alpha$	$+1''.0$	$+4''.0$	$-0''.7$
$\Delta\alpha$	$+0.5$	-0.2	$+0.3$

The elements show a close agreement with those of (982) *Franklina*. The ephemeris of (982) in *Kleine Planeten* 1924 represents the four observations of Y. O. 23 as follows:

$$\text{O—C} \quad \Delta\alpha = +2^m.4, \quad \Delta\delta = -4',$$

which corresponds to the predicted ephemeris change of $-1'.7$ in declination for $+1^m$ in right ascension for (982), whereby the identity of the two objects is confirmed.

CONSTANTS FOR THE EQUATOR 1925.0

$x = r\,[9.992091] \sin(29°\ 33'\ 09''.9 + v)$
$y = r\,[9.938418] \sin(293\ \ 13\ \ 14.3 + v)$
$z = r\,[9.725664] \sin(317\ \ 25\ \ 09.5 + v)$

EPHEMERIS FOR GREENWICH CIVIL MIDNIGHT

1925	True α	True δ	log Δ
Jan. 30.0	4^h 49^m 46^s	$+31°$ $32'.1$	
Feb. 3.0	00 21	31 15.6	0.489
7.0	00 17	30 59.9	
11.0	49 35	45.0	0.505
15.0	50 14	30.8	
19.0	51 12	17.6	0.521
23.0	52 29	30 05.1	
Feb. 27.0	54 05	29 53.5	0.537
Mar. 3.0	55 58	42.6	
7.0	4 58 08	32.5	0.553
11.0	5 00 32	23.1	
15.0	03 11	14.4	0.568
19.0	06 08	29 06.1	
23.0	09 09	28 58.3	0.583
27.0	12 26	51.0	
Mar. 31.0	15 55	44.0	0.597
Apr. 4.0	5 19 34	+28 37.3	

The magnitude of the planet in December was estimated to be 15. The computed magnitudes at the beginning and end of the ephemeris are 15 and 16, respectively.

BERKELEY ASTRONOMICAL DEPARTMENT,
January 29, 1925.
Issued February 9, 1925.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 360

OBSERVATIONS OF THE SATELLITES OF MARS

BY

H. M. JEFFERS

The observations contained in this paper were made with the 36-inch telescope. A 270 power eyepiece was used almost exclusively. No occulting bar was employed, though occasionally settings on the satellites were made with *Mars* out of the field, this being effected by sliding the eyepiece.

During most of the series of observations the quality of the seeing was only fair. Moreover, on some of the best nights from the point of view of steadiness the satellites were visible with difficulty. Also, on these nights when the satellites were particularly easy to see there was usually a good deal of boiling at the edge of the planet. This opposition of *Mars* was unfavorable because of the large zenith distance at which observations had to be made.

The observations of both *Phobos* and *Deimos* from August 4 to August 20 were made in position angle and distance. Except for the two measures of August 4 all the distances were measured from both sides of the planet, and are therefore free of assumption as to the planet's apparent diameter. The correction for semidiameter on August 4 was made by measures of it on the same night. The values of the distances given are nearly invariably the result of two micrometer settings, one on each side of the illuminated disk. When the position angle changed enough during the distance measures to affect the distance, correction has been made.

The position angle measures were made with respect to the estimated center of gravity of the illuminated surface of *Mars*. The given values depend generally upon the mean of two or three settings.

The corrections for refraction and phase were made by the usual formulae.

POSITION ANGLES OF *Phobos*

1924	Gr. M. T.	Obs. p	Refraction	Phase	Cor. p	No. of settings
Aug. 4	$22^h07^m52^s$	267°.96	0°.00	+0°.02	267°.98	2
	22 11 12	266.25	0.00	+0.01	266.26	2
	22 30 57	261.17	−0.01	−0.02	261.14	2
	22 34 12	260.25	−0.01	−0.03	260.21	2
8	21 27 07	95.38	0.00	−0.04	95.34	3
	21 41 25	88.15	0.00	0.00	88.15	3
11	22 39 25	265.51	−0.01	−0.06	265.44	3
	22 56 53	260.78	−0.01	−0.08	260.69	3
	23 21 44	254.23	−0.02	−0.11	254.10	3
	23 43 04	247.38	−0.02	−0.15	247.21	2
14	21 02 01	238.62	−0.01	−0.10	238.51	2
	21 04 50	238.15	−0.01	−0.10	238.04	2
	21 21 43	228.74	−0.01	−0.15	228.58	2
	22 44 56	98.54	−0.01	+0.01	98.54	3
	22 55 36	92.54	−0.01	+0.02	92.55	3
	23 27 18	81.54	−0.03	+0.04	81.55	3
	23 38 27	78.75	−0.03	+0.04	78.76	3
	23 56 38	74.65	−0.04	+0.05	74.66	2
19	21 25 56	277.95	0.00	−0.03	277.92	3
	21 38 02	271.90	0.00	−0.04	271.86	2
	21 40 47	271.17	0.00	−0.04	271.13	2
	22 02 48	265.20	−0.01	−0.04	265.15	3
	22 14 04	261.17	−0.01	−0.03	261.13	2
	22 24 41	259.30	−0.02	−0.03	259.25	2
	22 36 33	255.95	−0.02	−0.04	255.89	2
	22 51 22	253.68	−0.02	−0.04	253.62	2
	23 14 59	245.90	−0.02	−0.05	245.83	2
	23 27 03	240.65	−0.03	−0.05	240.57	3

DISTANCES OF *Phobos*

1924	Gr. M. T.	Obs. *s*	Refraction	Phase	Cor. *s*
Aug. 4	22^h 18^m 15^s	17″71	+0″00	0″00	17″71
	22 26 11	18.79	+0.01	0.00	18.80
8	21 32 15	25.20	+0.01	+0.16	25.37
	21 35 07	26.31	+0.01	+0.16	26.48
	21 37 45	27.09	+0.01	+0.16	27.26
11	22 44 28	32.48	+0.01	−0.11	32.38
	22 47 23	32.73	+0.01	−0.11	32.63
	22 50 42	32.71	+0.01	−0.11	32.61
	23 02 26	33.54	+0.02	−0.11	33.45
	23 05 36	33.58	+0.02	−0.10	33.50
	23 08 10	33.98	+0.02	−0.10	33.90
	23 13 18	33.71	+0.02	−0.10	33.63
	23 28 08	33.01	+0.02	−0.10	32.93
	23 30 42	32.99	+0.02	−0.10	32.91
	23 34 13	32.15	+0.02	−0.10	32.07
14	21 09 19	23.10	+0.01	−0.04	23.07
	21 13 32	21.62	+0.01	−0.03	21.60
	22 49 16	23.89	+0.01	+0.08	23.98
	22 51 43	24.21	+0.01	+0.08	24.30
	23 31 02	33.43	+0.02	+0.07	33.52
	23 33 31	32.92	+0.02	+0.07	33.01
	23 44 40	33.83	+0.03	+0.07	33.92
	23 48 25	34.30	+0.03	+0.07	34.40
15	0 01 03	34.11	+0.04	+0.06	34.21
	0 02 14	33.63	+0.04	+0.06	33.73
19	21 30 36	24.95	+0.01	−0.02	24.94
	21 33 20	25.91	+0.01	−0.02	25.90
	22 07 07	32.82	+0.01	−0.02	32.81
	22 09 56	32.74	+0.01	−0.01	32.74
	22 17 19	33.42	+0.02	−0.01	33.43
	22 20 21	34.55	+0.02	−0.01	34.56
	22 28 33	34.50	+0.02	−0.01	34.51
	22 31 05	34.97	+0.02	−0.01	34.98
	22 42 00	34.15	+0.02	−0.01	34.16
	22 46 26	33.53	+0.02	−0.01	33.54
	23 19 23	28.07	+0.03	−0.01	28.09
	23 22 31	26.96	+0.03	−0.01	26.98
	23 32 07	24.18	+0.04	0.00	24.22
	23 35 45	22.79	+0.04	0.00	22.83

POSITION ANGLES OF *Deimos*

1924	Gr. M. T.	Obs. *p*	Refraction	Phase	Cor. *p*	No. of settings
Aug. 11	21^{h}35^{m}20^s	235°73	−0°01	−0°07	235°65	3
	22 07 13	230.51	−0.01	−0.07	230.43	4
14	21 42 14	73.69	0.00	+0.02	73.71	3
	21 53 59	73.22	−0.00	+0.02	73.24	4
	22 11 15	71.92	−0.01	+0.02	71.93	3
	22 16 46	71.84	−0.01	+0.02	71.85	3
	23 01 36	68.72	−0.02	+0.03	68.73	3
	23 14 55	68.15	−0.02	+0.03	68.16	3
19	21 03 09	80.20	−0.01	+0.01	80.20	3
	21 16 34	79.52	−0.01	+0.01	79.52	2
	21 47 09	78.07	−0.01	+0.01	78.07	4
	22 56 15	73.93	−0.02	+0.02	73.93	3
	23 10 11	73.05	−0.03	+0.02	73.04	3
	23 40 47	71.22	−0.04	+0.02	71.20	2

DISTANCES OF *Deimos*

1924	Gr. M. T.	Obs. *s*	Refraction	Phase	Cor. *s*
Aug. 11	21^h 42^m 20^s	50″49	+0″02	−0″07	50″44
	21 48 10	49.51	+0.02	−0.06	49.47
	21 53 26	49.06	+0.02	−0.06	49.02
14	21 46 30	82.97	+0.03	+0.06	83.06
	21 49 41	82.75	+0.03	+0.06	82.84
	21 59 06	83.12	+0.03	+0.06	83.21
	22 03 04	82.42	+0.03	+0.06	82.51
	22 23 16	80.44	+0.04	+0.06	80.54
	22 26 38	79.98	+0.04	+0.06	80.08
	22 29 06	79.79	+0.04	+0.06	79.89
	23 06 48	75.82	+0.05	+0.05	75.92
	23 10 25	75.12	+0.06	+0.05	75.23
	23 18 42	73.78	+0.06	+0.05	73.89
	23 21 14	73.33	+0.06	+0.05	73.44
19	21 07 14	85.89	+0.02	+0.02	85.93
	21 12 27	86.36	+0.02	+0.02	86.40
	21 52 56	87.11	+0.03	+0.01	87.15
	21 55 20	86.37	+0.03	+0.01	86.41
	23 01 48	83.76	+0.07	+0.01	83.84
	23 04 48	83.38	+0.07	+0.01	83.46
	23 45 57	80.06	+0.13	+0.01	80.20

After August 20 measures of the satellites were made in rectangular co-ordinates, after the method employed by Aitken and Barnard in 1909. Usually the measures were made in the direction of the major axis of the apparent orbit, from the east and west edges of *Mars*, and perpendicular to this, from the north and south edges. Some observations were also made with the micrometer set parallel to the equator, and on August 30, through an oversight, the measures were made in angle 78°4 (and perpendicular to this) instead of 79°4. Each given distance is the result of two double distance measures, four micrometer settings.

The following table gives the apparent distances as measured; no corrections have been applied.

MEASURES OF *Phobos*

1924	Gr. M. T.	East Edge	West Edge	North Edge	South Edge	*p*
Aug. 21	21^{h}00^{m}31^s		17″31			78°0
	21 04 34	43″02				
	21 10 45			19″60		168.0
	21 15 12				6″63	
	21 22 36		10.78			78.0
	21 27 42	36.16				
	21 33 24				4.90	168.0
	21 38 02			22.08		
	23 19 04		39.61			78.0
	23 24 23	14.22				
	23 29 48				7.65	168.0
	23 33 25			17.95		

1924	Gr. M. T.	East Edge	West Edge	North Edge	South Edge	p
Aug. 22	$22^h00^m42^s$		33″.44			78°.2
	22 05 06	8″.56				
	22 10 13			20″.36		168.2
	22 14 34				6″.20	
	22 25 39		44.58			78.2
	22 29 55	16.44				
	22 34 42			17.62		168.2
	22 38 12				8.84	
	23 11 31	21.42				78.2
	23 16 18		47.99			
	23 21 46			10.80		168.2
	23 26 09				15.86	
Aug. 26	19 35 41			7.84		168.8
	19 41 26				19.49	
	19 49 41		40.38			78.8
	19 56 42	11.09				
	22 15 34			7.97		168.8
	22 19 21				17.90	
	22 24 57	45.60				78.8
	22 28 52		19.88			
	23 06 23	47.60				78.8
	23 11 46		20.06			
	23 17 59				9.27	168.8
	23 22 10			17.35		
Aug. 28	21 03 22				10.64	169.0
	21 07 11			15.40		
	21 13 37		18.55			79.0
	21 19 25	44.21				
Aug. 30	19 33 58				6.89	168.4
	19 39 28			19.85		
	19 47 06		6.94			78.4
	19 53 56	30.99				
	21 30 50	10.99				78.4
	21 36 37		38.93			
	21 44 42				7.33	168.4
	21 49 18			18.56		
Sept. 1	20 59 39	16.83				79.7
	21 04 05		42.29			
	21 09 21			6.34		169.7
	21 13 27				20.36	
Sept. 6	18 55 32		20.20			80.4
	19 01 37	46.01				
	19 08 48			15.20		170.4
	19 14 54				11.52	
Sept. 14	17 19 40		7.14			90.0
	17 23 24	35.19				
	17 28 14			8.58		180.0
	17 31 36				14.24	
	17 54 42	41.75				90.0
	17 59 16		18.58			
	18 03 07			14.14		180.0
	18 06 51				9.01	
	18 33 32	42.77				90.0
	18 37 49		17.70			
	18 41 49				4.79	180.0
	18 44 35			19.14		
	19 03 19		12.93			90.0
	19 07 21	35.70				
	19 11 23				1.89	180.0
	19 14 17			22.05		

MEASURES OF *Deimos*

1924	Gr. M. T.	East Edge	West Edge	North Edge	South Edge	p
Aug. 21	$21^h48^m56^s$		60″.05			78°.0
	21 55 41	85″.29				
	22 02 58			25″.65		168.0
	22 10 05				14″.14	180.0
	22 14 17			40.66		
	23 03 38			29.35		168.0
	23 07 26				3.52	
	23 57 54				5.71	168.0
	24 02 10			32.11		
Aug. 22	20 54 20		38.97			78.6
	20 58 46	66.52				
	21 05 43			4.96		168.6
	21 10 44				31.05	
	21 22 50	72.19				78.6
	21 27 51		46.52			
	21 32 52			3.61		168.6
	21 37 20				29.63	
	22 50 26	89.59				90.0
	22 54 56		63.01			
	22 59 34			17.19		180.0
	23 03 52				8.58	
	23 36 50	94.28				78.6
	23 42 55		68.36			79.3
Aug. 26	20 21 55		73.26			
	20 27 26	99.89				
	21 46 02		68.72			79.3
	21 56 03	95.10				
	22 02 55				4.39	169.3
	22 08 59			21.92		
	23 33 44	79.70				79.3
	23 39 18		52.17			
	23 46 26			28.31		169.3
	23 51 25				2.76	
Aug. 28	21 30 48	45.75				79.5
	21 37 36		71.34			
	21 45 06			4.25		169.5
	21 50 10				31.05	
	22 12 39		63.04			79.5
	22 18 39	34.77				
	22 25 13			6.28		169.5
	22 32 12				32.72	
Aug. 30	20 05 21	55.35				78.8
	20 10 04		27.54			
	20 14 44				5.96	168.8
	20 20 34			32.06		
	21 01 17			31.70		168.8
	21 08 42				7.97	
Sept. 6	19 26 38	13.95				80.8
	19 33 33		37.58			
Sept. 14	15 53 19		57.09			90.0
	16 02 28	81.99				
	16 15 24			13.77		180.0
	16 19 50				9.71	
	16 35 49		62.39			90.0
	16 43 10	86.56				
	16 50 23			15.83		180.0
	16 55 36				7.04	
	17 36 33				3.52	180.0
	17 40 15			20.37		

1924	Gr. M. T.	East Edge	West Edge	North Edge	South Edge	p
Sept. 14	$17^h44^m07^s$		65″90			90°0
	17 48 32	89″73				
	18 14 42				0″62*	180.0
	18 17 43			23″41		
	18 22 33		65.59			90.0
	18 27 15	89.73				
	19 17 55			27.63		180.0
	19 20 58				3.77	
	19 24 18		63.71			90.0
	19 27 59	86.99				

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* *Deimos* North of South edge.

The incompleteness of certain of the above sets of measures is due either to the fact that *Deimos* was not well enough visible at the proper times, or, in two cases, because the micrometer was obviously wrongly read.

The results below have been derived by taking the mean of one set of measures from the East and West (or North and South) limbs, and of the corresponding times, and applying to them the corrections for refraction. By x are designated distances measured parallel to the equator, or along the major axis of the apparent orbit, and by y those distances measured perpendicular to x. The corrections for refraction have been calculated by the formulae for refraction in right ascension and declination, except that the displacements in right ascension have not been multiplied by the factor sec δ. When the measures were not made parallel and perpendicular to the equator, the corrections have been transformed into those belonging to the particular position angle in which the measures were made. The signs of x and y are in the sense *Satellite—Mars*. The corrections for phase have not been applied.

Phobos

1924	Gr. M. T.	x	p
Aug. 21	21^h 02^m 32^s	−30″17	78°0
	21 25 09	−23.48	
	23 21 44	+26.94	
Aug. 22	22 02 54	+21.01	78.2
	22 27 47	+30.53	
	23 13 54	+34.72	
Aug. 26	19 53 12	+25.75	78.8
	22 26 54	−32.76	
	23 09 04	−33.88	
Aug. 28	21 16 31	−37.39	79.0
Aug. 30	19 50 31	−18.97	78.4
	21 33 44	+24.97	
Sept. 1	21 01 52	+29.58	79.7
Sept. 6	18 58 34	−33.11	80.4
Sept. 14	17 21 32	−21.17	90.0
	17 56 59	−30.16	
	18 35 40	−30.25	
	19 05 20	− 24.33	

1924	Gr. M. T.	y	p
Aug. 21	21^h 12^m 58^s	−6″48	168°0
	21 35 43	−8.59	
	23 31 36	−5.14	
Aug. 22	22 12 24	−7.08	168.2
	22 36 27	−4.39	
	23 23 58	+2.55	
Aug. 26	19 38 34	+5.82	168.8
	22 17 28	+4.95	
	23 20 04	−4.06	
Aug. 28	21 05 16	−2.39	169.0
Aug. 30	19 36 43	−6.49	168.4
	21 47 00	−5.62	
Sept. 1	21 11 24	+7.02	169.7
Sept. 6	19 11 51	−1.84	170.4
Sept. 14	17 29 55	+2.84	180.0
	18 04 59	−2.56	
	18 43 12	−7.19	
	19 12 50	−10.09	

Deimos

1924	Gr. M. T.	x	p
Aug. 21	21^h 52^m 18^s	−72″71	78°0
Aug. 22	20 56 33	−52.75	78.6
	21 25 20	−59.39	
	22 52 41	−76.36	90.0
	23 39 52	−81.43	78.6
Aug. 26	20 24 40	−86.59	79.3
	21 51 02	−81.96	
	23 36 31	−66.14	
Aug. 28	21 34 12	+58.58	79.5
	22 15 39	+48.95	
Aug. 30	20 07 42	−41.46	78.8
Sept. 6	19 30 06	+25.78	80.7
Sept. 14	15 57 54	−69.59	90.0
	16 39 30	−74.52	
	17 46 20	−77.84	
	18 24 54	−77.68	
	19 26 08	−75.38	

1924	Gr. M. T.	y	p
Aug. 21	22 02 58	−12.52*	168.0
	22 12 11	−27.44	180.0
	23 05 32	−16.49	168.0
	24 00 02	−19.02	
Aug. 22	21 08 14	+18.01	168.6
	21 35 06	+16.62	
	23 01 43	− 4.32	180.0
Aug. 26	22 05 57	− 8.79	169.3
	23 48 56	−15.67	
Aug. 28	21 47 38	+17.68	169.5
	22 28 42	+19.54	
Aug. 30	20 17 39	−19.03	168.8
	21 05 00	−19.86	
Sept. 14	16 17 37	− 2.01	180.0
	16 53 00	− 4.39	
	17 38 24	− 8.42	
	18 16 12	−11.40	
	19 19 26	−15.72	

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* With semidiameter of *Mars* = 13″16.

The diameter of *Mars* was occasionally measured:

1924	Gr. M. T.	Diam.	p	
Aug. 8	21^h 45^m	25″.88	90.0	Two double distances.
Aug. 11	22 15	25.95	0.0	Two double distances.
	22 16	26.56	90.0	Two double distances.
	23 59	25.91	0.0	Two double distances.
Aug. 14	21 30	25.81	0.0	Two double distances.
	21 32	25.75	90.0	Two double distances.
Aug. 19	23 50	27.32	0.0	One double distance.
	23 52	27.88	90.0	One double distance.
Aug. 21	22 34	26.92	78.0	Four double distances.
	22 36	26.32	168.0	Four double distances.
Aug. 26	22 39	26.71	169.3	Two double distances.
	22 41	26.72	79.3	Two double distances.

These diameters are somewhat larger than those in the ephemeris. No correction has been applied to them. They are not numerous enough to be of value in discussing the planet's diameter, but they may be useful in connection with the preceding measures of the satellites.

January, 1925.

MEASURES OF COMETS AND OF BAADE'S ASTEROID

BY

H. M. JEFFERS

Most of the following measures were made with the 12-inch refractor, a few, however, were made with the 36-inch. The positions given are all mean places for the beginning of the year, that is, the position of the object is that obtained by applying to the mean place of the comparison star the measured $\Delta\alpha$ and $\Delta\delta$. In most instances the $\Delta\alpha$ was measured directly; in the few cases where this was not possible transits were employed.

COMET *c* 1924 (FINSLER)

1924	Gr. M. T.	Star	No. of Comp.	Comet—Star $\Delta\alpha$	Comet—Star $\Delta\delta$	Comet's Mean α	Comet's Mean δ
Sept. 21	15^h 16^m 49^s	1	8, 10	$+28^s.64$	+1′ 50″.2	14^h 09^m $29^s.87$	+8° 23′ 44″.4
21	15 47 29	1	6, 10	+36.90	− 5.8	14 09 38.13	+8 21 48.4
23	15 50 19	2	8, 8	− 3.98	− 58.1	14 21 57.80	+5 27 44.4
24	15 33 51	3	9, 10	−44.94	+5 8.2	14 27 25.60	+4 07 15.3
26	15 33 47	4	6*t*, 7	−76.14	−1 24.6	14 37 24.88	+1 35 8.4
28	15 42 53	5	4, 7	− 2.84	−7 10.7	14 46 13.66	−0 43 48.6
30	15 24 46	6	7*t*, 6	−94.06	+1 24.0	14 53 58.85	−2 48 51.6
Oct. 1	15 09 30	7	10, 11	+20.38	+ 51.6	14 57 32.29	−3 46 57.6
1	15 41 22	7	3, 4	+24.25	− 31.1	14 57 36.16	−3 48 20.3
7	15 15 30	8	8, 8	+32.08	+3 27.6	15 15 47.89	−8 48 41.9

MEAN PLACES OF COMPARISON STARS, 1924.0

	α	δ	Authority
1	14^h 09^m $01^s.23$	+8° 21′ 54″.2	A. G. Leipzig II 6636
2	14 22 01.78	+5 28 42.5	Toul. Phot. +5° 14^h 20^m No. 38
3	14 28 10.54	+4 02 7.1	A. G. Albany 4981
4	14 38 41.02	+1 36 33.0	A. G. Albany 5026
5	14 46 16.50	−0 36 37.9	Algiers Phot. 0° 14^h 48^m No. 89
6	14 55 32.91	−2 50 15.6	A. G. Strassburg 5261
7	14 57 11.91	−3 47 49.2	A. G. Strassburg 5266
8	15 15 15.81	−8 52 9.5	A. G. Wien-Ott. 5354

On account of the very low altitude at which the above measures were made, and because of the well-condensed head which favored good micrometer settings, all the above measured differences are corrected for the differential refraction. Measures made by transits are indicated by a *t* after the number of comparisons. I am indebted to Mrs. M. L. Shane for the recording of several of the observations, and to Mr. E. F. Carpenter for recording and checking two of them. The observation on October 1 at 15^h 41^m 22^s was made by Mrs. M. L. Shane, and recorded by me. All measures were made with the 12-inch telescope.

COMET *b* 1924 (ENCKE)

			No. of	Comet—Star		Comet's Mean	
1924	Gr. M. T.	Star	Comp.	$\Delta\alpha$	$\Delta\delta$	α	δ
Sept. 28	$0^h\ 08^m\ 42^s$	20	8, 9	$+6^s.63$	$+4'\ 37''.2$	$9^h\ 09^m\ 43^s.71$	$+30°\ 39'\ 43''.7$

MEAN PLACE OF COMPARISON STAR, 1924.0

	α	δ	Authority
20	$9^h\ 09^m\ 37^s.08$	$+30°\ 35'\ 6''.5$	Oxf. Phot. No. 23051

The above measure was made with the 36-inch telescope. The comet was diffuse and hard to set on, with a diameter of about 2′. Four Crossley photographs of this comet were also obtained, on August 27 and September 2, which can be measured should the occasion arise.

BAADE'S ASTEROID, 1924 T. D.

			No. of	Object—Star		Objects's Mean	
1924–1925	Gr. M. T.	Star	Comp.	$\Delta\alpha$	$\Delta\delta$	α	δ
Oct. 30	$17^h\ 40^m\ 25^s$	9	8, 8	$-30^s.20$	$-3'\ 56''.5$	$21^h\ 41^m\ 24^s.70$	$+10°\ 37'\ 31''.5$
31	15 46 57	10	10, 10	−12.86	− 29.8	21 45 51.51	+10 02 35.4
Nov. 3	15 57 27	11	11*t*, 9	+78.57	+2 59.0	22 00 11.63	+ 8 11 54.7
3	19 38 47	12	8, 9	−25.04	+1 51.0	22 00 54.37	+ 8 06 21.3
10	19 03 44	13	10, 10	+ 6.95	+4 5.1	22 32 53.22	+ 4 14 43.6
12	16 34 58	14	10, 11	−16.55	+1 20.5	22 41 13.35	+ 3 18 46.9
20	15 06 12	15	10, 10	− 1.68	−4 21.4	23 14 11.67	+ 0 00 23.6
Dec. 2	15 06 00	16	10, 10	+ 1.68	−1 52.4	23 58 10.38	− 3 13 13.7
2	15 30 10	17	10, 10	− 0.80	+1 3.9	23 58 13.89	− 3 13 22.5
9	18 17 20	18	10, 10	+ 7.10	+3 8.3	0 21 17.58	− 4 16 55.0
Jan. 5	3 07 55*	19	10, 10	+ 5.58	− 33.7	1 31 33.09	− 4 41 26.8
5	3 20 00*	19	8, 8	+ 6.79	− 32.1	1 31 34.30	− 4 41 25.2

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* Greenwich Civil Time.

MEAN PLACES OF COMPARISON STARS, 1924.0

	α	δ	Authority
9	$21^h\ 41^m\ 54^s.90$	$+10°\ 41'\ 28''.0$	½ (Toul. Phot. +11°, $21^h\ 40^m$ No. 202+Microm. comp. with A. G. Leipzig II 8663.)
10	21 46 4.37	+10 03 5.2	½ (Toul. Phot. +9°, $21^h\ 48^m$ No. 12+A. G. Leipzig II 10966.)
11	21 58 53.06	+ 8 08 55.7	A. G. Leipzig II 11080.
12	22 01 19.41	+ 8 04 30.3	½ (Toul. Phot. +9°, $22^h\ 04^m$ No. 186+Toul. Phot.+7°, $22^h\ 04^m$, No. 3.)
13	22 32 46.27	+ 4 10 38.5	½ (Toul. Phot. +5°, $22^h\ 28^m$ No. 162+Microm. comp. with A. G. Albany No. 7821.)
14	22 41 29.90	+ 3 17 26.4	A. G. Albany 7869.
15	23 14 13.35	+ 0 04 45.0	½ (A. G. Nicolajew 5803+Microm. comp. with A. G. Nicolajew 5805.)
16	23 58 8.70	− 3 11 21.3	A. G. Strassburg 8196.
17	23 58 14.69	− 3 14 26.4	½ (San Fernando Phot. $23^h\ 56^m$, −3° No. 104+$0^h\ 0^m$, −4° No. 17.)
18	0 21 10.48	− 4 20 3.3	½ (San Fernando Phot. $0^h\ 16^m$, −4° No. 163+$0^h\ 20^m$, −5° No. 103.)

MEAN PLACE OF COMPARISON STAR, 1925.0

	α	δ	Authority
19	$1^h\ 31^m\ 27^s.51$	$-4°\ 40'\ 53''.1$	½ (San Fernando Phot. $1^h\ 28^m$, −4° No. 131+$1^h\ 32^m$, −5° No. 36.)

The position of comparison star 19 has been subsequently observed with the Meridian Circle, through the kindness of Professor Tucker. The star is B.D.—5°.284, of the 9.5 magnitude. Professor Tucker gives the position for 1925.0: $1^h\ 31^m\ 27^s.02$, $-4°\ 40'\ 53''.7$. The epoch of the observation is 1925.03.

Of the above observations of Baade's Asteroid, those up to and including November 20 were made with the 12-inch refractor. The ones in December and January were made with the 36-inch. Thanks are due to Miss Gladys Wallace for recording all of the observations.

January, 1925.

Issued February 20, 1925.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 361

THE CLUSTER MESSIER 11

BY

Robert J. Trumpler

Messier 11 (N. G. C. 6705; G. C. 4437) is a rich cluster of faint stars situated in the constellation *Scutum Sobieski*[1] in a dense cloud of the Milky Way. The positions of the stars in this cluster and its immediate surroundings were measured by Lamont[2] (1836–39), Helmert[3] (1869–70), Stratonoff[4] (1896), and Küstner[5] (1915). Shapley[6] and Küstner determined the photographic magnitudes of the stars by comparison with the North Polar Sequence and the former also gives photovisual magnitudes and color-indices. Adopting with Shapley the star Stratonoff 462 as center of the cluter, its coördinates are:

R. A.: $18^h\ 45^m\ 43^s$ Decl.: $-6°\ 23'.2$ (1900)
Galact. Long.: 355°, Galact. Lat.: $-3°.4$

Reproductions of photographs by Isaac Roberts[7] and Barnard[8] have been published, and the following are some of the descriptions of Messier 11:

N. G. C.[9]: "Remarkable cluster, very bright, large, irregularly round, rich, one star 9th magn., stars 11^m. . ."

Bigourdan[10]: "Remarkable cluster, very pronounced concentration, in the central part the stars are rather crowded, but well separated. The whole cluster is contained in a circle of 15′ diameter, but the principal part has only about 5′: it includes a star 8^m–9 and a very large number of stars 12^m or fainter."

Bailey[11]: "Globular cluster, rather open, somewhat irregular." Diameter 12′; 250 stars 12–14^m.

Melotte[12]: "Globular cluster, rather loose, fairly bright stars." Diameter 10′.

Shapley[13] classifies Messier 11 as an open galactic cluster and states: "It seems probable that the cluster proper is composed of not more than 200 stars, nearly all of which are brighter than magnitude 14.5. The diameter of the group seems to be less than 5′."

There is considerable divergence among these observers as to the diameter of the cluster and as to whether it should be classified among the globular or the open galactic clusters.

The lack of concentration toward the galactic plane as well as the large radial velocities of the globular clusters leave little doubt that as a class they are fundamentally different from the open galactic clusters. It is, however, a rather perplexing fact that, if all star clusters are arranged according to their appearance, they seem to form a continuous transition from some of the rather poor and irregular clusterings of the Milky Way to the extremely rich and highly concentrated globular clusters. No marked gap in this series suggests a natural line of division. Messier 11 in *Scutum* and Messier 37 in *Auriga* appear intermediate, and a study of these clusters seems particularly desirable for gaining a better understanding of the characteristic differences between the two classes.

The spectral classes of 59 stars within the cluster Messier 11 and its immediate surroundings were determined with two different instruments. On July 31, 1924, an exposure of 2½ hours was made with the slitless quartz spectrograph attached to the Crossley reflector, kindly put at my disposal by Mr. Wright. This instrument operates like an objective prism and gives the spectra of all stars on one exposure. The dispersion is such as to separate the $H\beta$ and $H\epsilon$ lines by 7.2mm; the spectra extend far into the ultraviolet and are well suited to estimate the intensity of the K line of calcium. This is especially of advantage in our case where most of the stars are of class A. On the other hand there is some disadvantage in the fact that the spectra are not all of the same intensity, the fainter stars being somewhat underexposed. At the densely crowded center of the cluster a great many spectra are necessarily superposed and cannot be used.

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[1] The cluster is sometimes attributed to the neighboring constellation *Antinous*.

[2] *Ann. der kgl. Sternwarte bei München*, **17**, 305.

[3] *Publ. der Hamburger Sternwarte*, No. 1, 1874.

[4] *Publ. de l'Obs. astron. et phys. de Tachkent*, No. 1, 1899.

[5] *Veröff. der Univ. Sternwarte zu Bonn*, No. 18, 1923.

[6] *Contr. Mt. Wilson Obs.* No. 126; *Ap. Jour.* **45**, 164, 1917.

[7] *A Selection of Photographs of Stars, Star-clusters and Nebulae*, vol. 1, 105, 1894.

[8] *Publ. Lick Obs.*, **11**, plates 62–65, 1913.

[9] Dreyer, *Mem. R. A. S.*, **49**, 1888. The description is by J. Herschel from visual observations.

[10] *Ann. de l'Obs. de Paris.* Observations 1891. Visual observations with a refractor of 12.5 inches (305mm).

[11] *H. A.*, **60**, 214, 1908. Description from photograph with 24-inch Bruce telescope.

[12] *Mem. R. A. S.*, **60**, 185, 1915. Description from Franklin Adams Chart plates.

[13] *l. c.*, p. 18.

For these reasons the results of the slitless spectrograph were supplemented by observations with a one-prism spectrograph attached to the 36-inch refractor. In order to reach such faint stars it was necessary to combine this spectrograph with one or the other of two short cameras of 6 inches and 3½ inches focal length. On these spectra the region between $H\beta$ and $H\epsilon$ is 5.6mm resp. 3.4mm long, but despite the small dispersion the principal absorption lines are quite well defined. The region of the K line of calcium is about at the end of the spectrum on these plates and does not show on all of them; this makes the classification of some of the class A stars difficult. Separate spectrograms of 14 stars near the center of the cluster were secured with this instrument with exposure times of 2–5 hours; they have He and H comparison spectra and will also be utilized for radial velocity measures.

TABLE 1

SPECTRAL TYPES OF 59 STARS IN AND AROUND MESSIER 11

Stratonoff	Küstner	Dist. f. center	Photogr. magn.		Color index	Spectral class		Color excess
			Shapley	Küstner		1-pr.	Slitless	
143	46	5′8		12$^{\rm m}$40			B9	
158	60	5.2		12.36			B9	
159	61	6.0		12.47			A0	
172	72	8.0		11.40			B?	
188	85	3.7		12.80			A4	
193	87	3.6		12.73			A3	
197	91	3.4		12.22			A3	
211	105	3.3	12$^{\rm m}$26	12.14	+ $^{\rm m}$64		A1	+ $^{\rm m}$60
221	116	2.8	13.07	12.84	+ .21		B6	+ .37
227	123	2.7	12.06	11.56	+ .75		A1	+ .71
232	128	2.8	12.22	12.00	+ .33		A0	+ .33
235	132	4.5		11.94			A2	
237	134	3.1	11.74	11.51	+ .72		B9	+ .76
245	141	2.3	12.15	12.14	+ .26		B9	+ .30
267	163	2.2	13.26	12.80	+ .48		F0	+ .08
273	170	1.9	13.00	12.66	+ .46		A0	+ .46
294	192	5.2		12.66			B9	
331	227	1.7	12.24	11.80	+1.10	A0		+1.10
336	233	1.4	12.08	11.55	+ .79	A0	A1	+ .77
345	243	1.9	12.06	11.89	+ .74		B9	+ .78
356	254	1.6	12.60	12.32	+ .66		A0	+ .66
375	272	1.1	12.54	12.21	+ .57		A1	+ .53
381	280	1.5	12.44	12.33	+ .63		A0	+ .63
385	283	2′2	12$^{\rm m}$96	12$^{\rm m}$61	+ .78		B5	+ $^{\rm m}$98
395	293	2.2	12.19	11.58	+ .84		B8	+ .92
407	308	1.2	11.93	12.10	+ .91	A2	A0	+ .87
415	317	3.8		12.54			A2	
423	326	0.3	13.46	13.31	+1.63	K2		+ .35
437	336	1.3	11.46	10.36	+ .77	B8	B9	+ .83
452	355	0.2	12.65	12.29	+ .76	A0		+ .76
456	358	1.1	12.30	12.14	+ .86		A0	+ .86
462	366	0.0	12.79	12.40	+ .55	A		+ .45
493	404	0.9	12.55	12.36	+ .63	B9		+ .67
501	414	1.7	11.96	11.60	+ .85	B9		+ .89
516	425	1.4	12.12	11.62	+ .62	B9	A0	+ .64
523	434	1.1	12.25	12.22	+ .72		A2	+ .64
544	461	1.4	11.97	12.18	+ .62	A5		+ .42
545	462	1.0	9.3	8.29	+ .8	B7		+ .9
563	480	1.1	13.12	12.66	+ .51		A4:	+ .35
566	487	5.4		12.14			A4	
568	490	1.3	13.31	12.62	+1.15		A0	+1.15
571	491	1.3	13.16	12.47	+1.00		A0	+1.00
577	499	1.5	12.22	11.89	+ .78	A5		+ .58
581	506	1.3	11.85	11.79	+ .77	B7		+ .89
620	550	3.1	13.33	13.05	+ .27		A2	+ .19
631	562	3.3	12.46	12.26	+ .52		A3	+ .40
647	580	2.4	12.66	12.40	+ .68		A1	+ .64
662	593	3.0	12.47	12.29	+ .47		A0	+ .47
663	594	2.7	11.90	11.98	+ .68		A5:	+ .48
664	595	3.0	12.79	12.47	+ .49		B8	+ .57
686	616	5.0		11.99			A0	
687	615	5.8		9.50			F3	
694	620	5.4		9.85			K0	
698	624	5.6	13.14*	12.83	+1.16*		K	− .04
705	630	4.8		12.21			A3	
710	636	6.4	13.23*	12.98	+ .39*		A2	+ .31
740	665	6.9	12.02*	12.22	+ .06*		B9	+ .10
780		9.0	11.60*		+ .93*		A1	+ .89
816		9.0					B9	

Table 1 gives the spectral class estimates (7th and 8th column) which should be accurate within one or two subdivisions of a class. The first and second columns contain the numbers of the stars in the catalogues of Stratonoff and Küstner, the third column the angular distance from the adopted center, the fourth and fifth columns the photographic magnitudes as determined by Shapley and Küstner, the sixth column, the color-index according to Shapley (the data marked with an asterisk are taken from Table 5 of *Mt. Wilson Contribution* No. 133). The last column shows the excess of the observed color-index over that which normally corresponds to the observed spectral type and which, according to Shapley, is calculated by the formula

Color-index = 0$^{\rm m}$4 (spectral class—A0).

The discrepancy between Shapley's color-indices and the observed spectral classes is well illustrated by the unusually large values of the color excess. Arranging the stars according to Shapley's photographic magnitudes and taking group means as in Table 2 it becomes evident that this discrepancy is largest for the brightest stars and diminishes somewhat for the fainter ones.

TABLE 2

Photogr. Magnitude (Shapley)		Color excess (Obs.-calc.)		Difference of photogr. mag. Shapley-Küstner	
Limits	Mean				
$<11^{m}5$	$10^{m}4$	$+^{m}88$	(2)	$+1^{m}06$	(2)
11.5–12.0	11.84	+.74	(7)	+ .09	(7)
12.0–12.5	12.22	+.61	(16)	+ .22	(27)
12.5–13.0	12.73	+.64	(9)	+ .27	(28)
13.0–13.5	13.23	+.42	(9)	+ .33	(66)
13.5–14.0	13.70			+ .18	(60)
14.0–14.5	14.25			− .07	(81)
14.5–15.0	14.71			− .22	(64)
15.0–15.5	15.17			− .08	(16)

In view of the great care with which Shapley's measures were made one might be tempted to accept this color-excess as real; possibly due to some selective absorption in the dense cloud of the Milky Way in which the cluster is situated. There are, however, a number of factors arguing against the reality of the effect and attributing it rather to a systematic error in Shapley's color determinations:

1. If the color excess were real, the intensity of the continuous spectrum should be considerably weakened in the ultraviolet. The slitless spectrogram is especially suited for this test. A comparison of the spectra of the cluster stars with those of a number of comparison stars of the same type situated in other parts of the sky shows no appreciable difference, and the above supposition must be abandoned.

2. There is some evidence that Shapley's photographic magnitudes are numerically too large, at least for the stars brighter than magnitude 13.5 with which we are here concerned. Such an error in the photographic magnitudes would, of course, also affect the color-indices.

a. Küstner's photographic magnitudes of the stars in Messier 11 are based on the same system and determined by the same method as those of Shapley. A comparison of the results of these two observers is found in the last column of Table 2 and shows the existence of a systematic difference between the two magnitude scales: Shapley observed the brighter stars fainter than Küstner, while for the fainter stars the reverse is the case. The amount of the difference is about half of the color excess, i. e., if color indices were derived by combining Küstner's photographic magnitudes with Shapley's photovisual magnitudes, the discrepancy would be reduced to about half its value. There is further a possibility that even Küstner's magnitudes of the brighter stars are numerically too large as his observations of the six brightest stars ($12^{m}25$–$13^{m}25$) of the North Polar Sequence all have positive residuals (average $+0^{m}17$).

b. The exposure times of the spectrograms with both instruments compared with those of other star clusters indicate that the stars in Messier 11 are at least $0^{m}5$ brighter photographically than Shapley's magnitudes.

3. In *Mt. Wilson Contribution* No. 133 Shapley determined the color indices of four star fields in the neighborhood of Messier 11 by comparing them with the cluster. For the first of these four fields he also made some check observations by Seares's method of exposure ratios, without, however, giving the individual results. The frequency laws of color types obtained by the two methods differ considerably and make it evident that the color indices obtained by exposure ratios for classes a–g are about $0^{m}3$ smaller than those derived by comparison with Messier 11. This amount refers to the average of all stars between magnitudes 12 and 15; it is, however, likely to be larger for the brighter stars and smaller or vanishing for the fainter ones. The results of the exposure ratio method would then be in fair agreement with our spectral class observations.

Shapley himself states that the observations of Messier 11 were difficult on account of its great angular distance from the reference field at the North Pole. Still it would be hard to explain a systematic error of the required amount and it seems highly desirable that the photographic magnitude scale and the color-indices in Messier 11 be redetermined independently.

According to Table 1 the spectral classes B6–A5 predominate near the center, where the stars are most likely to be physical members of the cluster. Only one class K star within 2′ from the center was observed with the spectrograph; the color indices of Shapley, however, show within this circle a distinct group of about 15 red stars with color-indices of $1^{m}3$–$2^{m}0$ and photovisual magnitudes 11^{m}–13^{m} (mostly too faint for the spectrograph); they must represent the class of red giant stars. With regard to spectral types Messier 11 resembles the majority of the open galactic clusters, and this supports the view that Messier 11 belongs to that class.

The spectral classes of the stars give a means of determining the distance of the cluster. Confining ourselves to the region within 2′ from the center and excluding the brightest star we are probably safe in assuming that nearly all these stars are physical members. In Table 3 they are divided according to their spectrum into four groups and for each of these the average spectral class (2d column) and the mean apparent photovisual magnitude (3d column) are given. The last group contains the red giant stars which are selected from Shapley's list; their average color index is 1.64 which should correspond to about class K. Lund-

TABLE 3

Spectral Parallax of Messier 11

Spectral Class Limits	Mean	Photovis. magnitude	No. of stars	Abs. magn. (Lundmark)	Absolute-appar. mgn.	Parallax
B7–B9	B9	$11^{m}27$	6	$+0^{m}7$	$-10^{m}6$	$\overset{''}{.}0008$
A0–A1	A0	11.76	11	+1.2	−10.6	.0008
A2–A5	A4	11.83	5	+2.0	− 9.8	.0011
Red giants	K2	11.64	15	+0.7	−10.9	.0007
				Mean	−10.5	$\overset{''}{.}0008$

mark[14] derived average absolute magnitudes for each spectral class from trigonometric parallaxes, moving cluster parallaxes, spectroscopic and hypothetical parallaxes. The data of the 5th column of Table 3 were interpolated from Lundmark's table for the mean observed spectral class. The sixth column shows the differences between these mean absolute magnitudes and the observed apparent magnitudes of column 3, and from these differences the parallax values of the last column are calculated. The good agreement of the three most reliable values shows that spectral classes and luminosities in Messier 11 conform with the well known Russell diagram of giant and dwarf stars. The mean parallax of the cluster is $0\overset{''}{.}0008$ to which corresponds a distance of 1250 parsecs or 4000 light-years.

TABLE 4

Star Counts in Messier 11 according to Küstner's Catalogue

Ring	Mean Distance from center	Area in .001 square degrees	Photographic Magnitude Limits $11^{m}0$	$12^{m}0$	$13^{m}0$	$14^{m}0$	$15^{m}0$	$15^{m}5$	Total
0′–1′	$0\overset{\prime}{.}7$	0.87	1 (0)	0 (0)	22 (0)	21 (1)	20 (1)	6 (1)	70 (4)
1 –2	1.6	2.62	1 (0)	8 (0)	44 (1)	39 (2)	47 (4)	31 (3)	170 (11)
2 –3	2.5	4.36	0 (0)	3 (1)	17 (2)	25 (3)	43 (7)	37 (5)	135 (18)
3 –4	3.5	6.10	0 (1)	2 (1)	19 (1)	20 (4)	23 (10)	27 (8)	91 (26)
4 –5	4.5	7.85	0 (1)	2 (1)	6 (3)	18 (6)	27 (13)	22 (10)	75 (33)
5 –6	5.5	9.60	2 (1)	1 (1)	17 (4)	6 (7)	19 (16)	18 (12)	63 (41)
>6	7.2	18.35	0 (2)	1 (3)	10 (7)	14 (13)	27 (30)	30 (23)	82 (77)
Total		49.75	4 (5)	17 (8)	135 (18)	153 (36)	206 (82)	171 (61)	686 (210)

The figures of the table give the numbers of stars counted in each region; those in brackets are the calculated numbers of background stars.

The distribution of the stars in Messier 11 was studied on the basis of Küstner's catalogue, which should be complete to the photographic magnitude 15.5 for a square field of $13\overset{\prime}{.}4$ side. Counting Küstner's stars for each magnitude interval in six rings each 1′ wide with the star Stratonoff 462 in the center, the data of Table 4 were obtained. The last region comprises the remainder of the field outside of 6′. The first column of Table 4 designates the ring, the second and third columns give the mean distance from the center and the area of the ring in 0.001 square degrees. The table figures, of course, include a number of stars which are not physical members but which, situated before or behind the cluster, appear projected on this region of the sky. We may call them the background stars. In order to estimate their number it was assumed that for our magnitude limits the cluster did not extend much beyond ring 6 and that the last region practically contains only background stars. The star numbers of this region were then compared with the average star numbers of the galactic latitude of the cluster according to Van Rhijn's tables[15] and they were found to be 3.7 times larger than the table values. It thus appears that the star cloud in *Scutum Sobieski*, in which the cluster is situated, has a star density nearly four times larger than the galactic belt on the average. This seems quite a reasonable figure and the numbers of background stars which Table 4 shows in brackets are accordingly calculated from the Groningen tables by multiplication with 3.7. This empirical factor also takes account of any small systematic difference that might exist between Küstner's magnitudes and those on which the Groningen tables are based.

The cluster, then, is made up of about 480 stars brighter than magnitude 15.5 and an idea of their distribution and concentration toward the center may be gained from Table 5 and its graphic representation in Fig. 1. Deducting the background stars from our star counts and dividing each figure by the area of the ring we obtain the surface density (number of stars per 0.001 square degree) in the projection of the cluster on the celestial sphere. In Table 5 and Fig. 1 the surface density of the cluster members is represented as a function of the apparent angular distance from the center, separately, for the stars brighter than photographic magnitude 14.0, for stars $14^{m}0$–$15^{m}5$, and

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[14] *Publ. A. S. P.*, **34**, 150, 1922.

[15] *Gron. Publ.*, No. 27, 62, 1917.

TABLE 5

SURFACE DENSITY LAW OF CLUSTER STARS IN MESSIER 11

Apparent angular Distance from Center	Stars brighter than $14^{m}0$	Stars $14^{m}0$–$15^{m}5$	All Stars
0′.7	49.2	27.0	76.2
1.6	33.7	26.9	60.5
2.5	11.2	15.4	26.6
3.5	5.3	5.3	10.6
4.5	1.9	3.3	5.2
5.5	1.3	1.0	2.3
7.2	0.0	0.2	0.2
Background stars	1.4	2.9	4.3

The table figures are the numbers of cluster members per 0.001 square degree.

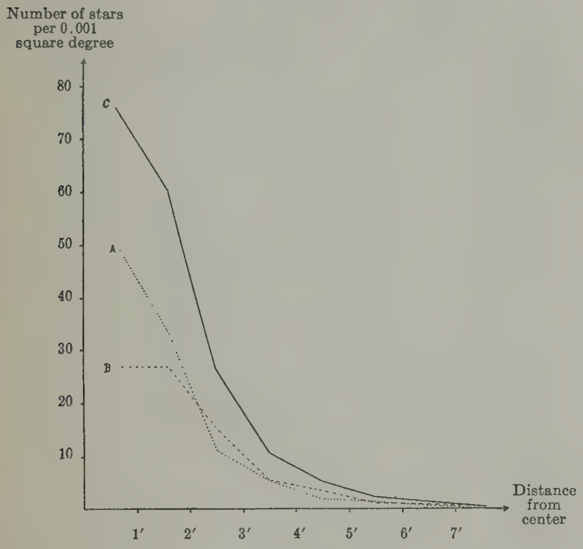

FIG. 1

SURFACE DENSITY LAW OF MESSIER 11

The star density per 0.001 square degree corrected for background stars is plotted as a function of the apparent angular distance from the center of the cluster:

A: for the stars brighter than $14^{m}0$ photogr. mg.
B: for stars $14^{m}0$–$15^{m}5$
C: for all stars.

for all stars together. It is quite evident that the brighter stars are more concentrated toward the center than the fainter ones. This same property was found to prevail in several other open clusters[16] and is probably a general characteristic of star clusters. V. Zeipel and Lindgren's investigations seem to throw some light on this phenomenon. In analogy with the kinetic theory of gases the dynamical effect of mutual approaches of the group members would be to produce a distribution of the stars and of their energy according to Maxwell's Law. The concentration of the stars toward the center then depends on their mass, the more massive stars having a higher concentration. On the other hand the mass of a star seems to be related to its spectral type and luminosity, the larger masses being found among the types of high luminosity. In Messier 11, for instance, the stars brighter than magnitude 14.0 are mostly of spectral class A; ranging probably between B8 and F2 (dwarf) and including a small number of red giant stars. The stars $14^{m}0$–$15^{m}5$, according to the known luminosity spectral type relation, should be dwarfs of classes F2 to GO. Our present information on stellar masses indicates that the latter stars are of smaller mass and the above theory then requires that they be less concentrated toward the center which is in agreement with the observations.

So far we have studied only the star distribution in the projection of the cluster on the celestial sphere, and we proceed to determine the star distribution in space: the volume density law. Assuming our cluster to be of globular symmetry, the problem is to find the volume density (number of stars per unit volume) in successive spherical shells around the center and to represent this volume density as a function of the true linear distance from the center. A general solution of this mathematical problem was given by v. Zeipel.[18] It is, however, possible also to derive the volume density law from the star counts by a purely numerical process of successive approximation. By this method, which will be treated in a later paper, the data of Table 6, referring to all stars brighter than photographic magnitude 15.5, were obtained. The results were checked by independent calculation according to v. Zeipel's method. For a parallax of 0″.0008, as derived above for Messier 11, an angular distance of 1′ in the cluster corresponds to 0.364 parsecs. By means of this value the distances from the center were expressed in parsecs and the star densities referred to the cubic-parsec as unit volume.

TABLE 6

VOLUME DENSITY LAW

Distance from center in parsecs	Stars per cubicparsecs
0.27	83
0.60	80
0.96	33
1.32	9.5
1.68	4.9
2.04	2.2
2.40	0.5

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[16] See R. Trumpler. Comparison and classification of star clusters. *Publ. All. Obs.*, **6**, 45, 1922.

[17] V. Zeipel und Lindgren, Photometrische Untersuchungen der Sterngruppe Messier 37: *Kungl. Svenska Vetenskaps-akademiens Handl.*, **61**, No. 15, 1921.

[18] *Ann. de l'Obs. de Paris*, **25**, 1908; *Kungl. Svenska Vetenskaps-akad. Handl.*, **61**, No. 15, 1921.

The central part of Messier 11 has a very high star density, especially if we keep in mind that only stars more luminous than absolute magnitude 4.5 (Sun = 5.0) are included, while the cluster may in addition contain a considerable number of fainter stars. More suggestive than the figures of Table 6 are perhaps the comparisons given in Table 7, in all of which only stars up to the same absolute magnitude limit were considered.

TABLE 7

Stars Brighter Than Absolute Magnitude +4.5 per Cubicparsec

Cluster	Parallax	No.
At center of Messier 11	($\pi = ''.0008$)	83
At center of Messier 37	($\pi = ''.0007$)	18
At center of Pleiades	($\pi = ''.008$)	2.8
Vicinity of Sun (within radius of 5 parsecs)		0.011

For Messier 37 the star density at the center is taken from v. Zeipel and Lindgren. For the *Pleiades* it was calculated from the data of *L. O. Bull.* **10**, 110, by omitting all stars fainter than photographic magnitude 10.0. Up to this limit our knowledge of the physical members of the *Pleiades* group should be complete. According to Eddington[19] there are 6 stars of the same luminosity as the Sun or brighter within a sphere of 5 parsecs radius around the Sun as center; this furnishes the last line of Table 7. The first three data are perhaps somewhat uncertain as they are greatly influenced by the adopted parallax values. These parallaxes which are all derived from the colors or spectral types of the stars can, however, hardly be so much in error as to double or halve the star densities.

In the central part of Messier 11 the stars more luminous than the Sun are, on the average, separated from their nearest neighbors by ⅓ of a parsec (this is equal to 1 light-year or to 70,000 astronomical units) and they are more than 7000 times more densely crowded than those in the vicinity of our Sun. An observer at the center of Messier 11 would find about forty stars with parallaxes of 2″ or more and which would appear about 3 to 50 times as brilliant as *Sirius* shines in our sky.

Even among galactic star clusters Messier 11 excels by great star density. From our three examples the conclusion might be drawn that the star density in open clusters is related to the star density in the vicinity of the clusters which is undoubtedly largest for Messier 11 and smallest for the *Pleiades.* A crowded star field surrounding the cluster may prevent the escape of cluster stars and facilitate the capture of neighboring stars by the cluster.

In a star cluster all members are practically at the same distance from us; their absolute magnitudes or luminosities are therefore obtained by deducting a constant amount from the apparent magnitudes. This constant, depending on the distance of the cluster, was found to be 10^m5 (Table 3) for Messier 11, corresponding to a parallax of $0''.0008$. The distribution of the cluster stars among the different degrees of luminosity, which is called the luminosity law, is illustrated by Table 8 and Fig. 2. The second column of Table 8 shows the mean apparent photographic magnitude of each magnitude interval and column 3 the corresponding visual absolute magnitude. The mean spectral class of the fourth column is taken from our observations in case of the first two lines; it is extrapolated for the others from the known relation between spectral class and absolute magnitude.[20] The data of the third and fourth columns are calculated with the omission of red giant stars. The fifth column gives the number of cluster members for each photographic magnitude interval (the number for the last line was doubled so as to refer it to a full magnitude interval), and the

TABLE 8

Luminosity Law of Cluster Stars in Messier 11

Magnitude Interval	Mean Photographic Magnitude	Visual Absolute Magnitude	Spectral Class	Number of Cluster Stars	Red Giant Stars
11^m0–12^m0	11^m5	$+1^m0$	B9	9	
12.0–13.0	12.5	+1.9	A3	117	(4)
13.0–14.0	13.5	+2.8	(F1)	117	(11)
14.0–15.0	14.5	+3.6	(F5)	124	
15.0–15.5	15.25	+4.3	(F8)	(218)	

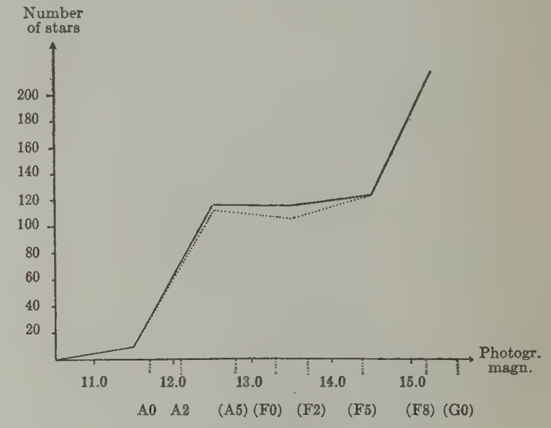

Fig. 2

Luminosity Law of Messier 11

The number of cluster stars in one magnitude interval is plotted as a function of the mean photographic magnitude of the interval. The dotted line gives the luminosity law when the red giant stars are excluded. The spectral classes corresponding to the magnitudes are indicated on the scale of abscissae; those in brackets are extrapolated.

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[19] *Stellar Movements*, p. 41.

[20] According to Lundmark: *Publ. A. S. P.*, **34**, 150, 1922.

last column indicates how many of the stars of the preceding column are probably red giant stars. In Fig. 2 the luminosity law is plotted for photographic magnitudes; if visual magnitudes had been used instead, the scale of ordinates would have been increased about 10% while the red giant stars would have been shifted toward the left. With such a change the luminosity law of Messier 11 would still less resemble a Gaussian error curve, which law had been derived by Kapteyn for the stellar system on the average. The irregularities in the luminosity law of Messier 11 are better understood if we exclude the red giant stars (dotted curve) and take account of the close relationship between spectral class and luminosity. We can then refer our luminosity curve to spectral classes as abscissae and obtain the frequency law of spectral classes. It is noticed that a frequency maximum is reached for class A stars; through class F there is a slight reduction in the frequency with a sharp rise toward class G.[21] The minimum around 14th magnitude evidently marks the division between the stars of the first and second type of Secchi. A quite similar effect was found in the *Pleiades*[22] and it looks as if in star clusters the luminosity law was of rather secondary importance, reflecting simply the frequency of spectral classes to which the luminosities are related. The analogy with the *Pleiades* and other star clusters as well as the last branch of our luminosity law suggests that Messier 11 contains a considerable number of stars fainter than magnitude $15^{m}.5$. These fainter members, more widely scattered and very little concentrated toward the center, may easily be as numerous as those included in our counts, because they fill a larger volume with more equal density. They are, however, not conspicuous to the observer and in no case is their number comparable with the star numbers in truly globular clusters, which may rise to 100,000 or more.

The peculiar distribution of the stars probably accounts for the discrepancy in the estimates of the diameter of the cluster by different observers. In good agreement with Bigourdan's description, the cluster appears as having a highly concentrated central part of 5′–6′ diameter containing a large proportion of the brighter members. This central part is surrounded by a region in which the cluster members are mostly faint and less numerous than the background stars. It is clear that this outer region is not conspicuous to the observer and can be noticed only on long exposures with a large field or by star counts. The density law of Table 5 and Fig. 1 indicates that this outer region extends at least to 6′ or 7′ from the center. Careful examination of some long exposure photographs with the Crossley reflector leads to the conclusion that if stars up to 18th or 19th magnitude are included the cluster may extend even as far as 8′ or 9′ from the center; a similar result is also obtained from Barnard's Milky Way photographs which include the region of Messier 11. The difference between Shapley's description of Messier 11 and ours is well explained by the fact that Shapley confines himself to the central part.

According to our study Messier 11 is a group of about 500 stars brighter than absolute magnitude $4^{m}.5$ (Sun = $5^{m}.0$) and may contain a considerable number of fainter stars. The brighter members are mostly of spectral class A, but also include about 15 red giant stars. The fainter stars must be dwarf stars and of smaller mass as they are not so highly concentrated toward the center as the bright ones. The cluster is situated at a distance of about 1250 parsecs (4000 light years, parallax $0''.0008$). Its diameter is about 15′ corresponding to 5.5 parsecs (18 light years) and may be as much as 18′ if fainter stars are included.

Of the stars more luminous than the Sun, our cluster contains at the center more than 7000 times as many as are found in the same volume in the immediate vicinity of the Sun. Messier 11 undoubtedly belongs to the richer class of open galactic clusters; this view is supported not only by its situation in the Milky Way, but also by its constitution: its diameter, the number and spectral types of its members, its luminosity law, etc.

Finally, attention should be drawn to the close resemblance between Messier 11 and Messier 37 studied by v. Zeipel and Lindgren. Both clusters seem to have about an equal number of members and the spectral classes of the stars are about the same. Messier 37, however, has nearly twice as large a diameter as Messier 11 and, accordingly, the stars are much less densely crowded.

February, 1925.

Issued March 10, 1925.

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[21] This last branch of the curve is perhaps somewhat uncertain on account of possible errors in the magnitude limits, but it is out of the question that such errors could entirely account for the sharp rise.

[22] See *L. O. Bull.*, **10**, 115, 1921.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 362

ON THE NUMBER OF SOLUTIONS IN LEUSCHNER'S DIRECT METHOD FOR DETERMINING THE ORBITS OF DISTURBED BODIES

By

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INTRODUCTION

In his "Direct Solution of the Orbits of Disturbed Bodies,"[1] Professor A. O. Leuschner has given an analytical method for determining the osculating orbit of a heavenly body of infinitesimal mass which moves under the attraction of two finite bodies. The efficiency of the method consists in rigorously taking into account all of the attracting forces affecting the infinitesimal body. Furthermore, the method yields the most general solution possible, i. e., one free from any assumptions as to the nature of the disturbed body, as is desired in the case of a newly discovered body, in apparent proximity to one of the major planets.

The six elements of the orbit are derived from the heliocentric or planetocentric coördinates and velocities of the infinitesimal mass at a given time. The solution is therefore obtained in exactly the same manner as in the ordinary Short Method,[2] i. e., the three original differential equations of motion are transformed into three algebraic equations involving five unknowns, namely the geocentric distance u, its first two derivatives with respect to the time, u' and u'', and the distances from the infinitesimal mass to the attracting bodies v and w. These three algebraic equations being linear in u, u', u'', can be solved with respect to these three unknowns as functions of the other two. Since on the other hand the last two unknowns v, w can be eliminated by geometrical relations between the distances, and since u' is expressible as a linear function in u, the problem evidently can be reduced to the determination of the roots of an algebraic equation containing the geocentric distance as the only unknown quantity. In the case of the "*problème restreint*" this equation is of the 28th degree, but reduces to the 8th (with one root equal to zero) in the ordinary Keplerian problem.

The geocentric distance u is given by an equation of the form

$$u = \frac{A}{[(u-ap)^2+(as)^2]^{3/2}} + \frac{B}{[(u-q)^2+t^2]^{3/2}} + C$$

which, when cleared of fractions and rationalized, is of the 28th degree in u. The preceding method has been successfully applied to the determination of the orbits of the seventh, eighth and ninth satellites of *Jupiter*. In the case of the ninth satellite, this equation has been solved by Dr. S. B. Nicholson by a graphical method proposed by Professor Leuschner,[3] namely by ascertaining the intersections of the curve

$$y = \frac{A/B}{[(u-ap)^2+(as)^2]^{3/2}} + \frac{1}{[(u-q)^2+t^2]^{3/2}}$$

with the straight line

$$y = u/B - C/B\ .$$

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[1] *Publ. Lick Obs.*, **7**, 459 *et seq.*, 1913.

[2] *ibid.*, 1 *et seq.*

[3] *L. O. Bull.*, **8**, 100, 1914.

The equation has five real roots, one negative, one very nearly equal to zero (corresponding to the Earth's solution) and three positive. The first two roots have no physical interpretation, the next two give two satellite orbits around *Jupiter* with the Sun as the disturbing body, whereas the third positive root corresponds to an orbit around the Sun with *Jupiter* as the disturbing mass.

The roots can be obtained analytically only by successive approximations. If two assumptions as to the nature of the infinitesimal body are made, i. e., if the action of the disturbing mass be neglected (as in the case of an asteroid or a comet) or is approximately evaluated (as in the case of a satellite) the degree of the general equation will be reduced respectively to the 7th or to the 8th.

Evidently the coefficients of these equations will be functions of the roots, since these coefficients are functions of the distance of the disturbing mass. Hence the solution involves two approximations, one for ascertaining the roots of the equation whose coefficients are only approximate, the other for determining their more exact values, as soon as the distances become known. This method of treating the equation has been employed by Professor R. T. Crawford and Professor W. F. Meyer in the determination of the orbits of *J VII* and *J VIII*.[4] They solved the equation with the Jovicentric distance as the unknown and obtained by successive approximations two of the roots of the original equation. But since in both cases, the equation has three real positive roots (it must necessarily have one or three) and since only two are given by this method, a complete analytical investigation of the equation seemed desirable. The present paper is a discussion of the limits and the number of real positive roots of this equation; by an inspection of these limits, the roots having no physical interpretation may be rejected and a process of approximation to the values of the coefficients may be derived.

I.—RELATION OF JACOBI'S INTEGRAL TO THE NUMERICAL ORBIT PROBLEM.

The direct determination by Professor Leuschner's method of the orbit of a heavenly body moving under the attraction of two masses leads to the following system of equations:

$$\left.\begin{aligned} \rho\cos\delta &= \frac{(\kappa)}{r^3}+\gamma\kappa\left[\frac{1}{[r]^3}-\frac{1}{R^3}\right]-\frac{[\kappa]}{(r)^3} \\ r^2 &= \rho^2+(\rho)^2-2\rho(\rho)\cos\psi \\ [r]^2 &= R^2+\rho^2-2R\rho\cos[\psi] \end{aligned}\right\}, \qquad (1)$$

where (κ), κ, and $[\kappa]$ are the ratios of certain determinants derived from the observational data; ρ, (ρ), and R the respective geocentric distances of the asteroid or satellite, planet, and Sun; r and $[r]$ the distances from the infinitesimal mass to the central and disturbing bodies respectively; ψ the angle between the lines joining the observer and the planet and the observer and the satellite; $[\psi]$ the angle between the line of sight to the object and the line connecting the observer and the Sun; γ the ratio of the masses of the two attracting bodies.[5] This system of equations will obviously give the most general solution of the problem, i. e., when no assumption is made regarding the nature of the infinitesimal mass—satellite or minor planet (or comet). On account of the smallness of the masses of the planets in comparison with that of the Sun, it is evident that in both cases one of the angles ψ will vary between certain limits, while the other, $[\psi]$ may have any value from 0 to 180°. In fact, the infinitesimal mass can move only in certain portions of space which are limited by the surface of zero relative velocity

$$2\Omega-C=0,$$

$$\text{where } \Omega=\frac{1}{2}(x^2+y^2)+\frac{1-\mu}{r}+\frac{\mu}{\rho}$$

and C is the Jacobian constant.[6] The shape of this surface varies with the value of the parameter C. For large values of C, the surface is made up of three separate folds, two being closed and surrounding the two attracting masses. The third fold is asymptotic to a certain cylinder whose equation is $x^2+y^2=C$. When C diminishes and reaches a certain value C_1, the first two folds coalesce, the point of contact being the first libration center L_1. For $C>C_1$, the small mass may be (1) a superior planet moving outside of the third fold, (2) an inferior planet moving within the first fold around the Sun, or (3) a satellite. The case $C=C_1$ gives superior limits for the *radii vectores* of stable inferior planets and of satellites which cannot sever their connections with their primaries. If $C=C_2$, we obtain the second libration center L_2; for $C_2<C<C_1$, the small mass may be a superior or inferior planet, or a satellite, or a body moving in an orbit which partakes of the characteristics of the last two.

For all satellites in the solar system (except *J VII* and *J VIII*) the constant C is larger than the critical value C_1, so that they are permanently attached to their primaries. The size of this limiting curve $C_1=2\Omega$ is given for *Jupiter, Saturn, Uranus* and *Neptune* in the following table:

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[4] *Publ. Lick Obs.*, **7**, 490 *et seq.*, 1913.

[5] *Publ. Lick Obs.*, **7**, 471, 1913.

[6] F. R. Moulton, *Introduction to Celestial Mechanics*, Rev. Ed., 280, 1914.

Table I.—Values of the Constants for Jupiter, Saturn, Uranus and Neptune.

Argument	Jupiter	Saturn	Uranus	Neptune	
Mass	1/1047.355	1/3501.6	1/22869	1/19314	
C_1 (Darwin)	3.042616	3.018959	3.005400	3.006045	
C_1' (Moulton)	3.039714	3.018097	3.005269	3.005890	
C_2 (Darwin)	3.041343	3.018578	3.005341	3.005977	
C_2' (Moulton)	3.038443	3.017716	3.005210	3.005821	
L_1	−0.0667	−0.0450	−0.0242	−0.0256	
L_2	+0.0698	+0.0463	+0.0246	+0.0261	
x_1'	+0.0581	+0.0400	+0.0221	+0.0234	
y_1	±0.0448	±0.0301	±0.0162	±0.0171	
z_1	±0.0430	±0.0289	±0.0155	±0.0164	
x_1	−0.3469	−0.4295	−0.4656	−0.7713	a. u.
x_1'	+0.3025	+0.3820	+0.4251	+0.7031	
y_1	±0.2332	±0.2879	±0.3112	±0.5158	
z_1	±0.2235	±0.2757	±0.2980	±0.4937	
ψ_1	4°.75	2°.88	1°.46	1°.52	
ρ	0.322	0.363	0.339	0.576	a. u.
ψ_2	4°.40	2°.43	1°.07	1°.13	

where C_i is the constant of Jacobi's integral; L_1, L_2 are the libration centers; x_1, x'_1, y_1, z_1, the intersections of the surface $2\Omega - C_1 = 0$ with three rectangular axes passing through the center of mass of the planet, and ρ, the radius of the "sphere of action".

The coördinates x_1, x'_1, y_1, z_1 have been computed from the equation

$$x^2+y^2+\frac{2(1-\mu)}{r}+\frac{2\mu}{\rho}=C_1 \quad ,$$

where

$$r^2=(x-X_1)^2+y^2+z^2,$$
$$\rho^2=(x-X_2)^2+y^2+z^2.$$

The y_1 and z_1 distances are obtained by solving the foregoing equation and after putting

$$\left.\begin{matrix} x=X_2, & z=0 \\ x=X_2, & y=0 \end{matrix}\right\} \text{ respectively.}$$

For the computation of x_1 and x'_1 the following transformed equation will be employed:

$$(1-\mu)\ (r^2+2/r)+\mu\ (\rho^2+2/\rho)=C_1+\mu\ (1-\mu) \quad ,$$

which form is equivalent to Darwin's, namely:

$$(r^2+2/r)+\nu\ (\rho^2+2/\rho)=C_1 \quad .$$

It will then be sufficient to put $r=1\mp\rho$ and to solve the two quartic equations

$$\rho^4\,(1+\nu)-\rho^3\,(3+\nu)+\rho^2\,(3-C_1)-\rho\,(3-2\nu-C_1)-2\nu=0,$$
$$\rho^4\,(1+\nu)+\rho^3\,(3+\nu)+\rho^2\,(3-C_1)+\rho\,(3+2\nu-C_1)+2\nu=0.$$

The units used for the computation of these quantities are

$$\left.\begin{matrix} \text{Distance from Sun to Planet} \ldots\ldots\ldots\ldots = 1 \\ \text{Gravitational Constant} \ldots\ldots\ldots\ldots k^2 = 1 \\ \text{Mass of the Sun+mass of Planet} \ldots = 1 \end{matrix}\right\} \quad ,$$

and

$$\left.\begin{matrix} \text{Distance from Sun to Planet} \ldots\ldots\ldots\ldots = 1 \\ \text{Gravitational Constant} \ldots\ldots\ldots\ldots k^2 = 1 \\ \text{Mass of the Sun} \ldots\ldots\ldots\ldots\ldots\ldots = 1 \end{matrix}\right\} \quad ,$$

according as Moulton's or Darwin's formula has been used.

From the previous analysis, it follows that for all stable satellites the values of the *radii vectores* are less than a certain limit and that the corresponding angles ψ_1, have an upper limit which has been given in Table I. *J VIII* and *J IX*, whose constants are $C'=3.02090$ and $C'=3.0165$ (in Moulton's units) are at present within the limiting surface $2\Omega - C_1' = 0$. Jacobi's integral is not a sufficient criterion for determining the stability of these satellites; it shows merely that they cannot become minor planets with the same mean motion as that of *Jupiter*. (Fig. 1.)

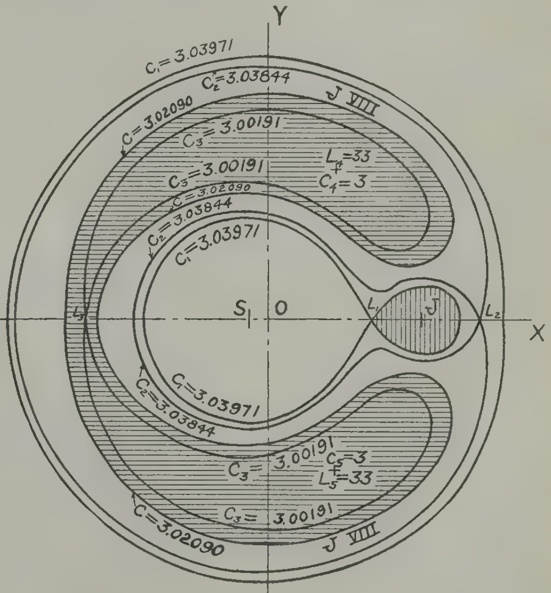

Fig. 1.—Intersections of the surfaces $2\Omega - C = 0$ with the plane of the orbit of *Jupiter* (the XY-plane).

The curve $C=3.02090$ is the curve for *J VIII*, and shows that the satellite is excluded from the region shaded horizontally. The curve for *J IX* will lie within the former and will be similarly shaped to that for *J VIII*. At present both *J VIII* and *J IX* are contained within the region bounded by the surface, whose cross-section with the XY-plane is shaded vertically. This figure which illustrates only qualitatively the character of the curves, was drawn by G. H. Darwin and corresponds to a ratio of the masses 1 : 10, (*Scientific Papers*, **4**, 12). For an accurate drawing of the limiting curves, see E. W. Brown's paper "On a New Family of Periodic Orbits in the Problem of Three Bodies," *Mon. Not. R. A. S.*, **71**, 444, plates 6 and 7. The constant C has been computed by the formula[7]

$$C+\mu(1-\mu)=(1-\mu)\left(r^2+\frac{2}{r}\right)+\mu\left(\rho^2+\frac{2}{\rho}\right)-V^2\ ,$$

in which the mean values of r, ρ, and V are to be substituted, giving

$$C+\mu(1-\mu)=(1-\mu)(1+2)+\mu\left(a^2+\frac{2}{a}\right)-a^2(1-n)^2\ .$$

Since, however, *J VIII* and *J IX* have made several revolutions around *Jupiter*, and since Jacobi's integral fails to give an upper limit for the *radii vectores*, the question arises whether the stability of these two retrograde orbits could be established by some other criterion. F. R. Moulton in a paper entitled "On the Stability of Direct and Retrograde Satellite Orbits" (*Monthly Notices, R. A. S.*, **75**, 40, 1914) fully investigates the case of *J VIII* and concludes that a retrograde periodic orbit having the period of *J VIII* is very stable, while he finds that a direct orbit having the same period as *J VIII* would be unstable, with "even stability" (G. H. Darwin's terminology).

Professor C. V. L. Charlier, in his paper "*Uber die Bewegung der Satelliten in unserem Planetensystem*" (*Meddelanden fran Lunds Astronomiska Observatorium*, Serie II, **5**, 27 *et seq.*, 1909) has also discussed the case of *J VIII* from a cosmogonic point of view. He first makes use of Jacobi's integral and finds for the value of the constant, $C=2.529$ (in Hill's units), showing that the corresponding limiting curve is not closed, and he is therefore led to the two following hypotheses: *J VIII* is either a captured body, in which case the initial rotation of *Jupiter* may be assumed to be direct, as shown by the motions of *J VI* and *J VII*,—or *J VIII* belongs from the beginning to the *Jupiter* system. In the latter case, Professor Charlier proposes a theory which will account for the actual configuration of the *Jupiter* system and for the differences in the motions of the satellites.

From a slightly different point of view, it will be advantageous in the computation of the perturbations of the small body to know which one of the attracting masses is the central one and which the disturbing mass. This discrimination can be effected by considering the points in space for which the ratios of the forces of attraction to the disturbing forces are equal.[8] In this way, a surface of revolution about the line joining the two attracting masses is obtained, the approximate equation of which in polar coördinates will be

$$\frac{\rho}{r'}=\left[\frac{m'^2}{(1+3\ \cos^2\omega)}\right]^{\frac{1}{5}}\ ,$$

where ρ is the distance from the planet to the object, r' that between the Sun and the planet, and m' the ratio of the mass of the planet to that of the Sun. This surface does not differ greatly from a sphere[9]

whose radius[10] is $\frac{\rho}{r'}=\frac{m'^{\frac{2}{5}}}{1.063}$.

The following enumeration gives the radii of the various spheres for the eight major planets:

Mercury	0.001 A. U.	Jupiter	0.322 A. U.
Venus	0.004	Saturn	0.363
Earth	0.006	Uranus	0.339
Mars	0.004	Neptune	0.576

The corresponding values of ψ_2, for time of opposition are given in Table I. A comparison between the limiting surface $2\Omega-C_1=0$ and the "sphere of action" shows that for all planets this surface lies within the "sphere of action," except the portion near the X axis.

When a complete solution involving the case where the body might be a satellite, an asteroid, or a comet, is encountered, it follows from the preceding considerations that the angle ψ is of the order of magnitude indicated in Table I, or is smaller; hence the parameter $p=\cos\psi$ is always nearly equal to $+1$.

The original equations may then be transformed as follows:

$$\text{Let}\quad\left.\begin{array}{lll}\lambda=-\dfrac{(\kappa)}{R^4\ \cos\ \delta} & \dfrac{\rho}{R}=u & \dfrac{(r)}{R}=b\\[2ex] \mu=-\dfrac{\gamma\kappa}{R^4\ \cos\ \delta} & \dfrac{r}{R}=v & \dfrac{(\rho)}{R}=a\\[2ex] \nu=-\dfrac{[\kappa]}{R^4\ \cos\ \delta} & \dfrac{[r]}{R}=w & \lambda\beta=\mu+\dfrac{\nu}{b^3}\\[2ex] p=\cos\psi & q=\cos\,[\psi] & \lambda=\dfrac{\mu}{\gamma}+\nu\\ s=\sin\psi & t=\sin\,[\psi] & \end{array}\right\}, (2)$$

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[7] J. Jackson, Retrograde Satellite Orbits, *Mon. Not., R. A. S.*, **74**, 77, 1913.

[8] F. Tisserand, *Mécanique Céleste*, **4**, 198.

[9] Laplace's "*sphère d'activité.*"

[10] G. H. Darwin, *Scientific Papers*, **4**, 436.

then we shall have

$$\left.\begin{aligned} u&=\left(\frac{\nu}{b^3}-\frac{\lambda}{v^3}\right)+\mu\left(1-\frac{1}{w^3}\right)\\ v^2&=(u-ap)^2+(as)^2\\ w^2&=(u-q)^2+t^2 \end{aligned}\right\}. \qquad (3)$$

The general solution of this system is obtained by the elimination of v and w, which elimination leads to a complete 28th degree equation in u, viz.,

$$[(u-\lambda\beta)^2\, v^6w^6-\lambda^2w^6-\mu^2v^6]^2-4\lambda^2\mu^2v^6w^6=0\ .$$

If, however, an assumption be made as to the nature of the body, the system of equations (3) can be solved by successive approximations involving only an eighth degree equation, which will be investigated hereafter.

II.—SATELLITE CASE.

In the case where the infinitesimal mass is assumed to be a satellite, the Sun becomes the disturbing body and the original system of equations can be solved by successive approximations, the solution involving an equation of the 8th degree, whose coefficients are variable since they are functions of the unknown distances v and w.

In the first approximation if $w=b$, the three fundamental equations reduce to the following form:

$$\left.\begin{aligned} u&=\lambda\beta\left(1-\frac{1}{\beta v^3}\right)\\ v^2&=(u-ap)^2+(as)^2\\ w^2&=(u-q)^2+t^2 \end{aligned}\right\}\ ,$$

$$\text{where}\quad \left.\begin{aligned} p&=\cos\psi,\ \text{always positive}\\ s&=\sin\psi,\ \text{a small quantity}\\ q&=\cos\,[\psi]\\ t&=\sin\,[\psi]\\ \lambda\beta&=\mu\left(1-\frac{1}{b^3}\right)+\frac{\nu}{b^3} \end{aligned}\right\}\ .$$

The elimination of either one of the unknowns, u, or v, leads to an equation of the 8th degree, namely:

$$\left.\begin{aligned} f(v)&=v^8-[(\lambda\beta-ap)^2+(as)^2]\,v^6+2\lambda(\lambda\beta-ap)\,v^3-\lambda^2=0\\ f(v)&=v^8-A^2v^6+2Bv^3-\lambda^2=0 \end{aligned}\right\}\ ,$$

or when v is eliminated,

$$f(u)=[(u-ap)^2+(as)^2]^3\,(u-\lambda\beta)^2-\lambda^2=0\ .$$

It is evident that in this case, it will be advantageous to discuss the equation $f(v)=0$, in finding first the number of its real positive roots, and secondly the number of these roots to which a solution of the problem will correspond.

The equation $f(v)=v^8-A^2v^6+2Bv^3-\lambda^2=0$ has either one or three real positive roots, which will now be investigated.

1st Case, $B<0$: there is only one positive root.

2nd Case, $B>0$: there are either one or three real positive roots (by Descartes' rule). Hermite's[11] and Sturm's[12] theorems give the exact number of real roots between two given numbers a and b. Although theoretically perfect, they lead in the present problem to a discussion of very complicated rational functions of the coefficients, and it is therefore not feasible to apply them. Cauchy's[13] general method for the determination of the number and the nature of the real roots of an algebraic equation of the nth degree involves the discussion of (n–4) rational functions of the coefficients of the given equation. Some of these functions (when $n \geqq 5$) result from the elimination of an auxiliary unknown, and this process is in general very laborious, except in a few simple cases.

Given an equation $f_n(x)=0$, of the nth degree in x, if its first derivative $f'(x)=n\,f_{n-1}(x)=0$ can be solved, then Rolle's theorem will furnish the number of real roots of $f_n(x)=0$, and the intervals within which these roots lie: for, let x_i and x_{i+1} be two numerically consecutive roots of $f_{n-1}(x)=0$, if $f_n(x_i)f_n(x_{i+1})<0$, then there will be one root, X_{i+1}, of $f_n(x)=0$, and only one, between x_i and x_{i+1}, so that $x_i<X_{i+1}<x_{i+1}$. Now Rolle's theorem cannot be applied directly to the equation $f(v)=0$, but it will be noted that this last equation has the same number of real roots as the transformed equation $F(1/v)=F(z)=0$, whose roots are the reciprocals of the roots of $f(v)=0$. The roots of the derivative equation $F'(z)=2z(4\lambda^2z^6-5Bz^3+A^2)=0$ can be readily determined, since the second factor is a quadratic in z^3.

If $25B^2-16\lambda^2A^2<0$, or $\frac{3}{4}<|\eta|$, where $\eta^2=\dfrac{(as)^2}{(\lambda\beta-ap)^2}$, then $F'(z)=0$ has only one real root, and consequently $F(z)=0$ has only one real positive root.

If $25B^2-16\lambda^2A^2>0$, or $\frac{3}{4}>|\eta|$, then $F'(z)=0$ has three real roots, namely $z'_1=0$, and two positive ones, z'_2 and z'_3. The necessary and sufficient condition that $F(z)=0$ shall have three real positive roots, z_1, z_2, z_3, is that

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[11] Hermite's method is based on the properties of homogeneous and integral functions of the second degree in n variables. These functions can always be expressed as the sum of the squares of m linear functions ($m \geqq n$).

J. A. Serret, *Algèbre Supérieure*, 1, 585, Sec. 259, 4th Ed.

[12] Sturm's theorem can be considered simply as a corollary of Hermite's theorem.

J. A. Serret, *ibid.*, 92 *et* 593.

[13] A. Cauchy, *Oeuvres Complètes*, *Série* II, 1, 170, *Mémoire sur la détermination du nombre des racines réelles dans les équations algébriques.*

$$D=F(z'_2)F(z'_3)<0 \quad ;$$

for, if $F(z)=0$ has three real positive roots, z_1, z_2, z_3, the first derivative $F'(z)=0$ must have three real roots, namely $z'_1=0$, and two positive, z'_2, z'_3, such that

$$0<z_1<z'_2<z_2<z'_3<z_3 \quad ;$$

hence $D<0$. Conversely, let us suppose that $D<0$ is satisfied, then $F(z)=0$ has a root, z_2, between z'_2 and z'_3. Furthermore, since $F'(z)$ is positive for any positive value of z not within the interval (z'_2, z'_3), and negative for any value of z between z'_2 and z'_3, it follows that the function $F(z)$ is decreasing when z varies from $z=z'_2$ to $z=z'_3$, and is increasing in the intervals $(0,z'_2)$ and $(z'_3,+\infty)$. Therefore $F(z'_2)$ is a maximum and $F(z'_3)$, a minimum, and since they have different signs, we must have $F(z'_2)>0$, and $F(z'_3)<0$. On the other hand, $F(0)=-1$, and $F(+\infty)=+\infty$; hence the equation $F(z)=0$ has three real positive roots, z_1, z_2, z_3, which are now completely separated by the roots of $F'(z)=0$, forming Rolle's sequence, 0, z_1, z'_2, z_2, z'_3, z_3, $+\infty$, or

$$+\infty,\ v_3,\frac{1}{z'_2},\ v_2,\frac{1}{z'_3},\ v_1,\ 0,$$

where $v_1=1/z_3$, $v_2=1/z_2$, $v_3=1/z_1$.

If the preceding results be summarized, they may be tabulated as follows:

$$\begin{array}{lll}
B<0 & \dots\dots\dots\dots\dots\dots & \text{one positive root} \\
B>0 \left\{ \begin{array}{l} |\eta|>3/4 \\ |\eta|<3/4 \left\{ \begin{array}{l} D>0 \\ D<0 \end{array} \right. \end{array} \right. & \begin{array}{l} \dots\dots\dots \\ \dots\dots\dots \\ \dots\dots\dots \end{array} & \begin{array}{l} \text{one positive root} \\ \text{one positive root} \\ \text{three positive roots.} \end{array}
\end{array}$$

The computation of the resultant D is very laborious when performed directly by means of the coefficients of the original equation, but in practice, since the computation of the roots z'_2, and z'_3 is necessary for the separation of the roots of $F(z)=0$ and also for a more accurate determination of the coefficients A, B, it will be sufficient to substitute directly the values of z'_2 and z'_3 into $F(z)$. In order to simplify this calculation, let us call $R(z)$ the remainder after $F(z)$ has been divided by $\varphi(z)=4\lambda^2z^6-5Bz^3+A^2$; then $R(z'_i)=F(z'_i)$, $(i=2, 3)$, and the two conditions for the existence of three real roots of $F(z)=0$ become

$$\left.\begin{array}{l} R(z'_2)=z'^2_2\,(A^2-Bz'^3_2)-4/3>0 \\ R(z'_3)=z'^2_3\,(A^2-Bz'^3_3)-4/3<0 \end{array}\right\} \quad .$$

The two roots of $\varphi(z)=0$ are given by the following expression:

$$z'=\sqrt[3]{\frac{1}{8\lambda^2}\,(5B\pm\sqrt{25B^2-16A^2\lambda^2})} \ .$$

Now let v_o and V be the exterior limits of all the positive roots v_i of $f(v)=0$; these limits will form, with the roots z'_i of $F'(z)=0$, the following intervals within which the roots v_i will be found:

$$0,\ v_o,\ v_1,\ \frac{1}{z'_3},\ v_2,\ \frac{1}{z'_2},\ v_3,\ V \quad .$$

It now remains to be discovered how many of the roots of $f(v)=0$ are possible as physical solutions of the problem. In general, the only roots v_i which will be acceptable as such are those which give positive geocentric distances u_i. Such roots v_i must satisfy certain inequalities, and the matter is reduced to finding the positions of the limits of these roots with respect to the values of all the roots of the equation $f(v)=0$.

If u_i is to be positive, then $z_i>\beta^{1/3}$, when $\lambda<0$ and $\lambda\beta<0$, or $z_i<\beta^{1/3}$, when $\lambda>0$ and $\lambda\beta>0$. The case $\lambda<0$ and $\lambda\beta>0$ requires no further consideration, since corresponding to any positive value of v, there is a positive value of u, and the number of possible solutions will be either one or three.

Consider the function $F(z)=\lambda^2z^8-2Bz^5+A^2z^2-1$ and form the quantity $F(\beta^{1/3})=\beta^{2/3}a^2-1$. Two cases have to be considered, according as $\beta>1/a^3$, or $\beta<1/a^3$.

1st Case: $F(\beta^{1/3})>0$.

(*a*) In the first place, let us examine the case where the equation $F(z)=0$ has only one positive root, z_1, and compare the signs of $F(0)$ and $F(\beta^{1/3})$; we shall have $F(0)=-1$, $F(z_1)=0$, and $F(\beta^{1/3})>0$, consequently, since $z_1<\beta^{1/3}$, there is no solution when $\lambda\beta<0$, and one solution when $\lambda\beta>0$.

(*b*) Secondly, let us suppose that $F(z)=0$ has three real positive roots, z_1, z_2, z_3. The function $F(z)$ will be positive for any value of z satisfying the following inequalities:

$$\left.\begin{array}{l} z_1<z<z_2 \\ z_3<z \end{array}\right\} \quad ;$$

hence $\beta^{1/3}$ will be found within one of the intervals (z_1, z_2) or $(z_3,+\infty)$, i. e., $z_1<\beta^{1/3}<z_2$, or $z_3<\beta^{1/3}$. In order to ascertain in what interval $\beta^{1/3}$ lies, let us compare $\beta^{1/3}$ with the roots of $\varphi(z)=4\lambda^2z^6-5Bz^3+A^2=0$, which equation will be transformed into $\varphi(x)=4\lambda^2x^2-5Bx+A^2=0$, by putting $z^3=x$. Further, if $x=\beta$, we have $\varphi(\beta)=a^2+3ap\lambda\beta$.

Subcase I. $a+3p\lambda\beta>0$.

β will not lie between the roots x'_2 and x'_3, and since the inequality $3\lambda\beta+5ap>0$ is always satisfied when $a+3\lambda\beta p>0$ is fulfilled, it follows that when $\lambda\beta<0$ and $\lambda<0$, then $\beta<x'_2$, or $\beta^{1/3}<z'_2$, and therefore

$$\begin{array}{l} z_1<\beta^{1/3}<z'_2<z_2<z'_3<z_3, \text{ or} \\ v_3>\beta^{-1/3}>z_2'^{-1}>v_2>z_3'^{-1}>v_1 \ , \end{array}$$

which latter sequence of values exhibits the presence of two physical solutions, namely v_1 and v_2.

When $\lambda\beta > ap$ and $\lambda > 0$, we shall have $z_1 < z_2 < z'_3 < z_3 < \beta^{1/3}$; hence it follows that all of the positive roots z_1, z_2, z_3 are acceptable as physical solutions of the problem.

Subcase II. $a+3p\lambda\beta<0$, then $\lambda\beta$ is necessarily negative, and β will lie between the roots x'_2 and x'_3, so that we shall have $z'_2<\beta^{1/3}<z'_3$. Since $F(\beta^{1/3})$ is positive, the only arrangement—i. e., the relative positions—of the roots will be $z_1<z'_2<\beta^{1/3}<z_2<z'_3<z_3$, giving thus but two physical solutions z_2 and z_3.

Case II.—$F(\beta^{1/3})=\beta^{2/3}a^2-1<0$.

This case is discussed exactly as Case I. The results of both are collected in the following table:

TABLE II.—NUMBER OF POSITIVE ROOTS OF $f(v)=0$ AND NUMBER OF PHYSICAL SOLUTIONS.

Conditions fulfilled by the coefficients					No. of positive roots	No. of solutions
CASE I: $\beta>a^{-3}$	$B<0$	$\lambda\beta<0$			1	0
		$\lambda\beta>0$			1	1
	$B>0$	$\lvert\eta\rvert>3/4$, or $\lvert\eta\rvert<3/4$ and $D>0$	$\lambda\beta<0$		1	0
			$\lambda\beta>0$		1	1
		$\lvert\eta\rvert<3/4$ and $D<0$	$\lambda\beta<0$		3	2
			$\lambda\beta>0$		3	3
CASE II: $\beta<a^{-3}$	$B<0$	$\lambda\beta<0$			1	0
		$0<\lambda\beta<ap$			1	1
		$ap<\lambda\beta$			1	0
	$B>0$	$\lvert\eta\rvert>3/4$, or $\lvert\eta\rvert<3/4$, and $D>0$	$\lambda\beta<0$		1	1
			$0<\lambda\beta<ap$		1	1
			$ap<\lambda\beta$		1	0
		$\lvert\eta\rvert<3/4$ and $D<0$	$\lambda\beta<0$	$a+3p\lambda\beta>0$	3	3
				$a+3p\lambda\beta<0$	3	1
			$0<\lambda\beta<ap$		3	3
			$ap<\lambda\beta$		3	2

It will be noted, however, that in the satellite case, $F(\beta^{1/3})=\beta^{2/3}a^2-1$ is in general positive. First, in most cases, μ and $\lambda\beta$ have the same sign; $\frac{\lambda\beta}{\mu}$ will be positive as long as the condition $\left(1-\frac{1}{b^3}\right)+\frac{\nu}{\mu b^3}>0$ is satisfied, which in virtue of the relation $\lambda=\frac{\mu}{\gamma}+\nu$ becomes

$$\left[1-\frac{1}{b^3}\left(1+\frac{1}{\gamma}\right)\right]+\frac{\lambda}{\mu b^3}>0\ . \qquad (4)$$

This inequality is always satisfied when λ and μ have the same sign; if they are of opposite signs, (4) may be written, after a few substitutions, in the form[14]

$$\gamma(b^3-1)-1>-\frac{\Delta.(\kappa)}{\Delta.\kappa}\quad ,$$

where $\Delta=\begin{vmatrix} -\alpha'^2 & 0 & 1 \\ \alpha'' & \alpha' & 0 \\ (\tan\delta)'' & (\tan\delta)' & \tan\delta \end{vmatrix}$.

Now, for *Jupiter*, $\gamma=1047$ and $b^3=141$; hence $1/\chi=147\,500$, where $1/\chi=\gamma(b^3-1)-1$.

For *Saturn*, $\gamma=3500$ and $b^3=868$; hence $1/\chi=304.\ 10^4$. These figures show that the ratio $\frac{-(\kappa)}{\kappa}$ must become very large in order that (4) will not be satisfied. Since the product $\Delta.(\kappa)$ may be written in the form

$$\Delta.(\kappa)=\begin{vmatrix} \epsilon & 0 & 1 \\ \epsilon' & \alpha' & 0 \\ \epsilon'' & (\tan\delta)' & \tan\delta \end{vmatrix}\ ,$$

where ϵ, ϵ', ϵ'' are small quantities of at least the same order of magnitude as the angle ψ, it is clear that this determinanent is bounded (in the satellite case all of its elements have finite values) and consequently the ratio $\frac{-\Delta.(\kappa)}{\Delta.\kappa}$ becomes very large only for exceedingly small values of the denominator, i. e., when one of the three following relations is very nearly satisfied:

$\left.\begin{matrix} A-\alpha=0,\ \pi \\ D=\pm\delta \end{matrix}\right\}$, or $\alpha'=(\tan\delta)'=0$, or $M\alpha'+N\delta'=0$,

where $M=\tan\delta\cos(A-\alpha)$, $N=\sin(A-\alpha)(1+\tan\delta)$.

Except for these special cases, in general $\chi<\frac{-\kappa}{(\kappa)}$, and μ, $\lambda\beta$ will have the same sign. Now $F(\beta^{1/3})$ will be positive as long as $\beta>a^{-3}$, or after multiplication by λ (or by $-\lambda$, if $\lambda<0$) and a few transformations, when

$$\mu\left[1-\frac{1}{b^3}\left(\frac{1}{\gamma}+1\right)\right]>\lambda\left(\frac{1}{a^3}-\frac{1}{b^3}\right)\ .$$

Since in general μ and $\lambda\beta$ have the same sign, if $\frac{\mu}{\lambda}<0$, λ and $\lambda\beta$ will have opposite signs, only when $\lambda<0$ and $\lambda\beta>0$; hence $\beta<0$, giving rise to a case where the quantity $F(\beta^{1/3})$ was not considered. If $\frac{\mu}{\lambda}>0$, and $a>b$, then $F(\beta^{1/3})$ will always be positive, whereas if $\frac{\mu}{\lambda}>0$ and $a<b$, $F(\beta^{1/3})$ will be positive for $\chi'<\frac{\kappa}{(\kappa)}$, where

$$1/\chi'=\frac{a^3}{b^3-a^3}[\gamma(b^3-1)-1]=\frac{a^3}{(b^3-a^3)\chi}\ .$$

For *Jupiter*, $\chi'=1/163\,000$ and for *Saturn*, $\chi'=1/77.10^5$; hence in the satellite case, $F(\beta^{1/3})$ will in general be positive. In numerical computation, it is sufficient to consider the relative order of magnitude of the determinants κ, (κ), and $[\kappa]$ in determining the sign of $F(\beta^{1/3})$.

Let us now find the limits of the roots of the equation $f(v)=0$, these limits being used for a first approximation of the roots and of the coefficients A and B, which are functions of them. The discussion will be divided into two sections, according as B is positive or negative.

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[14] *Publ. Lick Obs.*, **7**, 471, 1913.

With $B=\lambda(\lambda\beta-ap)$ negative, $\lambda\beta$ must be positive, since the case $\lambda\beta<0$, and $\lambda>0$ yields no solution for the problem. If $\lambda>0$, then $0<\lambda\beta<ap$; and if $\lambda<0$, then $\lambda\beta>ap$. These two cases are entirely analogous in their analysis, so that it will be sufficient to treat only one of them, for instance the case where $\lambda>0$. Let us apply the method of groups (each group having at most one variation in sign) to the equation $F(z)=\lambda^2z^8-2Bz^5+A^2z^2-1=0$; it is found that an upper limit of the positive roots of $F(z)=0$ is given by the two inequalities

$$A^2z^2-1>0, \qquad \lambda^2z^8-2Bz^5>0;$$

the second is always satisfied, since B is negative, hence $1/A$ is an upper limit of the positive root z_1, and consequently A will be a lower limit of the root v_1 of $f(v)=0$. An upper limit may be obtained by one of the well known methods, Cauchy's for example, which method gives for an upper limit the greatest of the three quantities $(3A^2)^{1/2}$, $[6\lambda\ (\lambda\beta-ap)]^{1/5}$, $(3\lambda^2)^{1/8}$.

It will be noted that the lower limit A agrees with the limit derived from physical considerations. It has been shown that when $\lambda\beta>0$ and $\lambda>0$, then v must be greater than $\beta^{1/3}$; hence $u<\lambda\beta$. Now the function $v^2=(u-ap)^2+(as)^2$ is a decreasing function of u, since $u<ap$; therefore $v^2>(\lambda\beta-ap)^2+(as)^2$, or $v>A$. The more important case where B is positive will be considered hereafter.

If $\lambda\beta>0$, $\lambda>0$, then $\lambda\beta>ap$, and $u<\lambda\beta$. The limits may be discovered by Laguerre's method, in considering the two functions $f_6=v^2-A^2$ and $f(v)=v^8-A^2v^6+2Bv^3-\lambda^2$. The first of these expressions will be positive for $v>A$, and $f(A)=2BA^3-\lambda^2$ will be positive for $A>\left(\frac{\lambda^2}{2B}\right)^{1/3}$, therefore A will be an upper limit of the positive roots of $f(v)=0$. It can be shown in a similar way that $\left(\frac{\lambda^2}{2B}\right)^{1/3}$ will be a lower limit of the positive roots of $f(v)=0$, by proving that $\left(\frac{\lambda^2}{2B}\right)^{-1/3}$ is an upper limit of the positive roots of $F(z)=0$. Now, if $f(A)$ is negative, then $A<\left(\frac{\lambda^2}{2B}\right)^{1/3}$, and since all of the Laguerre's functions will be positive for $v>\left(\frac{\lambda^2}{2B}\right)^{1/3}$, it follows that $\left(\frac{\lambda^2}{2B}\right)^{1/3}$ will be an upper limit and A, a lower limit of the positive roots of $f(v)=0$. Hence we shall have

$$\left(\frac{\lambda^2}{2B}\right)^{1/3}<v_i<A,\ \text{when}\ A>\left(\frac{\lambda^2}{2B}\right)^{1/3},$$

$$\text{with}\ 2ap-\lambda\beta<u_i<\lambda\beta,\ \text{or}$$

$$A<v_i<\left(\frac{\lambda^2}{2B}\right)^{1/3},\ \text{when}\ A<\left(\frac{\lambda^2}{2B}\right)^{1/3};$$

$$\text{with}\ u_i<2ap-\lambda\beta\ .$$

In either one of these cases the lower limit should be compared with the condition $v>\beta^{-1/3}$.

If $\lambda\beta>0$ and $\lambda<0$, then $\lambda\beta<ap$ and $u>\lambda\beta$. An exactly similar method of discussion as that just employed in the preceding case leads to the same limits for v, namely:

$$\left(\frac{\lambda^2}{2B}\right)^{1/3}<v_i<A,\ \text{with}\ \lambda\beta<u_i<2ap-\lambda\beta,\ \text{or}$$

$$A<v_i<\left(\frac{\lambda^2}{2B}\right)^{1/3},\ \text{with}\qquad u_i>2ap-\lambda\beta.$$

There remains the case $\lambda\beta<0$ and $\lambda<0$. It has been shown that $v<\beta^{-1/3}=\beta'$, this condition yields two limits u_o and U for the geocentric distance u. From the equation $v^2=(u-ap)^2+(as)^2$, it follows that $u_o=ap-(\beta'^2-a^2s^2)^{1/2}$ and $U=ap+(\beta'^2-a^2s^2)^{1/2}$, with $s<\frac{\beta'}{a}$. Then two new limits for v may be readily obtained by substituting successively u_o and U into the fundamental equation $u=\lambda\beta\left(1-\frac{1}{\beta v^3}\right)$; u_o is of course supposed to be positive. Performing these substitutions, we find the following results:

$$\left[\beta\left(1-\frac{U}{\lambda\beta}\right)\right]^{-1/3}<v_i<\left[\beta\left(1-\frac{u_o}{\lambda\beta}\right)\right]^{-1/3}.$$

The smaller the upper limit U and the larger the lower limit u_o, the closer will be the limits for v.

III.—THE CASE OF AN ASTEROID OR A COMET.

If the body be assumed to move around the Sun, and if the perturbations produced by the disturbing mass be neglected in the first approximation, the general system of equations will reduce to the form

$$\left.\begin{aligned} u&=\mu(1-1/w^3)\\ w^2&=(u-q)^2+t^2 \end{aligned}\right\},$$

or after elimination of u, to the equation

$$w^8-A^2w^6+2Bw^3-\mu^2=0,$$
$$B=\mu(\mu-q),\ \text{and}\ A^2=(\mu-q)^2+t^2.$$

In this case, it will be observed that B is always positive, and furthermore that $w_1=1$ is a root of the equation $f(w)=0$, which evidently has three positive roots, namely $w_1=1$ (the Earth's solution), w_2, which corresponds to the actual heliocentric distance of the body, and a third positive root w_3, since the equation must have either one or three positive roots.[15] The results for this case will be summarized in Table III.

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[15] C. V. L. Charlier, *Arkiv för Matematik, Astronomi och Fysik*; Band 7, No. 5, 7 *et seq.*; *Die Analytische Lösung des Bahnbestimmungsproblems.*

TABLE III.—LIMITS AND NUMBER OF SOLUTIONS IN THE CASE OF AN ASTEROID OR A COMET.

Conditions fulfilled by the coefficients		Limits for w	Limits for u
$\mu<0,\ q>0$		$\left(1+\frac{2q}{-\mu}\right)^{-1/3}<w<1$	$0<u<2q$
$\mu>0$	$q>0$	$1<w<(\mu^2-2\mu q+1)^{1/2}$	$2q<u<\mu$
	$q<0$	$1<w<(\mu^2-2\mu q+1)^{1/2}$	$0<u<\mu$

Number of Solutions

$$1+3\mu q<0 \quad \text{..........................one solution}$$

$$\left.\begin{array}{l} 1+3\mu q>0 \\ 5q+3\mu>0 \end{array}\right\} \quad \text{......................two solutions} \quad .$$

Before applying the preceding principles to the computation of illustrative numerical examples, we shall summarize the foregoing analytical results. In the case of a satellite, the solar perturbations of which are taken into account, the solution of the problem—i. e., the determination of the distance of the satellite from its primary—is obtained by successive approximations, each of which involves an eighth degree equation $f(v)=0$ with variable coefficients. Since the variations of the coefficients in the fundamental equation are small in passing successively from one approximation to another, it is necessary only to find the number, the limits and the values of the roots of $f(v)=0$ in the first approximation, the roots of the subsequent equations being obtained by differential corrections.

The substitution of $F(z)=0$ for $f(v)=0$ satisfactorily solves the problem, since this transformation gives not only the number of real positive roots of the original equation $f(v)=0$, but also the intervals within which they lie, furnishing thus by combination with the outer limits a method of determining their approximate values. All of the roots which do not satisfy the physical conditions of the problem may be rejected, while further approximations to the values of the others may be made. In the case of the existence of two or three possible solutions, the two or three orbits corresponding to them must be computed and the coördinates of the satellite obtained from these orbits compared with subsequent observations in order to ascertain which one is the true physical solution.

IV.—NUMERICAL EXAMPLES.

Example I.—Orbit of *J VII*.

In order to compare the following computational results with the numerical data given *Publ. Lick Obs.*, **7**, 490, *et seq.*, the coefficients λ, μ, ν, A^2, $2B$ will have the same meaning here that were assigned to them in that volume, i. e., v, u, w, will respectively represent r, ρ, and $[r]$.

From p. 491 we have, after putting $w=b$,

$$\left.\begin{array}{lll} \lambda=[7.72636]\,n & ap=[0.72425] & \lambda\beta=[0.80031]n \\ \mu=[0.78641]\,n & b=[0.69635] & A^2=[2.12995] \\ \nu=[6.71010] & (as)^2=[6.69109] & 2B=[9.09236] \\ \frac{\mu}{\lambda}=[7.76631]\,n & \lambda^2=[5.45271] & \beta=[3.07395] \\ a=[0.72426] & \lambda\beta-ap=[1.06497]n & \beta'=[8.97535] \end{array}\right\},$$

where the numbers in brackets are logarithms.

Since $\lambda\beta<0$ and $\lambda<0$, it has been shown that $v_i<\beta^{-1/3}=\beta'$, $\beta'=0.0945$, and

$$u_o=ap-[\beta'^2-(as)^2]^{1/2}<u_i<U=ap+[\beta'^2-(as)^2]^{1/2};$$

in as much as u_o is positive, two limits for v_i are at once obtainable, namely:

$$\beta'\left(1-\frac{U}{\lambda\beta}\right)^{-1/3}<v_i<\beta'\left(1-\frac{u_o}{\lambda\beta}\right)^{-1/3} \quad ,$$

or, numerically, $0.076913<v_i<0.077318$.

On the other hand, $|\eta|=\left|\frac{as}{\lambda\beta-ap}\right|<3/4$ is always satisfied when $\lambda\beta<0$, since for $\lambda\beta=0$, $|\eta|=|s/p|=\tan\psi$, which is a very small quantity; hence the equation $F'(z)=0$ has three real roots: $z'_1=0$, $z'_2=8.169$, and $z'_3=12.968$. Now the product $F(8.169)F(12.968)<0$, therefore the equation $f(v)=0$ has three real positive roots, which constitute the sequence

$\left(\frac{\lambda^2}{2B}\right)^{1/3}=0.0612$, 0.0769, v_1, 0.0771, v_2, 0.0773, $\beta'=0.0945$, v_3, $A=11.614$. The third root v_3 is very large and close to the upper limit A, since $f(v)=0$ can be written $v^6[(v^2-A^2)+2B/v^3-\lambda^2/v^6]=0$. For $v=A$, the expression within the bracket will become almost equal to zero; v_1 and v_2 are the two positive roots acceptable as physical solutions. Since the coefficients A^2 and $2B$ are functions of the roots of the equation in which they occur, a second approximation to the values of these coefficients will be obtained at once by taking

$$\left.\begin{array}{ll} v_1=[8.88600] \quad , & v_2=[8.88828] \\ v_1=\ 0.0769 \quad , & v_2=\ 0.0773 \end{array}\right\} .$$

The only variable quantity in the coefficients is $\lambda\beta=\mu\left(\frac{1}{R^3}-\frac{1}{w^3}\right)+\frac{\nu}{b^3}$, so that a new value of w_i will be secured from the equations

$$\left.\begin{array}{l} v_i^2=(u_i-ap)^2+(as)^2 \\ w_i^2=(u_i-Rq)^2+(Rt)^2 \end{array}\right\} ;$$

the elimination of u_i between the two preceding equations gives

$$w_i^2=(m\pm\sqrt{v_i^2-a^2s^2})^2+R^2t^2,$$

where $m=ap-Rq$. It is easy to avoid the ambiguity of the double sign in the following way: let us consider the two equations $u_i = ap \pm (v_i^2 - a^2s^2)^{½}$ and $u_i = \lambda\beta\left(1 - \frac{1}{\beta v_i^3}\right)$, the sign + will be taken if $u_i > ap$, and the sign −, if $u_i < ap$. But for $u_i > ap$, $v_i < \beta^{-⅓}\left(1 - \frac{ap}{\lambda\beta}\right)^{-⅓}$, and if $u_i < ap$, $\beta^{-⅓}\left(1 - \frac{ap}{\lambda\beta}\right)^{-⅓} < v_i$, so that no ambiguity as to the sign before the radical will arise. In the present case, $\beta^{-⅓}\left(1 - \frac{ap}{\lambda\beta}\right)^{-⅓} = 0.077113$; hence for $v_1 < 0.0771$, $u_1 = ap + (v_1^2 - a^2s^2)^{½}$, $w_1^2 = (m + \sqrt{v_1^2 - a^2s^2})^2 + R^2t^2$, and for $v_2 > 0.0771$, $u_2 = ap - (v_2^2 - a^2s^2)^{½}$, $w_2^2 = (m - \sqrt{v_2^2 - a^2s^2})^2 + R^2t^2$. Substituting the following numerical values in the expressions for w_i, namely $m = [0.68930]$, $(Rt)^2 = [9.90626]$, $+\sqrt{v_1^2 - a^2s^2} = [8.86718]$, and $-\sqrt{v_2^2 - a^2s^2} = [8.86967]n$, we shall obtain $w_1 = [0.70278]$, $w_2 = [0.69009]$, which values check with those listed in the second approximation on p. 492, the new values derived for the coefficients A^2 and $2B$ being precisely the same as before.

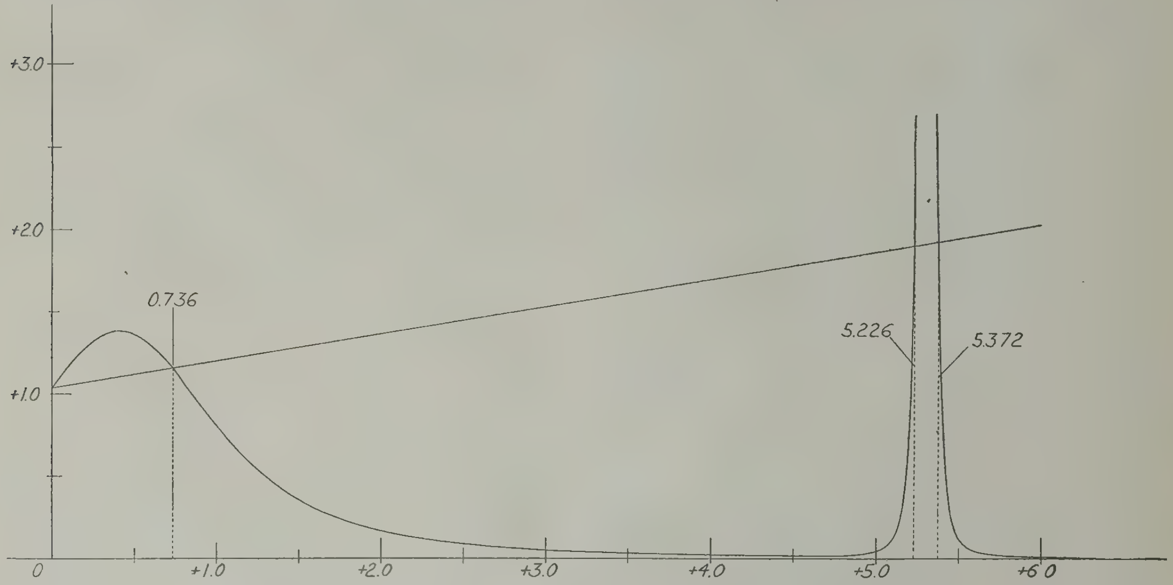

Fig. 2.—Graphical solution for geocentric distance, *Jupiter VII*, by Leuschner's method.

If the body be assumed to move around the Sun and if the perturbations caused by *Jupiter* be neglected, the former values of the coefficients will give

$\mu < 0$, $q > 0$; hence $0.9604 < w < 1$, and $0 < u < 0.8306$. Since, on the other hand, $1 + 3\mu q < 0$, there is only one solution w_1. An approximate value for the geocentric distance u_1 may be readily obtained from the Table, *Publ. Lick Obs.*, **7**, 337, *et seq.*; in the present case, with the arguments $[\psi] = 65^{\circ}.467$ and $1/\mu = -0.15506$, we find $u_1 = 0.7360$. The value of the root of $f(w) = w^8 - A^2w^6 + 2Bw^3 - \mu^2 = 0$, where $A^2 = [1.680755]$, $2B = [1.947121]$, $\mu^2 = 41.5899$, $q = 0.415284$, and $t^2 = 0.827539$, will be computed by two successive approximations by means of Newton's formula. In the application of this method, care must be taken in the use of an approximate value of the root. As Fourier has shown in his "*Analyse des Equations,*" this approximate value must be such that $\frac{f''(w_i)}{f(w_i)} > 0$, or in other words $f''(w_i)$ and $f(w_i)$ must have the same sign. The substitution of the equation $F(z) = \mu^2z^8 - 2Bz^5 + A^2z^2 - 1 = 0$ for the original one allows the determination of the sign of $F''(z_i)$ in a satisfactory manner. $F''(z) = 0$ being a quadratic in z^3, if its two roots be denoted by z_1'' and z_2'', the function $F''(z) = 28\mu^2z^6 - 20Bz^3 + A^2$ will be negative for any value of z lying between z_1'' and z_2'', and positive for any other value of z.

When this method is applied to the present case, the roots of $F''(z) = 0$ are found to be less than 1, and since the root z_1 lies between 1 and 1.0412, it follows that $F''(z_1)$ is positive. Assuming $z_1 = [0.01660]$ as a first approximation, we shall find $F(z_1) = +0.0397$ and $F'(z_1) = [1.26865]$; hence $\Delta z_1 = -0.00214$. A second approximation yields the following numerical values:

$$F(z_1) = +0.0023, \quad F'(z_1) = [1.21505]; \quad \Delta z_1 = -0.000140, \quad z_1 = 1.03668.$$

The corresponding heliocentric, geocentric, and jovicentric distances are $w_1 = 0.964612$, $u_1 = 0.736127$, and $v_1 = 4.63458$ respectively. The same distances expressed in astronomical units are

$$[r_1] = 0.951877, \quad \rho_1 = 0.726404, \quad r_1 = 4.57339 \ .$$

If the action of *Jupiter* be taken into account, it will be sufficient to put $\left(\frac{\nu}{b^3}-\frac{\lambda}{v^3}\right)+\mu=\mu\beta$, and the original system of equations reduces to the following equations:

$$\left.\begin{aligned} u&=\mu\beta\left(1-\frac{1}{\beta w^3}\right) \\ w^2&=\quad(u-q)^2+t^2 \end{aligned}\right\}.$$

In the case of *J VII*, the coefficient $\mu=0.809494$ is changed into $\mu\beta=0.809490$ and the introduction of *Jupiter's* action affects the computation in the sixth decimal place only.

Example II.—Orbit of *J VIII*.

If $w=b$, the approximate values of the coefficients are

$$\left.\begin{array}{rrr} \lambda=[7.02770]n & a=[0.65870] & \nu/b^3=[5.52310]n \\ \mu=[0.61589] & b=[0.72553] & \lambda^2=[4.05540] \\ \nu=[7.69968]n & ap=[0.65868] & \lambda\beta=[0.62191] \\ \frac{\mu}{\gamma}=[7.59580] & as=[8.65762] & \beta=[3.59421]n \\ \lambda\beta-ap=[9.56816]n & A^2=[9.14284] & 2B=[6.89689] \end{array}\right\}.$$

λ, $\frac{\mu}{\gamma}$, and ν are of the same order of magnitude, and $\lambda\beta$ has the same sign as μ. Since the condition $|\eta|=\left|\frac{as}{\lambda\beta-ap}\right|<\frac{3}{4}$ is satisfied, $F'(z)=0$ has two positive roots, $z'_2=4.374$, and $z'_3=7.016$; now the quantity $D=F(4.374)F(7.016)$ is negative, hence $f(v)=0$ has three real positive roots, forming the sequence

$$\left(\frac{\lambda^2}{2B}\right)^{1/3}=0.1129,\ v_1,\ 0.1425,\ v_2,\ 0.2244,\ v_3,\ A=0.3728,$$

each of the v_i giving a possible satellite solution.

For a second approximation of the coefficients, we shall take $v_1=0.13$, $v_2=0.18$, and $v_3=0.30$. In the formula $w_i^2=(m\pm\sqrt{v_i^2-a^2s^2})^2+R^2t^2$ the sign + will be used when $v_i<0.1423=\beta^{-1/3}\left(1-\frac{ap}{\lambda\beta}\right)^{-1/3}$; hence the new values are

$$\begin{array}{rrr} w_1=[0.73563] & w_2=[0.71155] & w_3=[0.70120] \\ A_1^2=[9.13852] & A_2^2=[9.14916] & A_3^2=[9.15459] \\ 2B_1=[6.89470] & 2B_2=[6.90010] & 2B_3=[6.90286] \end{array}$$

The corresponding values of the roots are

$$v_1=[9.11267],\quad v_2=[9.23810],\quad v_3=[9.54899].$$

The first two roots after a third approximation would lead to the same results as those given in *Publ. Lick Obs.*, **7**, 501, but the third root, v_3, having there been found by a graphical method, will here be derived analytically. In general, let a_k and v_i be the respective values of any coefficient and any root of the equation $f(v)=0$, in any approximation; if Δv_i be the correction to be applied to v_i, and Δa_k be the variation of the coefficient a_k, then by Taylor's theorem we shall have

$$f(v_i+\Delta v_i,\ a_k+\Delta a_k)=f(v_i,\ a_k)+\Delta v_i\frac{\partial f(v_i,\ a_k)}{\partial v_i} + \sum_k \frac{\partial f(v_i,\ a_k)}{\partial a_k}\ \Delta a_k+\dots\dots\dots\ ,$$

or, if the squares of the corrections be neglected,

$$\Delta v_i=\frac{-\sum\limits_k \frac{\partial f(v_i,a_k)}{\partial a_k}\ \Delta a_k}{\frac{\partial f(v_i,a_k)}{\partial v_i}}=\frac{v_i(v_i^3\Delta A^2-\Delta 2B)}{8v_i^5-6A^2v_i^3+6B},$$

where the mean values, $a_k+\frac{1}{2}\Delta a_k$, of any coefficient will be used. Applying this method to the third root v_3, we obtain from the second approximation the following results:

$w_3=[0.69650]$, $\lambda\beta_3=[0.62127]$, $A_3^2=[9.15684]$, $2B_3=[6.90400]$; hence the new values of the jovicentric and geocentric distances are

$$\Delta v_3=[7.0904],\ v_3=[9.55050],\ u_3=[0.62371],\ u_3=4.2045.$$

For a complete check, let us find the geocentric distances, u_1, u_2, u_3, of *J VIII* by the graphical method, i. e., by finding the intersections of the curve

$$y=\frac{E/F}{[(u-ap)^2+(as)^2]^{3/2}}+\frac{1}{[(u-Rq)^2+(Rt)^2]^{3/2}},$$

with the straight line

$$y=u/F-G/F\quad ,$$

where the constants have the following numerical values:

$$\left.\begin{array}{lll} E/F=[6.4118]\,n & G=4.214 & (as)^2=0.0021 \\ G/F=[0.00887]\,n & -G/F=1.021 & Rq=-0.7182 \\ 1/F=[9.38411]\,n & ap=4.557 & (Rt)^2=0.4706 \end{array}\right\}.$$

From the graph (Fig 3) we derive

$$\left.\begin{array}{lll} u_1=4.680, & u_2=4.390, & u_3=4.205 \\ v_1=0.1296, & v_2=0.1731, & v_3=0.3552 \end{array}\right\}.$$

The curve shows that the third root $u_3=4.205$ lies close to the second intersection $u_2=4.390$, and is undoubtedly what the footnote in *Publ. Lick Obs.*, **7**, 501, refers to, although this second solution really corresponds to the so-called "first solution," i. e., $r_1=[9.23826]$ in Sec.

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[16] J. H. Coolidge, The continuity of the roots of an algebraic equation, *Annals of Mathematics*, Series 2, **9**, 116, 1907–08.

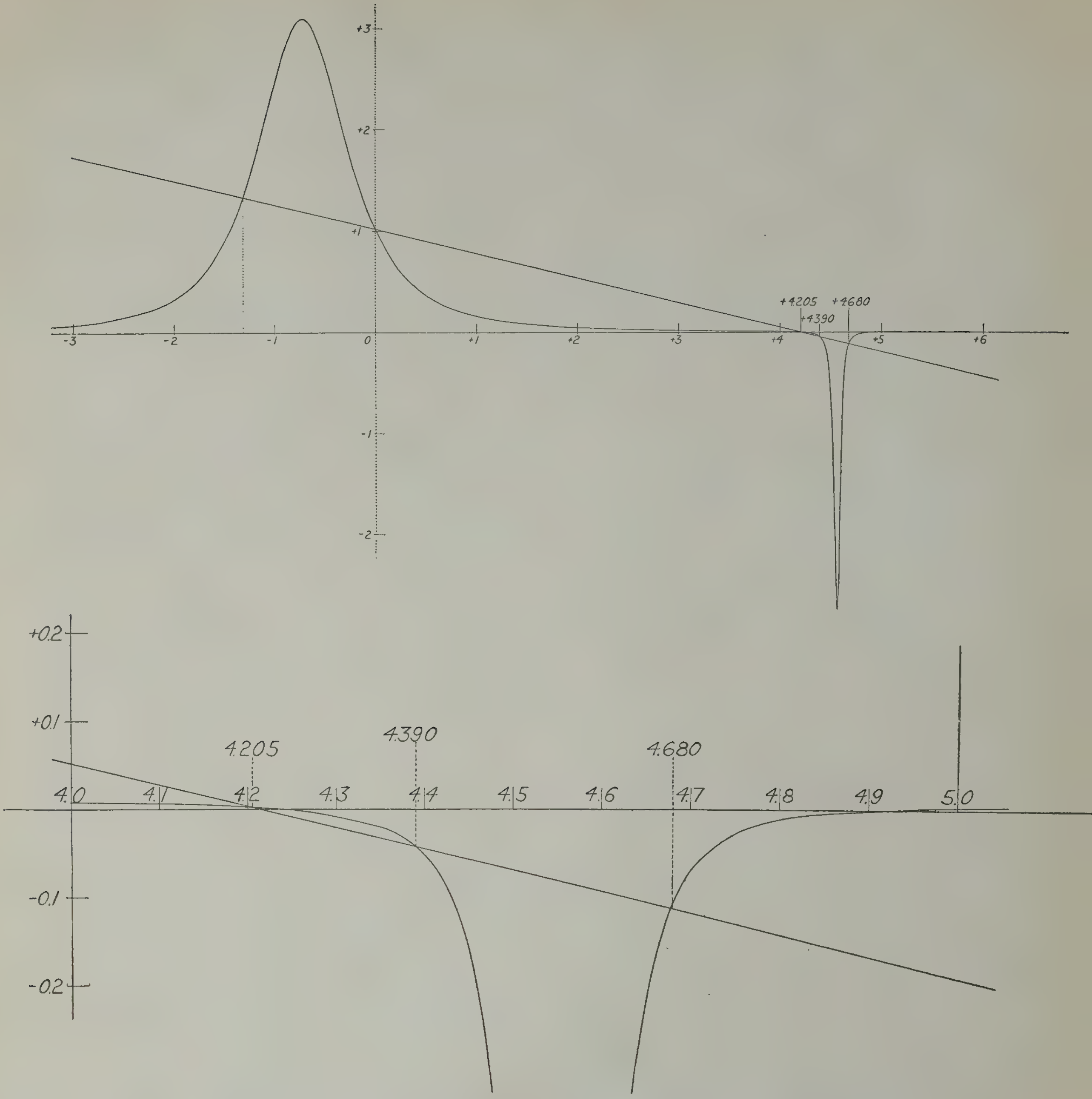

Fig. 3.—Graphical solution for geocentric distance, *Jupiter VIII*, by Leuschner's method.

III, on the same page; hence the ambiguity there arises simply from a difference in notation.

Since $\mu>0$, $q<0$, and $1+3\mu q<0$, there will be a unique solar solution in w; in this case, the limits for the heliocentric and geocentric distances will be

$$\left.\begin{matrix} 1<w<A \\ 0<u<\mu \end{matrix}\right\}, \text{ or numerically } \left.\begin{matrix} 1<w<5.0145 \\ 0<u<4.244 \end{matrix}\right\}.$$

The solutions for w and u in the first approximation are respectively $w=4.9805$, $u=4.2092$, and after three approximations have been made, they become $w=5.0043$, $u=4.2333$, or since in this calculation, the symbols w and u are respectively equal to r/R and ρ/R, the final value of the geocentric distance ρ will be 4.2046, which entirely agrees with the values previously obtained, when *Jupiter* was considered as the central body.

Example III.—Orbit of *J IX*.

In the first approximation ($w=b$), the values of the coefficients were found to be

$$\left.\begin{array}{rrr} \lambda=[8.30692] & \lambda\beta-ap=[0.43648] & ap=[0.60928] \\ \mu=[0.85586] & A^2=[0.87368] & as=[9.04638] \\ \nu=[8.12817] & 2B=[9.04443] & \lambda^2=[6.61384] \\ \dfrac{\mu}{\gamma}=[7.83576] & a=[0.60944] & \nu/b^3=[6.01632] \\ \lambda\beta=[0.83245] & b=[0.70395] & \beta=[2.52553] \end{array}\right\} .$$

$\lambda\beta$ has the same sign as μ, and furthermore $\lambda\beta>0$, $\lambda>0$, $\lambda\beta-ap>0$, $B>0$, $v_i>\beta^{-1/3}=0.144$. It is seen by inspection of the coefficients that $|\eta|<3/4$, so that the equation $F'(z)=0$ has two positive roots, namely: $z'_2=[0.50948]$, and $z'_3=[0.70978]$. Now $F(z'_2)$ is positive, while $F(z'_3)=-0.68$ is negative; consequently $F(z)=0$ has three real positive roots forming the series

$$1/A=0.366,\ z_1,\ z'_2=3.232,\ z_2,\ z'_3=5.126,\ z_3,\left(\frac{2B}{\lambda^2}\right)^{1/3}=6.46,$$

$$A=2.734,\ v_3,\ 1/z'_2=0.3094,\ v_2,\ 1/z'_3=0.1951,\ v_1,\left(\frac{\lambda^2}{2B}\right)^{1/3}=0.155.$$

The value $F(z'_3)$ shows that z'_3 is almost a double root of $F(z)=0$; consequently, we should expect the presence of two close roots of $f(v)=0$. On the other hand, since A is a large coefficient (in comparison to $2B$ and λ^2), the third root, v_3, will be very close to A. In the first approximation, let us take $v_1=0.190$, $v_2=0.200$, and $v_3=A=2.734$. The corresponding values of w_i will be obtained by the formula

$$w_i^2=(m\pm\sqrt{v_i^2-a^2s^2})^2+(Rt)^2,$$

where $m=5.0556$, and $(Rt)^2=0.05374$. The sign $+$ will be taken when $v_i>0.19505$, and the sign $-$, when $v_i<0.19505$. The resulting values of w_i will be

$$w_1=[0.69081],\quad w_2=[0.71823],\quad w_3=[0.89159]\quad .$$

Computing the new coefficients with the foregoing values of w_i, we obtain

$$\left.\begin{array}{rrr} \lambda\beta_1=[0.83211] & \lambda\beta_2=[0.83278] & \lambda\beta_3=[0.83502] \\ 2B_1=[9.04359] & 2B_2=[9.04525] & 2B_3=[9.05081] \\ A^2_1=[0.87200] & A^2_2=[0.87532] & A^2_3=[0.88642] \end{array}\right\} .$$

The values of the roots of $f(v)=0$ will be computed by Newton's formula, as in the case of *J VII* (solar solution). The two roots of $F''(z)=0$ are found to be $z_1''=1.94$ and $z_2''=4.47$, and since in the first approximation $v_1=0.1900$, $z_3=1/v_1=5.2632$, $v_2=0.200$, $z_2=1/v_2=5.0000$, $F''(z_3)$ and $F''(z_2)$ are both positive. Now $F(z_3)=+0.794$, $F'(z_3)=22.38$; hence $\Delta z_3=-0.0355$ and $z_3=5.2277$. Two other approximations yield the following numerical results:

$$\left.\begin{array}{l} F(5.2277)=+0.124,\ \Delta z_3=-0.0077; \\ F'(5.2277)=+16.154,\quad z_3=5.2200; \\ F(5.2200)=+0.0065,\ \Delta z_3=-0.00044 \\ F'(5.2200)=+14.863,\quad z_3=5.21956 \end{array}\right\} ,$$

giving for the jovicentric distance $v_1=0.19159$.

The correction of the second root, z_2, is found in a similar manner as follows:

$$\left.\begin{array}{l} F(5.0000)=+0.341,\ \Delta z_2=+0.0229; \\ F'(5.0000)=-14.90,\ z_2=5.0229; \\ F(5.0229)=+0.025,\ \Delta z_2=+0.00198 \\ F'(5.0229)=-12.60,\ z_2=5.0249 \end{array}\right\} ;$$

hence $v_2=0.19901$. The corresponding geocentric distances are $u_1=3.9110$, and $u_2=4.2321$.

As for the third root, $v_3=1/z_1$, it is very easy to compute it, since the value of z_1 lies very close to that of $1/A$. Let 0.36057 be the approximate value of z_1, then $F(z_1)=+0.0003$, $F'(z_1)=+5.542$, and a single differential correction will give $z_1=0.36052$, $v_3=2.7737$, $u_3=6.8385$, $w_3=7.8305$. With this value of w_3, it is found that the variation of $\lambda\beta$ is equal to $+0.000226$, so that the new values of the coefficients become

$$\left.\begin{array}{r} \lambda\beta_3=[0.83503] \\ A^2_3=[0.88644] \\ 2B_3=[9.05082] \end{array}\right\} ;\text{ hence } \left.\begin{array}{r} v_3=2.7737 \\ u_3=6.8386 \\ w_3=7.8306 \end{array}\right\} .$$

A first approximation would give for an orbit around the Sun the following results:

$$\mu>0,\ q<0,\ 1+3\mu q<0;\ 1<w<7.727;\ 0<u<6.751\quad .$$

The unique solution for w gives $w=7.7132$ for the heliocentric distance, $u=6.7360$ for the geocentric distance, and $v=2.7327$ for the jovicentric distance. A second approximation leads to the value $w=7.7120$. In this last computation, w, u, v, are symbols equal respectively to $[r]/R$, ρ/R, and r/R, so that, after the distances are expressed in astronomical units, we find for the values of the heliocentric distance $[r]=7.8307$, which value agrees with the heliocentric distance, w_3, found when *Jupiter* was considered as the primary.

The two orbits corresponding to the geocentric distances $u_2=4.2322$ and $u_3=6.8387$ have been found to be hyperbolic, the first, with *Jupiter* as central body, having an eccentricity $e=1.62$; and the second, with respect to the Sun as attracting mass, having an

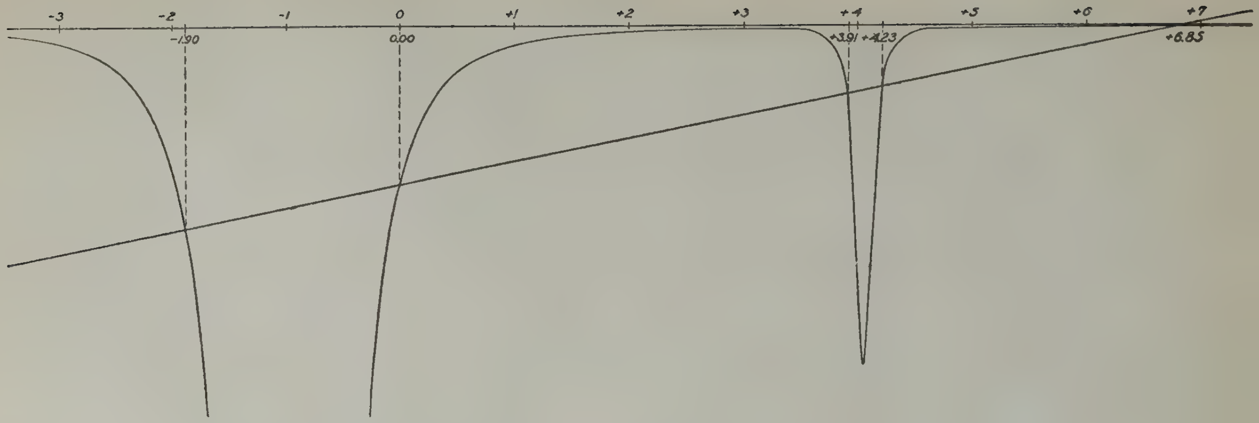

Fig. 4.—Graphical solution for geocentric distance, *J IX*, by Leuschner's method. (Copied from *L. O. Bull.*, **8**, 101, 1914.)

eccentricity $e=1.27$. Although these two orbits were highly improbable, the selection of the true orbit was made on the basis of the representation of subsequent observations.[17]

The numerical examples have been fully explained and developed according to the general principles set forth in the first part of this paper, and may be used as a synopsis of formulae. In each case, advantage has been taken of the particular and relative values of the numerical coefficients for simplification in the computation of the roots. For comparison Leuschner's graphical solution which was applied by Nicholson to *Jupiter IX* (fig. 4) is given also for *Jupiter VII* and *VIII* (figs. 2 and 3). The values of the abscissae corresponding to the roots are printed in the figures and agree with those derived analytically by the method here proposed.

In concluding, I wish to thank Professor A. O. Leuschner and Professor R. T. Crawford for valuable suggestions and criticisms.

Students' Observatory,
University of California,
Berkeley, Calif., May, 1923.
Issued May 15, 1925.

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[17] *L. O. Bull.*, **8**, 102, 1914.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 363

ELEMENTS AND EPHEMERIS OF COMET *a* 1925 (SHAJN-COMAS SOLÁ)

BY

HOWARD B. KASTER AND KATHERINE PRESCOTT

Comet a 1925 was discovered at Simeis by G. Shajn on March 22 and also by J. Comas Sola at Barcelona on March 23. The first accurate observation, received by telegram, was secured by Schorr at Bergedorf on March 23.95 Gr. C. T. Early observers described the comet as being of the 11th to 12th magnitude, round, with a coma of 1′.5 diameter. Jeffers of the Lick Observatory and others have observed a very faint nucleus. A first preliminary parabolic orbit was based on the following observations:

1925 Gr. C. T.	α (1925.0)	δ (1925.0)	Observer
Mar. 23.9470	$11^{h}\ 47^{m}\ 47^{s}.8$	+1° 43′ 57″	Schorr, Bergedorf
Mar. 26.2586	11 43 38.3	+1 55 14	Jeffers, Lick
Mar. 28.2715	11 39 59.7	+2 04 58	Van Biesbroeck, Yerkes

ELEMENTS I

T	1925 Sept. 17.37 Gr. C. T.	
ω	207° 37′.7	
Ω	357 29.8	1925.0
i	146 27.8	
log q	0.61719	

O—C	I	III
$\cos\delta\,\Delta\alpha$	−16″	−14″
$\Delta\delta$	− 1	00

As there was some doubt whether the first observation represented a mean or an apparent place no attempt was made to remove these residuals, the order of the reduction from one to the other being comparable with the residuals in right ascension and to a lesser degree with those in declination. It has since been announced that the first observation, which in accordance with international understanding was used in the orbit computation as an apparent place, was given for the beginning of the year, so that the actual residuals corresponding to the first preliminary orbit are negligible, provided the third observation is also a mean place which seems probable on account of a difference of designation of some Yerkes positions received by letter and by telegram.

The problem of determining the orbit from the first available positions proved to be one of unusual theoretical interest. The motion was fairly slow and practically uniform, in a great circle which passed nearly through the Sun. Such a condition makes the method of Olbers inapplicable. The unusual discrepancies between various published orbits may be due to the fact that computers accustomed to employ older methods adhered to that of Olbers instead of using that of Von Oppolzer which would have yielded far more accurate results. The ratio of accuracy in this case of the method of Olbers to that of Oppolzer is only .24. The first preliminary orbit given above was computed by Leuschner's parabolic method. On account of the uniformity of the motion the accelerations in a and δ became of the order of accelerations depending solely on the probable errors of observation and the critical determinant would have become indeterminate if Professor Leuschner had not recognized this condition at the outset and advised us to neglect the accelerations altogether. On account of this method of solution the resulting first preliminary orbit, which was obtained without difficulty from a direct solution, has proved to be one of unusual accuracy under the existing unfavorable conditions. The orbit and a short ephemeris are published in *Harvard College Observatory Bulletin* **816**.

A second preliminary parabolic orbit was based on the following observations:

1925 Gr. C. T.	α (1925.0)	δ (1925.0)	Observer
Mar. 25.9877	11ʰ 44ᵐ 07ˢ.3	+1° 53′ 54″	Vinter Hansen, Copenhagen
Apr. 3.1835	11 29 31.24	+2 32 40.5	Van Biesbroeck
Apr. 24.35021	10 56 00.84	+3 51 25.0	Jeffers

Elements II

T	1925, Sept. 6.3291 Gr. C. T.	
ω	205° 38′ 30″.5	
Ω	357 29 24.6	1925.0
i	146 42 00.8	
log q	0.621417	

Constants for the Equator 1925.0

$$x = r\,[9.999875] \sin (297^\circ\ 44'\ 23''.7 + v)$$
$$y = r\,[9.993584] \sin (\ 27\ \ 58\ \ 42.6 + v)$$
$$z = r\,[9.236246] \sin (199\ \ 50\ \ 12.1 + v)$$

O—C	I	III
$\cos \delta\, \Delta\alpha$	−0″.8	+0″.2
$\Delta\delta$	+1.1	+0.0

The elements and a short ephemeris have been distributed by telegram to the Harvard College Observatory and by air mail to several American observatories. An extension of the ephemeris follows:

Ephemeris for Greenwich Civil Midnight

1925	True α	True δ	log Δ	Br.
May 20.0	10ʰ 28ᵐ 24ˢ.5	+4° 29′ 04″	0.6032	0.77
24.0	25 27.4	29 15	0.6107	
May 28.0	22 50.5	28 02	0.6182	0.72
June 1.0	20 32.7	25 28	0.6255	
5.0	18 32.9	21 38	0.6326	0.68
9.0	16 50.1	16 35	0.6396	
13.0	15 23.1	10 22	0.6464	0.64
17.0	14 10.9	4 03 02	0.6529	
21.0	13 12.6	3 54 39	0.6592	0.61
25.0	12 27.2	45 14	0.6652	
June 29.0	11 53.6	34 52	0.6708	0.58
July 3.0	30.6	23 34	0.6762	
7.0	17.5	3 11 23	0.6812	0.55
11.0	13.4	2 58 21	0.6859	
15.0	17.3	44 31	0.6902	0.53
19.0	28.6	29 53	0.6942	
23.0	11 46.5	2 14 29	0.6979	0.52
27.0	12 10.1	1 58 22	0.7011	
July 31.0	10 12 38.6	+1 41 32	0.7040	0.50

The unit of brightness is that of March 25.

Although the comet is diffuse with only a faint stellar nucleus and has been described as difficult to observe, most of the following observations are in good agreement. Some evidently erroneous observations have not been included in the list. Fortunately no orbit computations were based on them. The observation of March 28.27 is bracketed because of uncertainty whether it is an apparent or a mean place. For April 13.18 a satisfactory comparison star was not available. The second preliminary orbit was based on the starred observations. It will be seen that the observation of May 1 by Jeffers taken eight days after the last orbit date is well represented.

Residuals, O—C,

Gr. C. T.	Observer	Δα	Δδ
Mar. 23.95	Schorr, Bergedorf	+0ˢ.2	−2″
*25.99	Vinter Hansen, Copenhagen	0.0	+2
26.16	Bower, Washington	+0.2	+2
26.18	Van Biesbroeck, Yerkes	0.0	+2
26.21	Jeffers, Lick	+0.1	+1
26.26	Jeffers, Lick	+0.1	+1
27.16	Pitman, Swarthmore	−0.1	−2
27.23	Van Biesbroeck, Yerkes	−0.1	0
28.27	Van Biesbroeck, Yerkes	[−1.0	+1]
28.37	Fox, Evanston	+0.8	−3
29.22	Fox, Evanston	0.0	0
Mar. 29.28	Comrie, Evanston	−0.1	−4
Apr. 1.18	Van Biesbroeck, Yerkes	0.0	+1
2.27	Jeffers, Lick	−0.1	−1
*3.18	Van Biesbroeck, Yerkes	0.0	0
13.18	Jeffers, Lick	+0.6	+2
17.17	Howe, Denver	−0.1	+3
17.19	Howe, Denver	+0.4	0
Apr. *24.35	Jeffers, Lick	0.0	0
May 1.26	Jeffers, Lick	−0.03	−0.5
May 1.30	Jeffers, Lick	−0.16	−1.1

The orbit is based on the starred positions.

Berkeley Astronomical Department,

1925, May 22.

ELEMENTS AND EPHEMERIS OF COMET *b* 1925 (REID)

BY

A. D. MAXWELL

Comet b 1925 was discovered by Reid at the Cape of Good Hope on March 24, 1925. The announcement of discovery together with an observation was received by telegraph at the Students' Observatory. No description of the object was given, except that it was described as of the eighth magnitude. The following observations by Jeffers at the Lick Observatory have been used to determine a parabolic orbit:

	1925		α (1925.0)			δ (1925.0)		
(1)	April	6.42333	13^{h}	16^{m}	$39^{s}.92$	−23°	31′	21″.5
		6.44215	13	16	38.42	−23	31	39.7
(2)	April	24.26271	12	51	01.43	−28	03	01.9
		24.27348		51	00.38	−28	03	13.2
(3)	May	2,25790		38	44.42	−29	53	44.1
		2.27211	12	38	43.11	−29	53	55.1

ELEMENTS

T	1925, July 30.0129 Gr. C. T.	
ω	258° 58′ 12″.8	1925.0
Ω	6 08 17.7	
i	27 12 13.5	
log q	0.214989	

THE CONSTANTS FOR THE EQUATOR 1925.0

$$x=r\,[9.999481]\sin(354^\circ\ 26'\ 01''.9+v)$$
$$y=r\,[9.804097]\sin(267\ \ 49\ \ 47.3+v)$$
$$z=r\,[9.887877]\sin(262\ \ 07\ \ 40.4+v)$$

The residuals were concentrated, according to our usual custom, into the declination of the first observation, and are as follows:

O—C	I	III
cos δ Δα	0″	0″
Δδ	−15	0

EPHEMERIS FOR GREENWICH CIVIL MIDNIGHT

1925		True α			True δ		log Δ	Br
May	25.0	12^{h}	11^{m}	54^{s}	−34°	24′.8	0.026	1.4
	27.0		10	32	−34	46.8		
	29.0		09	22	−35	08.7		
May	31.0		08	25		30.7		
June	2.0		07	40	−35	52.8	0.031	1.4
	4.0		07	08	−36	15.0		
	6.0		06	49	−36	37.5		
	8.0		06	42	−37	00.1		
	10.0		06	49		23.1	0.037	1.4
	12.0		07	09	−37	46.3		
	14.0		07	42	−38	09.8		
	16.0		08	28		33.8		
	18.0		09	28	−38	58.1	0.045	1.4
	20.0		10	42	−39	22.9		
June	22.0	12	12	09	−39	48.1		
June	24.0	12^{h}	13^{m}	50^{s}	−40°	13′.7		
	26.0		15	44	−40	39.8	0.053	1.4
	28.0		17	52	−41	06.3		
June	30.0		20	14	−41	33.3		
July	2.0		22	50	−42	00.7		
	4.0		25	39		28.5	0.062	1.4
	6.0		28	42	−42	56.7		
	8.0		32	00	−43	25.3		
	10.0		35	32	−43	54.1		
	12.0		39	18	−44	23.2	0.071	1.4
	14.0		43	18	−44	52.4		
	16.0		47	33	−45	21.8		
	18.0		52	04	−45	51.3		
	20.0	12	56	49	−46	20.8	0.080	1.4
	22.0	13	01	50	−46	50.3		
	24.0		07	06	−47	19.6		
	26.0		12	38	−47	48.7		
	28.0		18	26	−48	17.5	0.090	1.3
July	30.0		24	30	−48	45.9		
Aug.	1.0		30	49	−49	13.8		
	3.0		37	25	−49	41.0		
	5.0		44	16	−50	07.4	0.100	1.2
	7.0		51	23		33.0		
	9.0	13	58	46	−50	57.6		
	11.0	14	06	25	−51	20.9		
	13.0		14	19	−51	43.0	0.111	1.2
	15.0		22	27	−52	03.7		
	17.0		30	50		22.9		
	19.0		39	27		40.4		
	21.0		48	17	−52	56.1	0.124	1.1
	23.0	14	57	19	−53	10.0		
	25.0	15	06	31		21.8		
	27.0		15	54		31.5		
	29.0		25	25		39.1	0.139	1.0
Aug.	31.0		35	04		44.4		
Sept.	2.0		44	48		47.3		
	4.0	15	54	37		47.9		
Sept.	6.0	16	04	28	−53	46.1	0.155	0.9

An observation by Jeffers on May 26 is represented as follows:

O—C   Δα = +1ˢ.6   Δδ = +7″

The last ephemeris place was also computed for the following distribution of residuals for a parabolic orbit:

O—C	I	III
cos δ Δα	−1″	+2″
Δδ	−8	−3

This orbit would require the following correction to the ephemeris:

	Δα	Δδ
1925, Sept. 6.0	+15ˢ	+0′.1

Several orbits have been computed for this object, as observations became available in increasing numbers.

The first attempt was made from three observations, all erroneous, on March 24.90, 28.29, and 29.19. Both Leuschner's and Olbers' parabolic methods, as well as Leuschner's General Orbit Method, failed to give a solution from these three observations. A second orbit was based on observations taken on March 24.90, 28.29, and April 3.23, of which the first two have proved to be erroneous. Assuming the observations to be of reasonable accuracy, it was found impossible to represent them by a parabola. An elliptical orbit with a period fifteen years was derived, and this orbit was distributed by air mail to various American observatories. (*Harvard College Observatory Bulletin* **817**.) The failure of this orbit to represent subsequent observations led to the derivation of an orbit with a period of 81 years from observations on March 24.90, April 3.23, and April 7.45, of which the first is erroneous. (*Harvard College Observatory Bulletin* **818**.)

Finally, the receipt of later observations from the Lick Observatory made possible the derivation of the foregoing orbit and the identification of the erroneous observations, on which the previous orbits had been based. The discovery observation of March 24 was used in all of the earlier orbit determinations, and consistently considered as an apparent place, according to the usual understanding in regard to telegraphic positions. It is more accurately represented by the final orbit, however, if it be taken as a mean place.

Further details in regard to the computation of the orbit of this comet may be found in the June number of the *Publications of the Astronomical Society of the Pacific*.

BERKELEY ASTRONOMICAL DEPARTMENT,
1925, May 22.

ELEMENTS AND EPHEMERIS OF COMET *c* 1925 (ORKISZ)

BY

J. D. SHEA AND A. P. ALEXEYEVSKY

Comet c 1925 was discovered on April 3, 1925, by Lucien Orkisz at a provisional mountain observatory located on Mount Lysina in the Beskid range, near Cracow. The first accurate observation of the comet, received by telegram, was secured by Moeller at Copenhagen on April 5.12 Gr. C. T. Jeffers describes the comet as pretty well condensed with no visible nucleus. Its magnitude was variously given from 7 to 9. Some observers described the coma as round and compact with a trace of a tail.

A first parabolic orbit was derived from the following positions by means of a direct solution by Leuschner's method without differential correction:

1925, Gr. C. T.	α (1925.0)	δ (1925.0)	Observer
April 5.1199	22ʰ 26ᵐ 46ˢ.1	+16° 37′ 28″.0	Moeller, Copenhagen
April 6.5367	22 28 09.17	18 06 04.4	Jeffers, Lick
April 7.5083*	22 29 07.71	+19 07 49.3	Jeffers

* Mean of two observations.

ELEMENTS I

T	1925, March 28.3924 Gr. C. T.	
ω	31° 4′.0	1925.0
Ω	317 6.9	
i	98 34.7	
log q	0.04581	

O—C	I	III
cos δ Δα	+5″.2	+0″.8
Δδ	+1.1	+0.2

These elements and a short ephemeris were distributed as usual. (*Harvard College Observatory Bulletin* **818**.)

The selection of the foregoing observations for the computation of the first preliminary orbit was based on a comparison of the first four observations, which revealed an eastern telegraphic position of April 7.37 to be in error. Two subsequent eastern telegraphic positions on April 8.46 and on April 11.43 were also found to be in error.

A second preliminary parabolic orbit was based on the following observations:

1925 Gr. C. T.	α (1925.0)	δ (1925.0)	Observer
April 6.53669	22ʰ 28ᵐ 9ˢ.17	+18° 6′ 4″.4	Jeffers, Lick
April 12.52207	34 30.96	24 41 31.9	Jeffers
April 24.46390	22 50 45.45	+39 30 39.1	Jeffers

This orbit was obtained by differentially correcting the first preliminary orbit.

ELEMENTS II

T	1925, April 1.38850 Gr. C. T.	
ω	36° 2′ 25″.9	1925.0
Ω	318 1 20.0	
i	99 59 2.0	
log q	0.044952	

O—C	I	III
cos δ Δα	−0″.8	+0″.2
Δδ	+7.9	−0.4

CONSTANTS FOR THE EQUATOR 1925.0

$x = r\,[9.876444]\sin(134°\ 54'\ 22''.7 + v)$

$y = r\,[9.901988]\sin(266\ \ 18\ \ 02.7 + v)$

$z = r\,[9.950769]\sin(\ 18\ \ 41\ \ 48.6 + v)$

The following continuation of the ephemeris published in *Harvard College Observatory Bulletin 818*, has been computed by Mr. A. D. Maxwell:

EPHEMERIS FOR GREENWICH CIVIL MIDNIGHT

1925 Gr. C. T.	True α	True δ	log Δ	Br.
June 1.0	4ʰ 42ᵐ 00ˢ	+82° 16′.3	0.225	0.55
3.0	5 46 14	81 59.9		
5.0	6 40 50	81 13.4	0.234	0.49
7.0	7 23 37	80 06.7		
9.0	7 56 14	78 48.2	0.256	0.44
11.0	8 21 11	77 23.4		
13.0	8 40 39	75 56.0	0.273	0.36
15.0	8 56 11	74 28.0		
17.0	9 08 52	73 00.8	0.289	0.33
19.0	19 27	71 35.1		
21.0	28 26	70 11.3	0.306	0.29
23.0	36 13	68 49.9		
25.0	43 02	67 30.9	0.322	0.25
27.0	49 06	66 14.4		
29.0	54 33	65 00.4	0.338	0.23
July 1.0	9 59 31	63 48.9		
3.0	10 04 04	62 39.8	0.354	0.20
5.0	08 16	61 33.2		
7.0	12 11	60 28.9	0.369	0.18
9.0	15 51	59 26.8		
11.0	19 18	58 26.9	0.384	0.16
13.0	22 34	57 29.0		
15.0	25 41	56 33.2	0.398	0.15
17.0	28 40	55 39.3		
19.0	31 31	54 47.3	0.411	0.13
21.0	34 16	53 57.0		
July 23.0	10 36 55	+53 08.5	0.423	0.12

The unit of brightness is that of April 12. The residuals of the mean of two subsequent observations by Jeffers at the Lick Observatory are:

O—C	April 28.48
$\cos\delta\,\Delta\alpha$	$+1''.2$
$\Delta\delta$	$+13.2$

The residuals for the two rejected observations are:

O—C	$\Delta\alpha$	$\Delta\delta$
April 7.37	$+0'.6$	$+2'.7$
11.43	$+5.0$	$+0.0$

Attention has been called by Banachiewicz and Lundmark to the resemblance of the elements of this comet to those of the comet observed in the year 1500.

Berkeley Astronomical Department,

1925, May 23.

SECOND ELEMENTS OF COMET *c* 1919 (METCALF-BORRELLY)

BY

W. F. MEYER

Comet c 1919 was discovered by Rev. Joel H. Metcalf at Camp Idlewild, South Hero, Vermont, on August 22, in right ascension $11^h\ 05^m$, declination +27°, and independently by Borrelly at Marseilles, France, on August 23.

During its apparition, the comet was visible in a small telescope and had a well defined nucleus. It was under observation from the date of discovery to February 3, 1920.

The following observations were made the basis of a second orbit using as a first approximation the elements computed by the writer (*Lick Obs. Bull.* **10,** 325):

1919 Gr. M. T.	App α			App. δ			Observer
Aug. 24.60802	14^h	06^m	$18^s.98$	+26°	23′	43″.7	Barnard, Yerkes
Aug. 24.62847	14	06	21.43	+26	23	14.6	Barnard
Aug. 24.70990	14	06	27.80	+26	21	19.0	Aitken, Lick
Nov. 11.21715	17	12	56.56	−12	30	09.2	Palisa, Vienna
1920	α	1919.0		δ			
Feb. 3.25124	23	11	24.32	−32	59	04.4	Innes, Union
Feb. 3.26030	23	11	25.90	−32	58	53.8	Worssell, Union

The first three observations were formed into a normal place, the fourth was used as the middle place and the mean of the fifth and sixth formed the third place of the orbit.

With the geocentric distance and the heliocentric velocities derived for the new middle date, from the preliminary orbit referred to above, and the new middle place, the following initial residuals were obtained for the outer places:

O—C	I	III
$\cos\delta\,\Delta\alpha$	+2″.88	−89″.38
$\Delta\delta$	+0.65	0.00

These residuals indicate the close approximation of the preliminary parabolic orbit, which was based on a fifty-nine day arc. By means of Leuschner's closed expressions for ∂f and ∂g a single differential correction gave:

ELEMENTS

T	1919, December 7.31414 Gr. M. T.	
ω	185° 44′ 55″.75	
Ω	120 58 26.02	1919.0
i	46 23 20.05	
$\log q$	0.0473727	

O—C	I	III
$\cos\delta\,\Delta\alpha$	+0″.04	−0″.24
$\Delta\delta$	+0.88	−0.08

CONSTANTS FOR THE EQUATOR 1919.0

$$x=r\,[9.8943027]\sin(46^\circ\ 46'\ 44''.88+v)$$
$$y=r\,[9.9990124]\sin(313\ \ 42\ \ 52.34+v)$$
$$z=r\,[9.7954879]\sin(218\ \ 52\ \ 12.19+v)$$

As a check on the observations used for the orbit and as a test of the parabolic solution, selected observations were represented by means of the elements and constants for the equator, as follows:

RESIDUALS, O—C,

1919 Gr. M. T.	$\Delta\alpha\cos\delta$	$\Delta\delta$	Observer
Aug. 24.60802	−0″.2	+0″.6	Barnard, Yerkes
24.62847	+7.5	+0.9	Barnard
24.7099	−3.5	−3.2	Aitken, Lick
25.3629	+2.8	−0.7	Borrelley, Marseilles
25.6908	+0.5	−1.8	Aitken
26.5768	−4.5	+1.3	Burton, Washington
Aug. 26.6644	−1.5	−1.3	Aitken
Sept. 15.5926	−5.2	+0.3	Wilson, Northfield
16.55254	−1.4	+1.6	Yowell, Cincinnati
16.69606	−6.2	+6.0	Jeffers, Lick
17.64946	−4.4	+4.6	Jeffers
19.6291	−4.4	+5.9	Wilson, Northfield
Sept. 20.5562	+3.6	+3.9	Van Biesbroeck, Yerkes
Nov. 11.21715	−0.1	0.0	Palisa
Nov. 11.49359	−5.7	−1.6	Van Biesbroeck
Dec. 19.26205	−3.4	−3.3	Wood, Union Obs., So. Afr.
19.26205	+4.1	−9.4	Wood
20.25260	−1.7	+2.6	Wood
23.25933	−6.2	−4.3	Wood
Dec. 24.26282	−6.6	−3.2	Wood
1920			
Feb. 3.25124	+4.0	−3.1	Innes, Union
3.26030	−4.5	+2.9	Worssell

The somewhat uniform, though slight, departure of the September and December observations from the ephemeris indicates that the orbit is not strictly parabolic. Professor Leuschner has pointed out that parabolic orbits computed from three places over increasing arcs show a convergence, except of course for the eccentricity, of the elements to limiting values corresponding to the definitive elements of periodic orbits. Inspection of the elements in the order of the lengths of the arcs on which they are based indicates such a convergence in this case. As the determination of the exact departure from a parabola is of little or no practical value, it has not been deemed profitable to attempt further improvement of the orbit.

BERKELEY ASTRONOMICAL DEPARTMENT,
1925, May 26.

Issued June 6, 1925.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 364

ON THE DETERMINATION OF THE SYSTEMATIC ERROR IN THE BOSS PROPER MOTIONS IN DECLINATION

BY

PIET VAN DE KAMP

A few years ago Kapteyn[1] pointed out the possibility of the presence of a large systematic error in the proper motions in declination of Boss's Preliminary General Catalogue; his conclusion was that assuming the error to be proportional to the cosine of the declination, the Boss μ_δ's might need a correction of something like $+0''.013 \cos \delta$.

An error of this amount would have tremendous effects on the observed motions of faint stars, as it might equal or surpass any systematic motions of those stars. Accordingly results obtained from motions of faint stars (fainter say than magnitude 10) on the system of Boss and conclusions drawn from them would lose their significance, and even become illusory for very small motions.

Let us examine[2] the effects of a systematic error $-G \cos \delta$ on the solar motion as derived from the motions of faint stars assumed to be on Boss's system.

The usual (Airy's) formulae connecting the parallactic motion with the rectangular components of the solar motion (neglecting precession corrections) are:

$$X \sin \alpha - Y \cos \alpha = \mu_\alpha \cos \delta \qquad \text{(1)}$$

$$X \cos \alpha \sin \delta + Y \sin \alpha \sin \delta - Z \cos \delta = \mu_\delta \qquad \text{(2)}$$

where X, Y, and Z are the equatorial rectangular components of the solar motion expressed in arc in the usual manner.

To take account of the effect on the solar motion of a systematic error $-G \cos \delta$ in the proper motions in declination (2) must be written

$$X \cos \alpha \sin \delta + Y \sin \alpha \sin \delta - (Z+G) \cos \delta = (\text{obs'd})\ \mu_\delta \qquad \text{(3)}$$

from which it appears that the component Z of the solar motion and the coefficient G of the assumed systematic error are entirely of the same character. Employing polar instead of rectangular coördinates, a solution for the solar motion from stars whose μ_δ's require a correction of the form $G \cos \delta$ will give the following results:

$$\left.\begin{aligned} \tan A &= \frac{Y}{X} && \text{(a)} \\ \tan D &= \frac{Z+G}{\sqrt{X^2+Y^2}} && \text{(b)} \\ \left(\frac{\overline{h}}{\rho}\right)^2 - 2G\left(\frac{\overline{h}}{\rho}\right)\sin D + G^2 &= X^2+Y^2+Z^2 && \text{(c)} \end{aligned}\right\} \qquad \text{(4)}$$

in which A and D are the right ascension and declination of the solar apex and $\left(\frac{\overline{h}}{\rho}\right)$ is the mean secular parallax[3] of the group of stars[4] considered.

It is apparent from (4 b, c) that the neglect of the coefficient G will give a value D which will differ the more from the true value the smaller Z is in comparison with G, and that it will also lead to an erroneous value of $\left(\frac{\bar{h}}{\rho}\right)$.

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[1] On the proper motions of the faint stars and the systematic errors of the Boss fundamental system. *B. A. N.*, **1**, 69, 1922.

[2] See also *B. A. N.*, **1**, 69, 1922, and *B. A. N.*, **1**, 209, 1923; On the determination of the systematic error in Boss's system of proper motions by P. J. van Rhyn and P. van de Kamp.

[3] The secular parallax of a star may be defined as the angular motion of the Sun in a year seen perpendicularly to the direction of the Sun's motion from a point at a distance equal to the distance of the star.

[4] Group of stars must be understood as a collection of stars scattered all over the sky but having certain characteristics in common: a better expression might be "set" of stars.

Let us consider for instance three groups of stars: the 5th, 10th, and 13th magnitude stars respectively, and assume for the moment, that they have the same solar apex: that is, that

$$\frac{Z}{\sqrt{X^2+Y^2}}=\tan D=\text{constant.}$$

If the motions of these stars are on the system of Boss and that system is affected by a systematic error—$G \cos \delta$ in the declination, it is evident that we shall find an increase in the observed declination of the solar apex with increasing magnitude and correspondingly decreasing mean secular parallax. Also the observed mean secular parallax of any one group of stars will be in error; assuming the true apex of the various groups of stars to have a declination of about $+30°$, the observed $\left(\frac{\bar{h}}{\rho}\right)$ will be increased for each group of stars and the increase will be relatively greater the smaller the true value of $\left(\frac{\bar{h}}{\rho}\right)$ is. This is illustrated in fig. 1, in which the coördinates $\sqrt{X^2+Y^2}$ and Z of the solar motion are plotted for the three groups of stars, putting

$$OA=\sqrt{X^2+Y^2}\,,\quad AP=Z\,,\quad OP=\left(\frac{\bar{h}}{\rho}\right),\quad <AOP=D$$

If the Boss μ_δ's are affected by a systematic error $-G \cos \delta$, the component Z of the solar motion will be increased by the amount G, which is represented in the figure by the segment PQ.

$AQ=Z+G=Z'$ is thus the component of the solar motion perpendicular to the equator found from the motions on the system of Boss, affected by the systematic error $-G \cos \delta$. Ignoring this error, the angles AOQ and the quantities OQ will be the declination of the apex and the amount of the mean secular parallax.

The rapid increase of the declination of the apex, affected by the systematic error $-G \cos \delta$ in the Boss μ_δ's, with increasing magnitude and correspondingly decreasing mean secular parallax is illustrated in fig. 2 where the quantities for the three separate groups of stars have been plotted from the same origin.

Such an apparent increase of the declination of the solar apex with increasing magnitude has been found, but need not be interpreted as a variation in systematic motion with apparent magnitude (or distance).

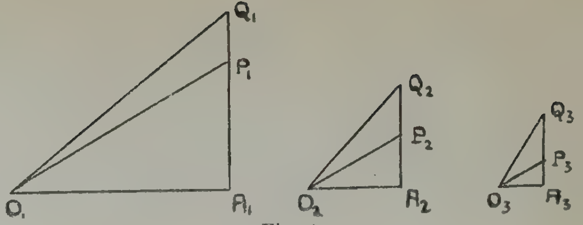

Fig. 1

Charlier[5] considers the change in the declination of the solar apex as real, but in view of the disturbing effect of systematic errors on small proper motions the security of that conclusion is open to question.

Information about systematic errors in proper motions can be obtained by several methods. A rediscussion of the systematic errors in the available star catalogues, for example, may throw some light upon them. Continued meridian observations will, of course, finally tend to decrease them. Radial velocity measures afford an independent and effective means of detecting them.

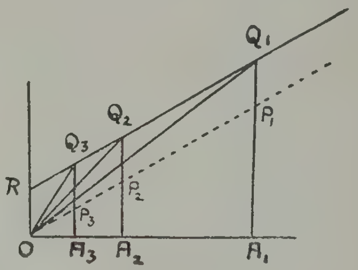

Fig. 2

If observers who have large telescopes at their disposal will observe a few hundred faint stars (of magnitude 10 or fainter, if possible) for radial velocity, a determination of the solar apex with respect to faint stars can be made, which will very probably be free from systematic errors. Such radial velocities of faint stars can be obtained in reasonable exposure times, if a small dispersion and short camera are used. The accuracy of the velocities of the individual stars will not be very great, but great accuracy is not necessary. For the determination of the solar motion, individual velocities act like "cosmical" errors;[6] thus it is not necessary to try to reach such an accuracy of the individual motions as is required in work on spectroscopic binary stars for instance. As the probable cosmical error in one coördinate as a consequence of peculiar motion (and orbital motion in the case of binaries) will be something like ±20 km/sec, it is not necessary for the determination of solar motion to reach an accuracy much greater than that given by the p. e. ±20 km/sec for the individual velocities.

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[5] On the different values obtained for the coördinates of the Apex of the Sun. *Publ. A. S. P.*, **36**, 198, 1924.

[6] Kapteyn, First Report on the Progress of the Plan of Selected Areas, page 12, 1911.

With a large telescope it is possible to get radial velocities of this accuracy in a comparatively short time. They might be affected by a rather large systematic error, but this probably would be mainly of a constant character (K term) and would not affect the solar motion coördinates.[7]

The value for the *position of the solar apex* thus obtained compared with the ones obtained from proper motions, will give definite information as to the size of the supposed systematic errors in the proper motions. The *amount of the solar motion* with respect to faint stars is also of the greatest importance, as it will give us the direct means of converting the mean secular parallax obtained from proper motions of faint stars into mean annual parallax of the same stars.[8] At present this is always done by assuming the solar velocity of about 20 km obtained for the brighter stars. But the solar velocity with respect to stars of faint apparent magnitude may very well be different[9] and the use of the same value of 20 km in reducing the secular parallaxes of different groups of stars to annual parallaxes, may make the latter systematically too small or too large.

The exact value of the mean annual parallax of stars of magnitude 10 or fainter is of great significance in problems concerning the structure of the universe. Moreover the knowledge of this quantity is necessary for establishing an absolute system of trigonometric parallaxes. Relative trigonometric parallaxes are reduced to absolute ones by adding the mean parallax of the comparison stars, which are of mean magnitude 10 for most observatories (of mean magnitude 13 for the Mount Wilson parallaxes). It is thus evident that in order to have our scheme of the structure of the universe and our absolute parallax system as free as possible from systematic errors, it is desirable to know the exact value of the mean annual parallax of the fainter stars, which is only possible if the solar velocity is determined with respect to stars of that magnitude.[10]

Another method of finding the value of G consists in the comparison of results of solar motion as derived from groups of stars having different secular parallaxes, assuming the direction of the solar motion for those groups to be the same.

This method was considered in *B.A.N.* 36[11] and will be described briefly here in a slightly different way.

Let $\left(\frac{\overline{h}}{\rho}\right)_1$, A_1 and D_1 be the mean secular parallax, right ascension, and declination of the solar apex with respect to one group of stars, and $\left(\frac{\overline{h}}{\rho}\right)_2$ A_2 and D_2 those for another group of stars.

A solution for solar motion for those groups of stars of which the proper motions are assumed to be on the system of Boss gives

GROUP I	GROUP II
$X_1=\left(\frac{\overline{h}}{\rho}\right)_1 \cos D_1 \cos A_1$	$X_2=\left(\frac{\overline{h}}{\rho}\right)_2 \cos D_2 \cos A_2$
$Y_1=\left(\frac{\overline{h}}{\rho}\right)_1 \cos D_1 \cos A_1$	$Y_2=\left(\frac{\overline{h}}{\rho}\right)_2 \cos D_2 \sin A_2$
$Z'_1=\left(\frac{\overline{h}}{\rho}\right)_1 \sin D_1+G$	$Z'_2=\left(\frac{\overline{h}}{\rho}\right)_2 \sin D_2+G$

or since

$$\sqrt{X_1^2+Y_1^2}=\left(\frac{\overline{h}}{\rho}\right)_1 \cos D_1 \qquad \sqrt{X_2^2+Y_2^2}=\left(\frac{\overline{h}}{\rho}\right)_1 \cos D_1$$

and calling

$$\sqrt{X^2+Y^2}=p$$

$$Z'=q$$

$$\left(\frac{\overline{h}}{\rho}\right)_1 \cos D_1=p_1 \quad , \quad \left(\frac{\overline{h}}{\rho}\right)_2 \cos D_2=p_2 \ldots\ldots\ldots\ldots\ldots(5)$$

$$\left(\frac{\overline{h}}{\rho}\right)_1 \sin D_1+G=q_1 \quad , \quad \left(\frac{\overline{h}}{\rho}\right)_2 \sin D_2+G=q_2 \ldots\ldots\ldots(6)$$

Assuming the solar apex to be the same for both groups of stars, we find both D and G from equations (5) and (6), since in this case $D_1=D_2=D$, writing them in the form

$$\left.\begin{aligned} G+p_1 \tan D&=q_1 \\ G+p_2 \tan D&=q_2 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots(7)$$

In order to have a good determination of the unknowns we must consider two or more groups of stars of which the secular parallaxes are sufficiently different, and for which we safely can assume the same apex.

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[7] When practicable, greater accuracy is obviously desirable, to add to our knowledge of the individual motions of the fainter stars.

[8] Secular parallax has for its baseline the distance over which the solar system travels in one year (length h). Let $V\odot$ be the Sun's velocity in km/sec with respect to the stars under consideration. Then

$$\frac{\overline{\pi}}{\left(\frac{\overline{h}}{\rho}\right)}=\frac{4.74}{V\odot}$$

[9] See e. g. *Seares*. The Sun's motion and the mean parallax of stars of different apparent magnitudes. *Ap. J.*, **60**, 50; *Contrib. M. W. Obs.* No. 281, 1924.

[10] One might object that the present trigonometric parallaxes are affected by systematic errors of unknown source, so that the exact reduction from relative to absolute parallax would not matter much. Still we ought to do our best to eliminate possible systematic errors of known origin.

[11] On the determination of the systematic error in Boss's system of proper motions. *B. A. N.*, *I*, 209, 1923.

In *B. A. N.* 36 four groups of stars were considered: the stars of spectral classes c, O, and N, and Boss stars between magnitudes 5.45 to 5.80 and galactic latitude 0° and ±20°.[12] The c, O and N stars were used as their secular parallaxes are very small and the uncertainty in the assumption of a common apex would not affect the value of G very seriously. The resulting value for G was $+0''.0064 \pm 0''.0013$.

In the same paper it was advised to use as soon as possible two groups of stars for which observations in the near future would enable us to give a determination of the solar motion, and consequently two equations of the form of (7), and for which the assumption of a common apex could be fairly safely made, or for which a small difference in apex would not affect G appreciably. These two groups of stars are the 10th and 13th magnitude stars used as comparison stars in parallax regions of which the motions on the system of Boss are found by referring them to the parallax star if it happens to be a Boss star.

Parallax plates taken before, say 1920, and repeated now will furnish us with fairly accurate values (probable error of about $\pm 0''.005$ in each coördinate) for the mean motion of the faint stars occurring in such a region, and this can be reduced to absolute motion by means of the central star. As most parallax stars are Boss stars and, by means of the rotating sector, have been reduced to the mean magnitude of the faint stars occurring on the plates, we thus have a very good means of finding accurate values for the mean parallactic motion of faint stars in different parts of the sky.

A solution for solar motion for 10th magnitude stars has been made at the McCormick Observatory,[13] where the proper motions in right ascension and declination of nearly 2000 10th magnitude stars in 287 regions well distributed over the sky north of −30° declination were measured. This solution was combined with one made from motions in right ascension of about 3000 10th magnitude comparison stars obtained as a "byproduct" from the parallax solutions at the Allegheny, McCormick, Sproul, and Yerkes Observatory.

At the thirty-second meeting of the American Astronomical Society, J. A. Oort and Hannah M. Marsh presented a paper on the proper motions of stars of the thirteenth magnitude, which Dr. van Maanen had put at their disposal;[14] their results being derived from the measurement of the proper motions in right ascension and declination of about 300 13th magnitude stars on Mt. Wilson parallax plates.[15] Assuming the coördinates of the solar apex with respect to those faint stars to be 18^h and +30°, they find an average correction of $+0''.0054$ to the northern Boss proper motions in declination.

In order to make their results directly comparable with those obtained at the McCormick Observatory, a solution of the Mount Wilson proper motions was made similar to the one used for the McCormick motions. The main difference between this method and that adopted by Oort and Mrs. Marsh is that the right ascension of the apex was considered unknown and to be determined from the proper motions.

It is well known that on account of the different distribution of stars of different spectral types over the sky, the mean (secular or annual) parallax of stars of a certain apparent magnitude for all spectral classes together depends upon the galactic latitude. In *A. J.* 843 the dependence of the mean secular parallax of 10th magnitude stars on the galactic latitude was derived. It was found that its value is about twice as large near the galactic poles as in the galactic plane. If for magnitudes 10 to 13 we assume the proportion of the numbers of stars between galactic latitudes 0° and ±20°, ±20° and ±40°, ±40° and ±90° to be about 4 : 2 : 1,[16] we find, assuming the above dependence on the galactic latitude, that the weighted mean parallax all over the sky is about equal to the mean parallax at ±30° galactic latitude.

Thus in what follows we shall reduce all mean parallaxes to galactic latitude ±30°. This procedure involves multiplying G by a factor the value of which may vary from about 0.8 to 1.6 in different parts of the sky, but the final value of G will not be changed to any appreciable extent.

The dependence of the secular parallax on galactic latitude of 13th magnitude stars was assumed to be the same as for 10th magnitude stars, and the quantities Q_2 given in *A. J.* 843 Table II were used in order to reduce all proper motions to galactic latitude 30°. The 39 available regions, in which mean motions of the comparison stars were known, were combined into 17 areas, each of them extending on the average less than one hour and at most two hours in right ascension, and less than 10° and at most 20° in declination. All areas were given equal weight. The mean motions of the 13th magnitude stars were on the Boss system corrected for precession corrections.[17]

Additional material for determining X and Y was furnished by the motions in right ascension obtained as a "byproduct" in the determination of stellar parallaxes by Dr. van Maanen with the 60″ reflector.[18] Omitting the regions for which the larger interval measurements had given much better proper motions, and those for which the probable error of the relative proper motion exceeded $\pm 0''.015$, 66 regions were available, distributed over 24 areas for the determination of X and Y.

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[12] This is the faintest and remotest group of Boss stars dealt with in *Gron. Publ.*, **29**.

[13] The Solar motion derived from the proper motions of tenth magnitude stars by H. L. Alden and P. van de Kamp, *A. J.*, **36**, 17; *Pop. Astr.*, **32**, 542; *Publ. Amer. Astr. Soc.*, 32nd meeting, 160, 1924.

[14] On the proper motions of stars of the thirteenth magnitude. *Pop. Astr.*, **32**, 559; *Publ. Amer. Astr. Soc.*, 32nd meeting, 178.

[15] The Mount Wilson regions are practically all north of the equator.

[16] See e. g. *Gron. Publ.*, **30**, Table 19, page 96.

[17] *Boss P. G. C.* Introd. page XXVIII.

The resulting values for X, Y, and Z' are given in Table I.

TABLE I

VALUE OF SOLAR MOTION COÖRDINATES DERIVED FROM MOUNT WILSON PROPER MOTIONS

	X	Y	Z'
From $\mu_\alpha \cos\delta$ and μ_δ in 39 regions (300 stars)	$+0''.0053\pm0''.0020$	$-0''.0044\pm0''.0022$	$+0''.0089\pm0''.0023$
From $\mu_\alpha \cos\delta$ in 66 regions ($\pm$500 stars)	-0.0060 ± 0.0024	-0.0066 ± 0.0023	
Weighted mean	$+0.0008\pm0.0015$	-0.0055 ± 0.0016	$+0''.0089\pm0.0023$

The results for X, Y, Z' and for A, p, and q, derived from the McCormick and Mount Wilson data are given in Table II.

TABLE II

VALUE OF SOLAR MOTION COÖRDINATES

(reduced to galactic latitude 30°)

	McCormick	Mount Wilson
X	$+0''.0018\pm0''.0009$	$+0''.0008\pm0''.0015$
Y	-0.0115 ± 0.0006	-0.0055 ± 0.0016
$q=\left(\frac{\bar{h}}{\rho}\right)\sin D+G\ =\ Z'$	$+0.0115\pm0.0008$	$+0.0089\pm0.0023$
A	$279^\circ.3\pm4^\circ.5$	$279^\circ\pm16^\circ$
$p=\left(\frac{\bar{h}}{\rho}\right)\cos D$	$+0''.0117\pm0''.0006$	$+0''.0055\pm0''.0016$

Substitution of the p and q from Table II in (7) gives

$$G+0''.0117\ \tan D=+0''.0115 \quad\text{............(8a)}$$
$$G+0''.0055\ \tan D=+0''.0089 \quad\text{............(8b)}$$

from which

$$G=+0''.0065\pm0''.0045$$
$$D=+23^\circ\pm16^\circ$$

The probable errors show that we cannot draw any definite conclusions about the correction G from this material. This was to be expected. The probable errors of the Mount Wilson quantities are rather large compared with the McCormick ones, which was to be expected from the limited data available. After a few years when Dr. van Maanen will have completed his program, the accuracy of the solar coördinates will be much greater and a better determination of G and D from intercomparison of solar coördinates with respect to 10th and 13th magnitude stars will be possible. Besides this, results from other observatories may be available by that time.

If we assume the solar apex to be the same for the brighter stars as for the 10th and 13th magnitude stars, we have an additional equation for determining G and $\tan D$:

$$G+0''.0330\ \tan D=+0''.0212 \quad\text{............(8c)}$$

This equation holds for the same group of Boss stars as was used in *B. A. N.* 36, viz.: between visual magnitude $5^{m}.45$ and $5^{m}.8$ and galactic latitude $0^\circ\pm20^\circ$, for which

$$p=+0''.0330\pm0.0027$$
$$q=+0''.0212\pm0''.0020$$

This group was used, as it is the group of Boss stars with the smallest mean secular parallax, and thus most nearly comparable with the other two groups.

Weighting the equations (8): 1, 6, 1 somewhat in accordance with the accuracy of the quantities p and q involved, we find

$$G=+0''.0061\pm0''.0014$$
$$D=+25^\circ\pm5^\circ$$

This solution is equivalent to drawing a line through the points Q (see fig. 2) for the segment OR of the q axis evidently is equal to the quantity G. This graphical solution is illustrated in fig. 3, where the probable errors of each point Q are indicated by circles whose radii equal the mean of the probable errors of p and q for the particular point. The deviations of the points from the straight line are much smaller than could be expected from the size of the probable errors, and may be considered to give additional support to the assumptions of a common apex and of a systematic error of the form $-G\cos\delta$.

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[18] *M. W. Contrib.* Nos. 111, 136, 158, 182, 204, 237, 270.

[19] Probable errors are used throughout this paper.

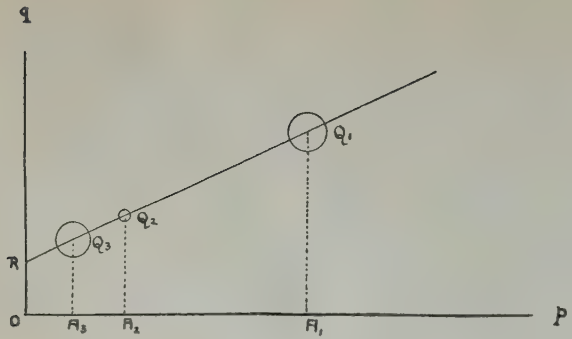

Fig. 3

The value of G agrees very well with the one obtained in *B. A. N.* 36, viz:

$$G = +0''.0064 \pm 0''.0013$$

Thus, on the assumption of a common apex of the solar motion for stars having different secular parallaxes we find that proper motions in declination on the Boss system need a correction of about $+0''.006 \cos\delta$.

This value is in good agreement with the results derived from data published in Boss's P. G. C. by *Varnum*[20] who gives $+0''.0059 \cos\delta$ as the correction to the Boss μ_δ's and also with the amount $+0''.0042 \cos\delta$ found by Raymond and Wilson[21] from a comparison of proper-motion and radial-velocity declinations of the solar apex, the solution being based on a homogeneous list of 2962 stars for which both Boss's proper motions and radial velocities are available.

Table III gives a summary of the various determinations of G.

TABLE III

VALUES OF THE COEFFICIENT G OF THE SYSTEMATIC CORRECTION $G \cos\delta$ TO BOSS'S μ_δ'S

Reference	G
B. A. N. No. 36 (1,209)	+0″.0064 ± 0″.0013
Varnum	+0″.0059
Raymond and Wilson	+0″.0042
This paper	+0″.0061 ± 0″.0014
Mean	$+0''.0056^5$

It thus seems that Kapteyn's estimate of $G = +0''.013 \cos\delta$ is too high. Still an error of about $-0''.006 \cos\delta$ has its effects. Table IV shows the declination of the solar apex for the groups of stars used (1) ignoring the existence of a systematic error, and (2) applying a systematic correction +0″.006 to the μ_δ's.

TABLE IV

VALUE OF THE DECLINATION OF THE SOLAR APEX FOR STARS OF DIFFERENT MAGNITUDE DERIVED FROM UNCORRECTED AND CORRECTED PROPER MOTIONS ON THE SYSTEM OF BOSS

Group of Stars		*Declination Apex*	
		Uncorrected	Corrected
Boss	vis. mag. $5^m.7$	+33° ± 3°	+25 ± 3°
McCormick	vis. mag. 10^m	+45° ± 3°	+25 ± 4°
(Mt. Wilson	vis. mag. 13^m	+58° ± 10°	+28 ± 20)

Proper motions do not enable us to separate the two phenomena: *systematic motions* (Z) and *systematic errors in the motion* (G) unless we make some kind of an assumption. The assumption of common apex for bright and faint stars appears to be rather unavoidable, if we consider the results obtained by Varnum, Raymond and Wilson, and the view of many authorities[22] that the declination of the solar motion for more distant stars should be much higher, is not supported. A small change in the declination of the apex may be real too, of course, but cannot be established from the proper motions now available, which may be affected by systematic errors to such a serious extent.

Altogether it seems advisable for the present to assume that the Boss μ_δ's need a correction of about $+0''.006 \cos\delta$, with a probable error of about ±0″.0006 for the numerical coefficient, and that the declination of the solar apex does not depend on the apparent magnitude (down to about 13^m) and is approximately $D = +30°$.

Mean Parallax of faint stars

Assuming $D = +30°$ and $G = +0''.006$ we find from the quantities given in Table II the mean secular parallax for 10th and 13th magnitude stars:

TABLE V

VALUE OF THE MEAN SECULAR PARALLAX $\left(\frac{\bar{h}}{\bar{\rho}}\right)$ FOR GALACTIC LATITUDE 30° FOR STARS OF MAGNITUDE 10 and 13 DERIVED FROM TABLE II ON THE ASSUMPTION

$D = +30°$ and $G = +0''.006$

$\left(\frac{\bar{h}}{\bar{\rho}}\right)$ ╲ Vis. mag.	10	13
from $p = \left(\frac{\bar{h}}{\bar{\rho}}\right) \cos D$	+0″.0135 ± 0″.0007	+0″.0063 ± 0″.0020
from $q = \left(\frac{\bar{h}}{\bar{\rho}}\right) \sin D + G$	+0″.0110 ± 0″.0020	+0″.0058 ± 0″.0048
weighted mean $\left(\frac{\bar{h}}{\bar{\rho}}\right)$	+0″.0132 ± 0″.0007	+0″.0062 ± 0″.0017
Gron. Publ. **29**	+0″.0142	+0″.0055

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[20] *Publ. of the Amer. Astr. Soc.* 30th meeting, page 96, 1923; *Pop. Astr.*, **32**, 34; *A. N.*, **222**, 241, 1924.

[21] *Pop. Astr.*, **33**, 26, 1925.

[22] See e. g. Shapley, *M. W. C.* No. 157, page 25c, 1918; Strömberg, *M. W. C.* No. 245, page 4 and 5, 1922; Charlier, loc. cit. (*Publ. A. S. P.*, **36**, 198, 1924).

In the last row of Table V are given the values for $\left(\frac{\bar{h}}{\rho}\right)$ derived at Groningen.[23] The agreement between the two sets of values is very satisfactory.

In order to convert mean secular parallaxes into annual parallaxes we must know the value of the solar velocity with respect to the groups of stars which we want to consider. At present no determination of the solar velocity with respect to faint stars is available; it is to be hoped (as has been mentioned) that within the near future this obstacle will be overcome. Seares has found indications of an increase of the solar velocity with the apparent magnitude. Still for the present, considering other uncertainties and assumptions, we may as well assume a constant solar velocity of 20 km/sec and thus convert mean secular parallaxes of faint stars into mean annual ones by multiplying the former by $\frac{4.74}{20}=0.237$, as in Table VI.

TABLE VI

VALUE OF THE MEAN ANNUAL PARALLAX $\bar{\pi}$ FOR GALACTIC LATITUDE 30° FOR STARS OF MAGNITUDE 10 AND 13, ASSUMING A SOLAR VELOCITY OF 20 KM/SEC.

Vis. mag.	10	13
$\bar{\pi}$	+0″.0031±0″.0002	+0″.0015±0″.0004

On account of the various assumptions, the probable errors in Tables V and VI should be regarded merely as giving some idea of the order of accuracy of the values of $\left(\frac{\bar{h}}{\rho}\right)$ and $\bar{\pi}$.

In the case of the mean secular parallax, probable errors 50% larger than the ones given should be nearer the truth on account of the assumption of the position of the solar apex. For the mean annual parallax, the additional assumption of the amount of the solar velocity should be accounted for by increasing the tabular probable errors by about 100%.

We thus see that values for mean secular and annual parallax of 10th magnitude stars are pretty well established; this is not yet true for fainter stars.

It may be well to point out that according to these recent results for the mean parallax of 10th magnitude stars, the amount 0″.005 usually applied in order to reduce relative trigonometric parallaxes to absolute ones, probably is too large. Of course there still remains the uncertainty of the solar velocity with respect to 10th magnitude stars. But accepting the value 0″.0134 for their mean secular parallax; the value 0″.005 for their mean annual parallax can be obtained only on the basis of a solar velocity of 13km/sec with respect to the 10th magnitude stars. At present there are no indications for such a small solar velocity; on the contrary Seares finds reasons to assume a larger solar velocity. Altogether the value 0″.003 seems a much better value for the reduction from relative to absolute trigonometric parallaxes, so far as this reduction is due to the comparison stars only, than the value 0″.005 hitherto usually assumed. This would mean that results from recent determinations of systematic errors in the present trigonometric parallaxes may have to be interpreted somewhat differently.

I wish to express my gratitude to Dr. van Maanen for putting his proper motions at my disposal.

SUMMARY

Proper motions of 10th and 13th magnitude stars on the system of Boss derived at the McCormick and Mount Wilson Observatories furnish us with values of the rectangular components of the (angular) solar motion with respect to those stars. Assuming a common solar apex for brighter and fainter stars, values of +30° for the declination and 270° to 280° for the right ascension of the solar apex, seem about the best we can derive at present. This assumption of a common apex is supported by evidence from other sources and all together we are led to believe that the motions in declination of the system of Boss need a correction of about +0″.006 cos δ. Accepting these results, values of the mean secular parallax of 10th and 13th magnitude stars for galactic latitude 30° are obtained, which are 0″.013 and 0″.006 respectively. On the assumption of a solar velocity of 20 km/sec the corresponding values of the mean annual parallax are 0″.003 and 0″.001^5.

April 24, 1925.

Issued July 13, 1925.

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[23] *Gron. Publ.*, **29**. The secular parallax of the stars of different magnitude, galactic latitude and spectrum, by J. C. Kapteyn, P. J. van Rhyn and H. A. Weersma. Table 26, page 55.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 365

THE ORBIT OF KRUEGER 60

BY

R. G. Aitken

The system known as Krueger 60 is one of the most interesting of the visual binary stars because of its large and well determined parallax and proper motion, and of the faintness, both apparent and absolute, and small mass of the two components. Both components are class M dwarfs and have been regarded as stars nearing the final stage of stellar existence.

Burnham made the first measure of the close physical pair in 1890 when observing the wider perspective pair, now known as Krueger 60, AC, and to him is really due the credit for its discovery. The parallax was not then known and there was no indication that the pair would be of greater interest than any other faint pair with an angular separation of more than 2″. Unfortunately, therefore, Burnham measured it on one night only and no further attention was paid to it until Doolittle measured it in 1898 and found a great change both in angle and in distance. Two years later, in 1900, Barnard began his remarkable series of measures of it, which extended through the year 1921. He measured it in every year in that interval except 1914, sometimes on as many as 18 to 21 nights. Burnham measured it in 12 of these years; no other observer has published measures in more than six years. In the last few years the apparent motion has been quite rapid and the observed arc now extends over 224°, which should be sufficient to define the apparent ellipse quite accurately, for the measures, especially those of position angle, are almost uniformly good.

In forming the annual means, weight one was assigned to the mean of from two to four measures by one observer, weight two to the mean of from five to nine measures, and weight three to the mean of ten or more measures. Even when this had been done it was apparent that the resulting curve would have been essentially the same if Barnard's measures alone had been used for the period from 1900 to 1921. Interpolating curves for angle and distance were drawn and the apparent ellipse constructed in the usual manner, to satisfy the law of areas and to represent the observed distances. The elements were then determined by graphical methods, with the following results:

Elements

$P =$ 44.27 years
$T =$ 1925.82
$e =$ 0.38
$a =$ 2″.46
$\omega =$ 171°.0
$i =$ 26.0
$\Omega =$ 113.8
$n =$ −8.1319
App. $m =$ 9.3, 10.8
Abs. $M =$ 11.3, 12.8

Ephemeris

Date	θ	ρ
1925.82	301°.9	1″.52
26.82	284.5	1.54
27.82	267.7	1.58
28.82	251.8	1.64
29.82	237.2	1.71
30.82	223.9	1.81
31.82	212.0	1.92
32.82	201.6	2.04
33.82	192.3	2.17
34.82	184.1	2.31
35.82	176.9	2.44

The table of observations which follows shows the residuals resulting from a comparison with the positions computed from these elements. Their systematic character indicates that improvements could be effected by applying differential corrections to the elements, but since the residuals are not large the corrections to the elements would be small, and it seems better to wait for further data before making a least squares solution, especially since the motion in the next few years will continue to be rapid.

The greatest immediate value of a double star orbit, from the astrophysical point of view, is that, in conjunction with the parallax, it leads to a knowledge of the mass of the system. The parallax of Krueger 60 is +0″256, and this, with the elements here derived, makes the mass of the system 0.45 that of the Sun.

			O—C		
Date	θ_o	ρ_o	$\Delta\theta$	$\Delta\rho$	
1890.79	178°8	2″32	−3°6	−0″02	β1n
98.45	140.7	3.19	−0.8	+0.01	Doo5n
1900.84	133.7	3.22	+1.5	−0.10	Doo4n, Bar4n
01.57	130.9	3.33	+1.4	−0.01	β4n, Bar7n
02.78	126.6	3.36	+1.4	−0.01	β4n, Bar4n, Doo3,0n
03.57	123.5	3.37	+1.2	−0.02	β4n, Bar15n, Doo6n
04.58	119.8	3.36	+1.1	−0.03	β3n, Bar19n
05.60	115.9	3.28	+0.8	−0.10	β6n, Bar2n
06.37	112.7	3.26	−0.5	−0.09	β4n, Bar7n
07.63	107.6	3.27	0.0	−0.04	β4n, Bar12n, v.Bies5,0n
08.53	103.8	3.15	−0.5	−0.12	β5n, Bar12n, v.Bies3n, Lau4n, L.J.3n
09.70	98.6	3.12	−1.1	−0.07	β4n, Bar16n, Lau1n, Sy1n
10.54	95.2	3.12	−1.0	−0.01	β4n, Bar21n
11.65	90.4	3.00	−1.0	−0.02	β4n, Bar10n, v.Bies4n, Doo5n, Fox3n
12.73	86.7	2.86	+0.3	−0.06	Bar11n, v.Bies3n, Abetti3n
13.63	79.8	2.91	−2.1	+0.10	Bar8n, Doo5n
14.02	80.2	2.76	−0.3	0.00	v.Bies3n
15.69	69.2	2.55	−1.1	+0.01	Bar18,17n, Doo7n
16.71	62.4	2.43	−0.9	+0.04	Bar10n, M.&O.4n
17.67	55.5	2.25	−0.5	+0.01	Bar10n, M.&O.9n
18.69	47.8	2.13	+0.7	+0.04	Bar9n, M.&O.9n
19.66	38.7	1.96	+1.4	+0.02	Bar12n, M.&O.5n
20.57	28.2	1.83	+1.5	+0.02	Bar.10,9n, A3n
21.54	16.7	1.69	+2.9	−0.01	Bar9,8n, A4n
23.81	341.6	1.56	+4.1	+0.03	A2n
1924.93	314.6	1.32	−3.1	−0.19	A3.1n

There have been two prior investigations of the orbit, the first by Russell,[1] in 1916, when the observed arc was only 115°, the other by Pavel and Bernewitz,[2] in 1920, when the observed arc was about 140°. Russell gives two solutions, with periods 46.0 years and 54.9 years, and values 2″49 and 2″86 for the semi-major axis. Pavel and Bernewitz give 46.64 years and 2″55. The mass values, with the parallax 0″256, are, respectively,0.44, 0.47 and 0.45 that of the Sun. It is therefore improbable that future investigations will result in any material change in this quantity. Dr. H. L. Alden[3] has recently determined the relative masses of the two components on the basis of photographic measures extending over eight years, taken in conjunction with Barnard's measures of AC. Taking the mass of the system as unity, the mass of the smaller component is 0.455±0.0048. The mass of the larger component is then approximately one-fourth, that of the smaller component approximately one-fifth the mass of the Sun.

In the *Monthly Notices R. A. S.* for March, 1925, Dr. W. M. Smart has a paper on "The Cosmogonic Time Scale and Binary Stars" in which he argues that visual binary systems of periods up to 400 years and masses not less than the Sun's mass might develop from close spectroscopic binary systems of much greater mass within one "cosmogonic unit," this unit being defined as the time required by the Sun to shrink to its present mass from an indefinitely great mass through loss by radiation. Approximately, the unit is 7x10^{12} years. To fall to a mass of 0.25☉ would require at least 16 units of time on this scale, and systems like Krueger 60 must therefore be, regarded, in Dr. Smart's words (in speaking of the dwarf double stars), "as intruders of venerable age formed outside the bounds of the stellar system at a very much more remote time," if the Sun and the majority of the stars had a more or less simultaneous origin.

It is a consequence of the theory developed by Smart, on the basis of Jeans's and Eddington's investigations, that the mass ratio in a binary system must increase with the time. I have been interested to tabulate the mass ratios, according to spectral class, for the systems showing two spectra which are listed by Moore in the "Third Catalogue of Spectroscopic Binary Stars."[4] Rejecting six systems[5] as uncertain or abnormal, we have 68 such binary stars, the data for which are given in the following table:

No.	Class	Av. P.	Av. m/m′	Range	
16	B	5.16 days	0.73	0.39–1.00	
29	A	9.58	0.69	0.17–0.99	
11	F	8.73	0.93	0.51–1.00	
(11	G	17.70	0.88	0.71–1.00)	
10	G	9.06	0.89	0.71–1.00	(Omitting Capella)
1	G5	5.4	0.88	—	
68		9.65	0.77	0.17–1.00	
67		8.25	0.78	0.17–1.00	
45	B–A	8.09	0.704	0.17–1.00	
22	A5–G5	8.73	0.908	0.51–1.00	
(23	A5–G5	12.87	0.903	0.51–1.00)	

If the stars here listed in classes F and G may be assumed to be older than those in classes B and A there is in this table some support for the theory.

May, 1925.

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[1] *Publ. Am. Astron. Soc.* **3**, 277, 1918.

[2] *A. N.* 211, **121**, 1920.

[3] *Pop. Astron.* **33**, 164, 1925.

[4] *L. O. Bull.* **11**, 141, 1924.

[5] These systems are: *φ Persei*, 9 *Camel.*, *ε Hydrae*, *ε Urs. Maj.*, *β Lyrae*, and *κ Pegasi*. Their inclusion would not seriously affect the mass ratios.

THE ORBIT OF β 581

BY

R. G. AITKEN

Recent measures of the binary star β 581 have shown that the motion in this system is more rapid than that predicted by my earlier orbit[1] and that the companion has now completed a revolution since its discovery by Burnham in 1878. A new investigation of the motion has therefore been made which has resulted in the following elements:

ELEMENTS	EPHEMERIS		
	Date	θ	ρ
$P=$ 44.0 years	1926.25	206°.4	0″.40
$T=$ 1909.75	27.25	212.6	0.41
$e=$ 0.39	28.25	218.6	0.42
$a=$ 0″.38	29.25	224.3	0.43
$\omega=$ 292°.3	30.25	229.7	0.44
$i=$ 47.7	31.25	235.0	0.45
$\Omega=$ 116.1	32.25	239.9	0.46
$n=$ +8.18182	1933.25	244.6	0.47

Sproul Observatory obtains for the parallax of the system +0″.073±0″.014; Adams and Joy give the spectroscopic parallax as +″.008. The former, combined with the elements here given, leads to the mass value 0.07⊙; the latter, to the value 55.4⊙. Both are improbable for a star of class G5, and we may venture the assertion that the hypothetic parallax +0″.025, based on an assumed mass of 1.7⊙ is more nearly correct than either the trigonometric or the spectroscopic value. The period and eccentricity in this system are approximately the same as those in the system of Krueger 60, but if the mass here assumed is correct, the linear dimensions of the orbit are greater.

Interpolating curves were drawn for position angles and distances on the basis of all available measures. A few obviously discordant observations were then rejected before forming the annual means on which the orbit computation rests. The representation of these annual means is given in the following table:

				O—C	
				$\Delta\theta$	$\Delta\rho$
1878.17	178°.0	0″.40	β2, Dem1	−1°.0	+0″.04
1883.37	205.2	0.30	Engel5	−8.7	− .11
1886.26	229.6	0.46	HΣ2	−0.4	+ .03
1889.23	249.8	0.44	Sp4	+5.3	− .03
1890.21	253.8	0.50	Sp4	+4.9	+ .02
1891.97	259.4	0.46	β4	+2.9	− .04
1893.24	263.2	0.50	Sp9	+1.6	± .00
1894.09	262.7	0.50	Bar1, HΣ4, Sp4	−2.4	− .01
1895.25	271.9	0.45	Sp3, HΣ1, L1, Com3,0	+2.4	− .06
1896.21	272.8	0.48	L3, Com3	−0.4	− .03
1897.14	275.2	0.50	A3, Com2	−1.1	± .00
1898.17	279.8	0.52	β1, Br3, L1, Com2, Sp1	−1.1	+ .01
1899.23	284.5	0.53	A3, Com2	−0.8	+ .04
1900.15	286.6	0.51	Doo2, Bry3, L2	−2.9	+ .04
1901.03	291.4	0.48	See4, Doo3, A2, Bry3,0	−2.1	+ .03
1902.18	294.0	0.46	Doo5, L3	−5.4	+ .04
1903.17	304.5	0.39	Com3, Bow1, Wz2	−0.7	± .00
1904.13	309.2	0.41	VanBies1	−2.8	+ .05
1906.17	326.0	0.33	A3, Com1	−7.0	+ .05
1908.10	21.3	0.25	Doo1	+10.0	+ .05
1909.27	46.0	0.20	A2	−1.3	− .05
1910.22	69.6	0.25	Bry2, A1	+1.1	+ .04
1911.07	88.5	0.2±	Doo1	+3.2	− .05
1912.27	100.4	0.30	A2, Bry1,0	−1.9	+ .01
1913.02	114.4	0.30	Greenwich2	+3.8	− .02
1914.22	126.2	0.27	Greenwich1	+4.4	− .08
1915.27	129.8	0.30	Rabe4, Com2, Greenw.3	−0.3	− .06
1916.16	135.9	0.40	A2	−1.0	+ .03
1916.94	144.1	0.37	Com2, Dool3,0	+1.5	− .01
1919.29	164.7	0.32	Com2	+5.6	− .06
1921.87	172.2	0.35	A1	−4.3	− .03
1924.17	190.5	0.34	A3	−2.2	− .05

May, 1925.

Issued July 13, 1925.

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[1] *Publ. Lick Obs.* **12**, 53, 1914.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 366

PHOTOGRAPHS OF MARS MADE WITH LIGHT OF DIFFERENT COLORS

BY
W. H. WRIGHT

The observations described in this paper were made during the close approach of *Mars* to the Earth in the fall of 1924. They consist of photographs made by light of a number of different colors. An untechnical account of them, with some tentative interpretations, has already appeared.[1]

The purpose of the present communication is to describe the observations somewhat more fully than was there practicable, though the presentation is still of a preliminary nature, for that is the character of the observations themselves. Although the planet will not then be so close to the Earth, the opposition of 1926 will probably offer to observers in the northern hemisphere a better opportunity for the study of *Mars* than was provided by the last one. The diameters at the two oppositions are 25″ and 20″ respectively, while the declinations are −17°6 and +14°9. The higher declination at the next approach will probably more than offset the disadvantage of a smaller diameter. With this circumstance in view, the observations of 1924 were undertaken as an exploratory measure, largely for the purpose of determining whether the method might be expected to yield data of value. The outcome of the reconnaissance has been favorable for it has, I believe, proved the promise of the method as a means of approach to the study of *Mars*. The aim in this paper is to give account enough of the observations, and to sketch sufficiently some of their implications, to assist anyone who may desire to engage in an investigation of this character in preparing a program against the next approach of the planet; a more critical consideration of the present photographs can probably better be undertaken in conjunction with the more complete investigation which it is hoped will then be possible.

Since the publication of the earlier paper I have had the advantage of criticism and discussion of some of the subjects with which it deals both through correspondence and personal conversation with astronomers. This has been extremely helpful in defining issues and especially in serving to develop items of information which might not otherwise have been brought into the discussion. For this and other reasons it has seemed expedient to regard the present paper as an amplification of the earlier one, and I shall therefore take the liberty of quoting freely from the latter where no special purpose would be served by altering the phrasing.

As already stated, the observations consist of photographs of the planet made by light of different parts of the spectrum. They were secured by the common enough method of exposing plates of suitable sensitivity behind light filters, or "color screens" of selected hue. Pictures taken in this way not infrequently exhibit differences among themselves, and it is sometimes possible through these differences to learn something of the body under observation. The method has had a wide application in astronomical practice, and has been used in the study of such diverse objects as the diffuse nebulae, the spiral nebulae, star clusters, single stars, and the planets. With respect to the latter, yellow and orange screens have long been used in their photographic observation, but not for their physical study; the purpose being to overcome the limitation of refractors which is imposed by chromatic aberration. The only attempts of which I am aware that have been made to secure screened pictures of the planets for analytical purposes are those of Professor R. W. Wood, whose pioneer work is more completely referred to in the earlier paper.

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[1] *Publ. Astron. Soc. Pac.* **36**, 239, 1924.

The photographs were made with the Crossley reflector, on which was mounted a microscope objective that served to magnify the image. The focal length of the lens is 13 mm. and the magnification 5.4 fold. It will assist in our later estimate of the effect of chromatic aberration of the microscope objective on the scale of enlargement if record is made of the conjugate focal lengths. They are approximately 15.4 and 83 mm. The arrangement has proved extremely convenient, except that the glass lens is, of course, not transparent to ultra-violet light, and for wave-length 3600A, which was sometimes used, it was necessary to dispense with enlargement and photograph directly in the focus of the mirror. It is of importance to note that the back focus, that is to say the distance between the enlarging lens and the photographic plate is fixed and not subject to alteration. This insures invariability of the enlarging factor of the camera, and in consequence, uniformity in the scale of the photographs. Adjustment in focus is accomplished by moving the enlarging camera in the axis of collimation.

As the matter of scale is of critical importance, a reference will be made to the effect upon it of chromatic aberration and faulty focal adjustment. Let us assume in the first instance that the enlarging lens is without achromatism but that there is a slight maladjustment in focus. It is convenient to regard the image formed on the photographic plate as an enlargement of the image in its conjugate plane, which plane is, under our assumption, slightly displaced from the focal surface of the mirror. If we consider the case of a double star, it is easy to see that the separation of the centers of the images in the plane referred to will be proportional to its distance from the mirror. But the variation in that distance is, in practice, so small that the effect on the linear separation of the stars will be negligible. Accordingly the sole effect of a maladjustment in focus, in the photography of a double star with a telescope and enlarging apparatus such as that used here, will be to enlarge the stellar images; the separation will not be altered appreciably. In the case of a planetary disk there will be some enlargement due to blurring of the edge but if the measuring wires are set, not at the farthest visible edge of the disk but on a peripheral contour of medium density, the effect of focal maladjustment on the measured diameter should be small.

A more serious difficulty is presented by chromatic aberration of the enlarging lens which we shall now assume to exist. If F be the focal length of the mirror or primary objective, f_1 the front conjugate focus of the enlarging lens, and f_2 the back focus, then:

$$\text{Scale} \propto \frac{F f_2}{f_1} \qquad (1)$$

On account of the shortness of f_1 (15 mm. in the present instance) a small change with color, in amount too little to be detected in a focal test of the instrument as a whole, might seriously affect the magnifying factor as between two of the colors used in photography. The procedure obviously indicated as a guard against error is to photograph a double star in the several colors it is proposed to use and to derive the scale factor for each color separately. Photographs of two double stars, θ *Serpentis* and 61 *Cygni* have been made with our apparatus by infra-red and by violet light and the magnification is, within the limits of accuracy of determination, the same for each.

The color filters are the commercial Wratten filters of colored gelatine cemented between glass plates of medium optical quality (grade B). In planning future work it would probably be well to provide filters of a better grade of optical glass. Irregularities of composition or of surface might produce a slight alteration in the effective focal length of the microscope and thus change the scale of the photograph, though the definition would not necessarily be injured. The filters are placed between the focal plane of the telescope and the enlarging lens, the first glass surface being 1 cm. behind the focus. The insertion of the screen or filter alters the focal setting but should not affect the magnification, provided, of course, that the sides are plane, parallel, and the material of the screen is homogeneous. The filters used in the present work are sensibly of the same thickness (approximately 4½ mm.) and may therefore be substituted for one another without altering the focal adjustment. In making the ordinary or "violet" photographs a filter is, of course, not necessary, but it is convenient to use one of clear glass having the thickness of the others in order to obviate the necessity of altering the focal setting, and that practice has usually been followed.

Scale of photographs.—As stated above the scale was determined by photographing the double stars θ *Serpentis* and 61 *Cygni* which have separations closely approximating that of *Mars* at opposition. Only one of the photographs was made while the observations were in progress and two have been made since. It would have been advantageous to have numerous check plates for scale made during the progress of the work, but the photographs were taken without thought that they would be subject to exact measurement. With a fixed back focus in the enlarging camera there does not, however, seem to be any way in which the magnification could suffer alteration, and there appears to be no reason to doubt that the scale value as determined is applicable to the observations.

Measurements made with the apparatus depend, of course, on the separations assumed for the double stars. They are as follows:

Star	Mean Date	Separation
θ Serpentis	1925.04	22″.15
61 *Cygni*	1925.36	24.45

In outlining the program of observation, it was at first proposed to use five different regions of the spectrum, the approximate wave-lengths of which are given in Column 1 of Table I. The second and third columns of the table record the plates and filters used to isolate these parts of the spectrum. The fourth position was never actually used, and the first on only a few occasions. The photographs taken with it were very similar to those made on Lantern Slide plates without use of a filter, though they bring out the "cloud forms" better. They were, however, somewhat troublesome to make, since the mounting had to be changed whenever this region was used, and it was finally discontinued. When observing *Mars* again this region will be employed systematically.

TABLE I

Wave-length	Plate	Filter (Wratten No.)
3600 A	Lantern Slide	18
4400	Lantern Slide	0 (clear glass)
5600	Pinaverdol Bathed	16
6600	Pinacyanol Bathed	29 (F)
7600	Kryptocyanin	70

Observations were occasionally made at 5600 A, but not regularly. The photographs taken there were found to give representations intermediate between those at 4400 A and 7600 A and they consequently provided little information of their own; furthermore, the region was kept under regular photographic observation by Dr. Trumpler with the 36-inch refractor. The two regions of which principal use was made were, therefore, that ordinarily employed in photography the center of which is assumed to be 4400 A and a comparatively narrow range of spectrum in the vicinity of 7600 A. The discussion of results rests very largely on photographs made in these two parts of the spectrum, but deductions are checked on the photographs taken in the yellow (5600 A) and on the appearance of the planet in the telescope. I think that in planning observations such as these for another occasion it would be well, if it could conveniently be done, to enlarge the image by a mirror rather than by a lens. The ultra-violet plates could then be taken without difficulty and absolute freedom from the effects of chromatic aberration would be assured.

The plates used for wave-length 7600 A were especially made by the Eastman Kodak Company for this work. The sensitizing agent was incorporated in the emulsion during manufacture. The plates have a narrow region of great sensitivity at about 7600 A. It is perhaps worth while to point out that the strong band A, due to oxygen in the Earth's atmosphere, lies within the region of sensitivity of these plates, and it undoubtedly reduces the amount of active light that reaches them. As no increase in the intensity of the oxygen bands has been observed in the spectrum of *Mars*, it is assumed that, in the photography of that planet, the effect of the band is only to cause the terrestrial atmosphere to act as a color filter. There appears to be no reason to fear that the character of the Martian image is altered by the occurrence of the band within the region of sensitivity, but it might nevertheless be advisable in future observations to secure check plates, if practicable, with light of a neighboring part of the spectrum. The plates have a very high contrast factor and for securing comparable photographs in the violet region "Lantern Slide" plates were generally used.

The following is a digest of the results of observation:

1. Infra-red light records the permanent surface markings with high contrast; yellow light renders them with contrast much reduced; while in the images made by violet light all contrast relating to permanent markings is lost.[2] (Plate II, figures 10, 11, 12.)

2. The above comment does not refer to the south polar cap and certain impermanent configurations found only on the violet photographs, notably a bright area recorded at times in the neighborhood of the north pole, and an irregular light that sometimes outlines the limb. There are, furthermore, certain streaks and mottlings which, it is hoped, will be discernible in some of the half-tone reproductions. The polar cap becomes larger and stronger, and the other configurations just referred to develop as the wave-length of the light used in recording them approaches the violet end of the spectrum. (Plate I, figures 5, 6, 7.) It may be noted that on the infra-red photographs the polar cap is not brighter than some other parts of the planet.

The several varieties of impermanent configuration referred to are illustrated in the accompanying cuts. The "mottlings" are irregular areas of brightness and are to be found on any part of the disk (Plate I, figures 5, 6), but more commonly near the edge, where they sometimes merge with the limb light. They come and go from the pictured surface of the planet. It has been asked whether any one mottling or streak is visible on more than one photograph and positive assurance can be given that such is the case. Successive exposures were made on each night of observation; it is, accordingly, an easy matter to detect a peculiarity of a single observation. By way of illustration, figures 1 and 2 of Plate II were made on separate plates. They show the equatorial band and the light on the right limb. A still more convincing proof of reality is supplied in figures 5 and 6 of Plate I. Figure 6 was made in the enlarging camera attached to the telescope while figure 5 was photographed in the primary focus using the ultra-violet screen, and was subsequently magnified in copying to approximate equality with figure 6. These two photographs, taken under very different conditions, display the same general features, especially the bright blotch to the right of the northern (lower) cap. Differences in contrast, as well as pictorial discrepancies, are to be laid in part to difference in color of the light used in photographing, but in several respects to changes occurring during the interval of an hour between the times of observation; changes which can readily be followed on other plates taken during the course of the evening. The impermanency of the spots or mottlings is further shown by the comparison of plates taken on different nights and is quite as certainly established as their actual existence. They appear to favor locations over dark markings.

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[2] In abstracts and reviews of the original paper it has been implied that the surface markings occur in the violet photographs in *reduced intensity*. So far as I have been able to ascertain from a close examination of the photographs there is no observable trace of the well-known markings, though other configurations referred to in following paragraphs are easily discernible.

Streaks.—A typical streak is shown on the photograph of September 21 (Plate II, figure 6) where it crosses the middle of the disk in the direction upward to the left. It causes a brightening in the limb light where it passes over the edge, presumably because it is there viewed ''end on.'' Something of the same sort is shown on August 21 (Plate II, figure 1) when the planet presented approximately the same face to the Earth, and its appearance there suggests it to be the remnant of a great "cloud" on the rising (right hand) limb which is carried across undispersed by sunlight through the Martian day. I can find no trace of it on the plate of October 31 where the surface presentation is approximately the same but something very like it occurs on a plate of December 1 taken by Professor Pease with one of the Mt. Wilson telescopes and kindly sent to me. On that photograph it lies a little farther south than on the earlier ones. The conclusion drawn is that during the period of the observations there was a more or less persistent streak of variable position and width hovering slightly north of the edge of the dark area known in its parts as the *Mare Sirenum* and the *Mare Cimmerium*. On August 21 when it could be followed to the eastern or oncoming limb the streak appeared to terminate in an extensive white area there, but the fact may of course be without significance. The object was not far from the planet's equator.

Rim light.—The rim light appears to be of two kinds, characteristic respectively of the rising and the setting limbs. When the planet was near opposition I could find no rim light on the edge on which the Sun was setting, but there was invariably a bordering light on the rising edge. This light showed a tendency toward discontinuity and was apparently associated with the mottlings. As phase developed, and the rising terminator encroached upon the face of the disk, the light became less pronounced, disappearing at times, though something of the sort is visible on Pease's photograph of December 1. On the other hand, the rim light of the setting, or western limb, not marked near opposition, developed with advancing phase (Plate II, figure 6), and eventually became the most striking feature of the violet images (Plate II, figure 13, November 16). It is strong on Pease's photograph of December 1.

3. The infra-red images show a falling off in intensity as the edge of the planet is approached. This is especially marked in the photographs taken close to opposition. The effect is hardly sensible in the photographs made with yellow light, and is altogether absent from the violet images which, except for the polar caps and occasional impermanent markings noted under (2), present a surface of monotonous uniformity. After opposition, with the development of phase, the edge-fading shown in the infra-red images becomes very much less at the limb, but continues of course on the terminator. On the later plates the limb is, in fact, more sharply defined on the infra-red images than on the others. There is always fading, but in the plates showing considerable phase the exposure must be correctly timed to show it. The fading is unquestionably an *edge effect*. If it were found on only a single plate it might conceivably be due to an actual falling off in the brightness in the parts of the planet which happen at the moment to be near the edge, but photographs made during the four-hour period of an evening's observation, during which time the planet rotates approximately 60°, show the condition to persist. The central region is consistently the brightest, and surface configurations, in their diurnal march, emerge from obscurity on one side and fade at the other.

4. The infra-red images are appreciably smaller than those made with yellow and with violet light. To the casual view, there is an effect of this sort due to the fading of the infra-red images at the edge, but making the best allowance possible for that there remains an appreciable excess in diameter of the violet images over those made with infra-red light. When the two are brought together in the field of a Hartmann photocomparator, the difference in size is strikingly shown. In figures 3 and 4, Plate II, which show images halved on each other, the inequality is emphasized by the fading of the red image at the edge. The yellow images are about equal in size to the violet. In the first paper they were stated to be a very little smaller,

which, in fact, they are if violet photographs of August 19, 20, and 21 are given due weight. The observations of those dates give somewhat greater diameters than the others, a circumstance which is, as we shall see, probably to be ascribed to the use of different plates on those occasions. If we consider violet images taken on Lantern Slide plates, the violet and yellow images are sensibly of equal diameter. The observations have been carefully scrutinized in a search for possible systematic error. Chromatic aberration in the enlarging lens might conceivably be troublesome, but the aberration has been found to be inappreciable. Double stars of separation very nearly equal to the diameter of *Mars* were photographed in violet and infra-red light as a check, and yielded the same separation in each case. Photographs of *Venus* made in both colors exhibit no apparent difference in size (Plate II, figure 9). Bad seeing might emphasize the inequality by reason of the difference in the character of the two images, but, as shown in the discussion of the measures of diameter, it can hardly contribute seriously to it. Related considerations are referred to in the following paragraphs. I can think of no valid reason for regarding the effect as instrumental or spurious, but it should be confirmed, if possible, by an independent method, perhaps by that of drift and triangulation suggested elsewhere in this article.

The considerations of paragraph 4 suggest the propriety of a somewhat careful examination of the method of observation in a search for the possibility of systematic error. Differences in diameter are noted between images made with light of different colors on plates of different quality and with different filters. Furthermore, by reason of the characteristic peculiarities of the several kinds of images the measuring practice had to be different for each color. It seems appropriate to inquire to what extent the observed anomalies may be due to any or all of these circumstances. We shall begin with a description of the system adopted in making the measurements as it is in reference to it that the effect of other circumstances of observation must be considered.

The violet and yellow photographs show fairly uniform disks with of course a little blurring or indeterminateness at the edge. Here the practice was adopted of setting the wire on a part intermediate in intensity between the outermost visible edge and the arc on which maximum intensity is established. It is usually possible to distinguish on normally exposed plates an arc of maximum contrast where the strength is about half that of the disk surface, and the measuring wire was set as closely as possible to that.

In measuring the infra-red plates a different procedure had to be adopted for the reason that there is a true fading of the image as the edge is approached. If the center of the disk is normally exposed one can not be sure that the true edge is actually visible (Plate II, figure 4). If the exposure is timed for full normal density at the edge, there is danger of irradiation from the brighter central region. The practice was therefore adopted of selecting images of a density which seemed to outline the true edge faintly, and of placing the wire tangent to this outermost visible edge. It will be seen that the tendency of this method of measurement is to reduce the apparent inequality between violet and red images, for the measures are made within the edge of the apparently larger images and at the outermost visible limit of the smaller ones. Away from opposition the edge fading of the infra-red images was greatly reduced (compare figures 7 and 8 of Plate II) and the system of setting used for the violet photographs was adopted.

What we shall term the *direct* measurement of diameter, by way of distinguishing it from another presently to be described, was made for the greater part along the line joining the cusps, while in instances in which that seemed inadvisable, a comparatively close diameter was selected and the phase correction applied. On some of the infra-red photographs, the cusp was found to be practically invisible through the fact of its lying on a dark area of the planet, though an apparently true arc of considerable extent was discernible. In such instances the chord of the arc and its middle ordinate were measured, and the diameter was calculated from these lengths. The measurements are very conveniently made with the double-slide micrometer microscope of the Repsold measuring engine. In discussing the measurements of diameter this will be designated as the *chord and ordinate* method (abbreviated c & o), as distinguished from the direct method (abbreviated d). A check, as between the two methods, is provided by the measurement of a number of good images, eighteen in all, by both of them. A comparison of the diameters thus secured shows the chord and ordinate measures (as made, and not as reduced to unit distance) to be systematically about 0″.58 larger than those made the direct way. This correction has been applied to all the chord and ordinate diameters.

We may now consider the effect of instrumental and other imperfections on observations of diameter made as described above. A reference has been made, in the description of the enlarging camera, to the effect of focal maladjustment and chromatic aberration. As the second of these items has been shown to be negligible for the lens used in this investigation we need only concern ourselves with the first, the effect of which is to blur the image and not to influence appreciably the scale of enlargement. One is likely to think of blurring as enlarging the apparent diameter of an extended source, such as a planet, but it is apparent on second thought that blurring at the edge is in both directions—inward as well as outward—and it would appear that if the part of the planet's edge selected for measurement were a contour of intermediate intensity, observations might not be systematically affected by influences which appreciably increase the apparent or spurious diameter of a star. There are of course many agencies besides faulty focal adjustment that produce blurring, for example: imperfect resolution, bad seeing, atmospheric dispersion and poor following of the object, to mention a few, and it appears convenient to consider their aggregate effect in its influence on measurements of diameter.

If we consider the formation of the image of an extended and approximately uniform source by an optical system which produces symmetrical blurring it is very easy to show that the true edge of the image is the peripheral contour of half intensity and that it is also the contour of maximum light contrast. This is doubtless a well recognized principle and it follows from the fact that the illumination curve of a section perpendicular to the edge of the image is the integral of the curve which expresses the intensity distribution in the blurred image of a line source. The intensity at the theoretical edge of the image is the definite integral over one side of the blur curve and the intensity on the body of the image is the integral over the entire curve. Since the curve is, by assumption, symmetrical the areas of its two sides are equal and the intensity at the edge is therefore half that of the image. To show that the theoretical edge is also the line of maximum light contrast it is only necessary to recall that contrast is measured by the steepness of the curve of illumination, and that in turn by its derivative. The derivative is the intensity curve of the blurred image of a line, and if we assume its maximum to be at the center, that is to say that the blurring takes place around a strong central core, the principle of maximum contrast stated above follows at once.

The foregoing principle or theorem is pertinent to any consideration of planetary disks and seems especially so to the present discussion in view of its bearing on the matter of the apparent difference in size of the violet and infra-red images. Attempts have been made to explain this difference as an effect of atmospheric refraction. Short waves, being more readily deflected than long ones, are more sensitive to "bad seeing." It is in fact the experience of several observers that star images photographed by violet light are larger than those taken by yellow or orange. Purely photographic factors probably contribute to this effect, but there is no reason to doubt that irregular atmospheric refraction is very largely responsible for it. But it does not follow that a planetary diameter, measured as described above, would be systematically too large, for bad seeing merely causes blurring. In attempting to set on the contour of half density, or of maximum contrast, one may err on one side or the other. As a matter of fact measures of diameter of the violet image of *Mars* made on two different brands of commercial plates yield values above and below the best telescopic determinations. A somewhat similar consideration is suggested by atmospheric dispersion. The atmospheric spectrum to which an ordinary plate is sensitive subtends at a zenith distance of 55°, which is approximately that at which the photographs were taken, an angle of about 1ʺ.8. This is equivalent in its effect to a vertical blurring. The amount seems large and the spectral intensity is probably not symmetrical with respect to the position of greatest strength, still the measures of diameter do not appear to have been affected. It is possible that the polar cap, being of small angular diameter, has suffered spurious enlargement through this cause.

Until now we have spoken only of light intensity and not of photographic density, while as a matter of fact one has, in dealing with photographs, to do with density. If we accept density formulae of the Hurter and Driffield type, in which density is represented as being simply related to the logarithm of the intensity, the contours of half density and half intensity in the one case, and of maximum photographic contrast and maximum light contrast in the other will not, in general, coincide. The logarithmic law of variation would tend to make the photographed image too large while plate inertia or reluctance to initial action would operate in a contrary sense. Plates of high contrast would probably render relatively small images, and no doubt other photographic characteristics are capable of influencing the size, so that emulsions of different quality may be expected to render images differing somewhat in diameter; nevertheless photographic observations of diameter appear to be very much freer from systematic error than measures ordinarily made with the telescope.[3] Probably the only way to evaluate the systematic error of photographic measures is to make observations of a planet at large and small apparent diameters as has been done in telescopic investigations of the diameter of *Mars*.

Measures of diameter.—When the present work was undertaken, and in fact during the period of observation, it was not planned to attempt absolute measures of diameter, the purpose in view being a comparative study of the photographs by different colors. But the differences in size brought out by the comparisons suggested that the photographs be examined as critically as possible for the purpose of determining whether the apparent inequalities could be due in any degree to systematic errors of the several groups of photographs, such as might conceivably result from irradiation, bad seeing, or whatever cause.

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[3] This is shown not only by the observations described in this paper but also by a series of measurements of the diameter of *Mars* made by Mr. van de Kamp with the refractor of the Leander McCormick Observatory. (*Publ. Astron. Soc. Pac.* **37**, No. 218, 1925.)

As has already been explained, the measures of diameter were made by two systems: *direct*, and *chord and ordinate*, designated in the tables by d and c & o, respectively. The direct system was used for all ultra-violet, violet, yellow, and for most of the infra-red photographs. The method of chord and ordinate was used on a comparatively few photographs taken in infra-red. A systematic correction of −0″.58 has been applied to all measurements made by the latter plan as is noted in the earlier reference to the subject. The measures are recorded in column 4 of Tables 2, 3, 4, and 5, and diameters reduced to unit distance in column 5. Under *quality* are entered numbers intended to indicate the observer's judgment of the quality of the photographs. The weight is a number which is

TABLE 2

DIAMETER OF MARS*

VIOLET IMAGES

Mean Date G. M. T.	Number of Obs.	Method	Diameter Apparent	Diameter Reduced	Quality	Weight	Remarks
1924 Aug. 19.86	2	d	25″.45	9″.50	⅔	2.4	Seed 23, no filter
Aug. 20.86	1	d	25.14	9.38	1	2.5	Seed 23, no filter
Aug. 21.89	1	d	25.28	9.43	1	2.5	Seed 23, with filter
Aug. 21.89	1	d	24.84	9.26	1	2.5	L. S., no filter
Aug. 26.82	1	d	25.17	9.42	1	2.5	Seed 23, with filter
Aug. 26.83	1	d	24.03	8.99	1	2.4	L. S., with filter
Aug. 30.81	3	d	24.22	9.15	1	4.2	L. S., with filter
Sept. 2.79	2	d	23.92	9.13	1	3.4	L. S., with filter
Sept. 4.78	2	d	23.72	9.14	1	3.4	L. S., with filter
Sept. 5.80	1	d	23.76	9.20	1	2.4	L. S., with filter
Sept. 11.78	3	d	22.56	9.06	1	4.1	L. S., with filter
Sept. 16.77	2	d	22.05	9.18	1	3.1	L. S., with filter
Sept. 21.73	5	d	20.93	9.09	1	4.7	L. S., with filter
Sept. 28.70	2	d	19.01	8.81	½	1.3	L. S., with filter
Nov. 16.70	2	d	11.67	8.98	1	1.6	L. S., with filter
Mean: Seed 23				9.43			
L. S. except Sept. 28 and Nov. 16:				9.13			

* Photographs taken on August 30 and thereafter were made on Lantern Slide (L. S.) plates with a focal compensating filter of clear glass, while those taken between August 19 and 26 are on both Lantern Slide and Seed No. 23 plates and were made with and without filter. The comparatively large values derived from the Seed 23 plates is commented upon in the text.

TABLE 3

DIAMETER OF MARS

INFRA-RED IMAGES

Mean Date G. M. T.	Number of Obs.	Method	Diameter Apparent	Diameter Reduced	Quality	Weight	Remarks
1924 Aug. 19.84	1	d	23″.64	8″.82	½	1.2	
20.81	3	d	23.86	8.90	1	4.1	
21.85	3	d, c&o	23.74	8.85	1	4.1	
23.88	1	d	24.16	9.01	1	2.4	
26.79	5	d	23.99	8.97	1	5.6	
30.84	2	d	23.76	8.97	1	3.4	
Sept. 2.79	3	d	22.99	8.78	1	4.0	
4.77	3	c&o, d	22.85	8.80	1	4.0	
5.80	2	d, c&o	22.55	8.73	1	3.2	
11.77	6	d, c&o	22.10	8.87	1	5.4	Values scattering
16.77	4	d, c&o	21.30	8.87	1½	6.3	Excellent
21.75	3	c&o, d	20.38	8.86	1	3.5	
28.70	4	d, c&o	18.58	8.62	1	3.7	
Oct. 31.69	4	d	12.64	8.28±	1	0*	
Nov. 16.71	4	d	10.78	8.30±	1	0*†	
		Mean		8.86			

* These measures were not used in striking the mean.

† The image is small and there is no assurance that the northern cusp is visible. See fig. 14 of Plate II.

TABLE 4

Diameter of Mars

YELLOW IMAGES

Mean Date G. M. T.	Number of Obs.	Method	Diameter Apparent	Diameter Reduced	Quality	Weight
1924 Aug. 20.85	3	d, c&o	24″.83	9″.26	1	4.3
Aug. 23.84	1	d	24.84	9.27	½	1.2
Sept. 4.84	2	d, c&o	23.74	9.15	½	1.7
Sept. 5.79	3	d	23.29	9.02	1	4.0
Sept. 10.78	3	d	23.12	9.22	1	4.0
		Mean		9.17		

TABLE 5

Diameter of Mars

ULTRA-VIOLET IMAGES*

Mean Date G. M. T.	Number of Obs.	Method	Diameter Apparent	Diameter Reduced	Quality	Weight
1924 Aug. 23.88	1	d	24″.48	9″.13	1	2.4
Aug. 26.80	2	d	25.03	9.36	1	3.5
		Mean		9.27		

* These photographs were taken with the ultra-violet color filter used in conjunction with the enlarging lens. The color equivalent of the filter is probably altered by the glass lens but the effective wave-length is probably not far from 3600 A.

proportional to the product of the quality index, the apparent diameter, and the square root of the number of observations. Below the tables are given the weighted means.

There are a number of discrepancies in Table 2 which call for comment. We may refer first to the relatively large diameters in the upper part of the table. These remained unexplained for a long time but a clue was suggested by the fact that the photographs were secured before a definite observing routine had been established. Consultation of the note book developed the facts recorded in the note following the table and the observations were segregated both with respect to the use of the filter and the kind of plate employed. As the reader may see for himself the indications are that the determining factor is the plate and not the use of the filter. In striking the mean, the measures have been separated with respect to the brand of plate on which they were made. There seems to be a systematic difference of 0″.3 between the two groups. By a curious accident the mean for the two, 9″.28 is exactly the diameter derived from the most reliable telescopic measures of *Mars*.[4]

The abnormally low value of the diameter for September 28 is difficult to account for. It is without doubt partly due to underexposure though the smallness of the diameter on the infra-red plate of the same day suggests that both observations were vitiated in some way.

The outstanding result for November 16 is only apparently discrepant. If we assume that the measures on Lantern Slide plates are affected by a systematic error sufficient to reduce the diameter at unit distance from 9″.27 to 9″.13, the error is simply calculated as −0″.35. This applied to the calculated apparent diameter of *Mars* for November 16 gives 11″.67 in exact agreement with observation.

In glancing over the values of the diameter of *Mars* provided by eye observation with the telescope one finds among them a surprisingly large range. Diameters are reported between 9″ and 11″. Until very recently the American Ephemeris used 10″.10. See, in the paper already referred to, made a selection from the work of the most trustworthy observers and divided their measures into two groups according as they were made with the filar micrometer (at night) or with the heliometer. The mean of the micrometer measures is 9″.68 and of those made with the heliometer 9″.34. Both of these methods of measurement are subject to systematic error, the most serious one resulting, in the case of the filar micrometer, from irradiation of the bright planet when projected against the night sky. This contributes toward the large diameters measured with that instrument. Perhaps the most reliable determination is that by Campbell who made a series of observations during daylight thereby reducing the effect of irradiation.[5] He eliminated systematic error by an adjustment of observations made at large and small apparent diameter

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[4] A comprehensive review and summary of measurements of the diameter of *Mars* to the year 1901 is given by See, *A. N.*, **157**. 97, 1901.

[5] *Astron. Journ.* **15**, 145, 1895.

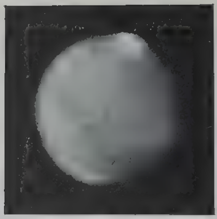

Fig. 1, Sept. 11

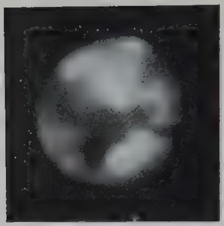

Fig. 2, Sept. 11

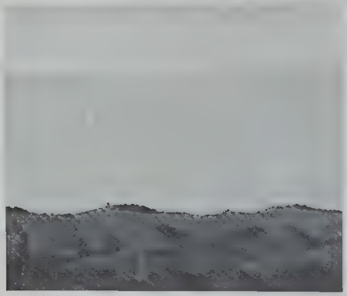

Fig 3

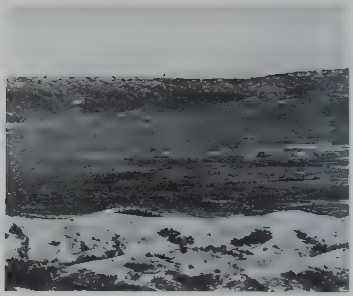

Fig. 4

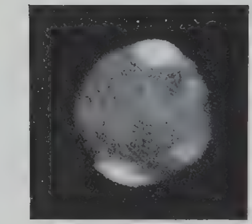

Fig. 5, Aug. 30

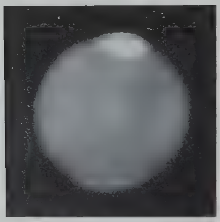

Fig. 6, Aug. 30 *

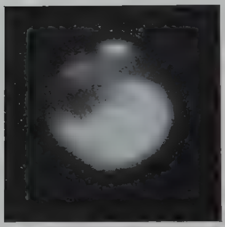

Fig. 7, Aug. 30

Figs. **1** to **4**: Mars and San Jose as photographed from Mount Hamilton; **1** and **3** with violet, **2** and **4** with infra-red light. The obliteration in **3** is due to the Earth's atmosphere and the comparison is suggestive of the presence of an atmosphere of considerable density on Mars. San Jose is distant 13½ miles. G. M. T. of Mars observations: Sept. 11; Fig. **1**, 18^h 50^m; Fig. **2**, 18^h 30^m.

Figs. **5**, **6**, and **7**: photographs of *Mars* by ultra-violet, violet, and infra-red light.

**CORRECTION*

The cuts for figure 6 of Plate I and figure 10 of Plate II, have, through error, been interchanged in the make-up.

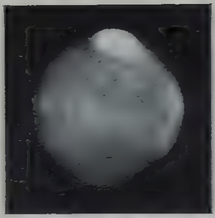
Fig. 1. Aug 21. v

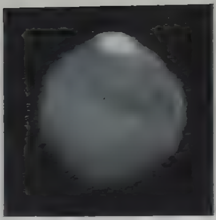
Fig. 2. Aug 21. v

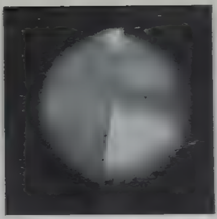
Fig. 3

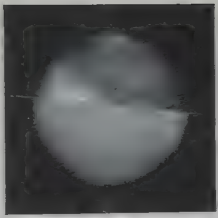
Fig. 4

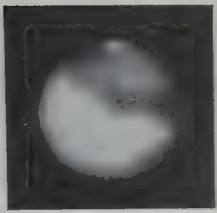
Fig. 5. Aug 21. i-r

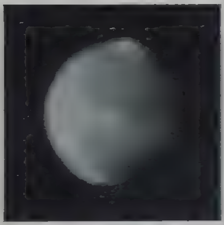
Fig. 6. Sept. 22. v

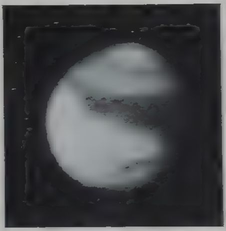
Fig. 7. Sept. 21. i-r

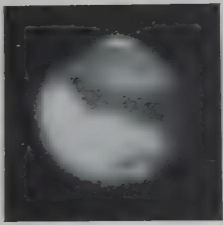
Fig. 8. Aug 22. i-r

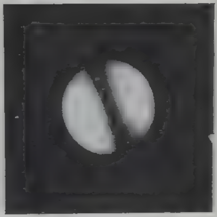
Fig. 9. Venus. v & i-r

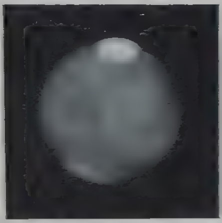
Fig. 10. Sept. 3. v

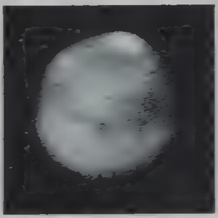
Fig. 11. Sept. 3. i-r

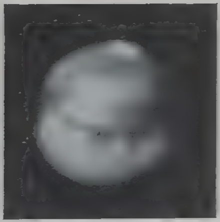
Fig. 12. Sept. 3. i-r

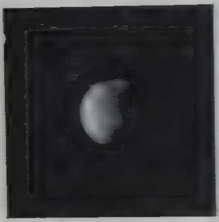
Fig. 13. Nov. 18. v

Fig. 14. Nov. 18. i-r

Photographs of Mars (excepting fig. 9) made by light of different colors. The letters v and i-r denote violet and infra-red. The G. M. T. of observations are:

Fig. 1. $21^h 33^m$; 2. $21^h 27^m$; 5. 21^h [illegible]; 6. 17^h [illegible]; 7. 17^h [illegible]; 8. 22^h [illegible]; 10. $18^h 21^m$; 11. 18^h [illegible]; 12. 18^h [illegible]; 13. [illegible]; 14. [illegible].

Figs. 3 and 4 are 2 and 5 halved and combined. The difference in quality between Figs. 1 and 2 is due to the [illegible] exposure of 2.

Fig. 9 consists of photographs of Venus taken in violet and in infra-red light. The violet image is at the [illegible] left. They are approximately of equal size. Compare with Figs. 3 and 4.

and derived the value 9″.25 for the polar diameter. Assuming a flattening of 1/195 the corresponding mean diameter may be taken as 9″.27. If we accept this as the true value and assume that the planet is seen as large by the eye as it would be in a perfectly rendered violet image, it will appear that the measures made on Seed 23 plates are 0″.16 too large, those on Lantern Slide plates, 0″.14 too small. The few measurements by yellow light (made principally on Seed 23 plates bathed in pinaverdol) are 0″.10 too small. Determinations by ultra-violet on two nights yield the residual 0. All of these results indicate that the systematic error of photographic measures of diameter is generally small,[3] much smaller than for ordinary telescopic measures, but suggest that plates of one kind or another may have characteristic idiosyncrasies in their rendering of a diameter. This is in accord with an earlier conclusion of this paper.

The diameters of the infra-red images are not measurable with the same degree of precision as those taken in violet or yellow light. A number of circumstances contributing to the difficulty of observation have been emphasized. All of the photographs by infra-red light are on plates of the same quality so that the observations are homogeneous in that respect. Excluding the last two the measures are as consistent among themselves as one might well expect considering their general difficulty. How faithfully their mean represents the actual diameter it would be impossible to say with any degree of certainty. We have already encountered what appears to be a systematic error of observation in violet images of approximately 0″.15 (reduced to unit distance) and in addition found substantial evidence that the amount of the error is dependent on the kind of plate used. The diameters recorded in Table 3 are consistently smaller than those in any of the other tables and it is conceivable that this is due to a large negative systematic error in the infra-red measures. The low values of October 31 and November 16 might appear to lend support to such an assumption. But an inequality of 9″.27−8″.86 = 0″.41 would imply a systematic error in the actual measurement of a little more than a second of arc, and even one of that size would not explain the last two values of Table 3. In view of the care taken to measure only fully exposed photographs in this color, and to set the measuring wire as far out as conscience would permit, I am reluctant to admit the likelihood of a negative systematic error of that size.

A spurious inequality between photographs of the planet made in light of two colors such as violet and infra-red might conceivably result either from difference in plate quality or difference in the general character of the image. An effect of the first kind we have already encountered but a number of photographs of *Venus* made in both colors supply pretty convincing evidence that we do not have to deal with it here. The infra-red and violet images of *Venus*, made on Kryptocyanin and Lantern Slide plates respectively, are, as nearly as one can tell, of the same size. Two photographs are reproduced in Plate II, figure 9. They were taken when the planet was close to quadrature and one is turned to the other so as to facilitate the making of the comparison. The right hand is the infra-red image.

From the evidence at hand I am disposed to draw the conclusion that there is a difference in size between the violet and infra-red images which in amount may be assumed to be 9″.13−8″.86 = 0″.27. This is much less than the estimate given in the earlier paper. The difference between the two estimates is to be ascribed in part to the fact that the first was based on hasty comparisons of photographs, some of which were made on Seed 23 plates which record a large image, and in part to the practice adopted in the present measurements of setting the wires on the extreme edge of the infra-red images and within the visible border of the other, which has, of course, reduced the apparent inequality.

General remarks on the photographic determination of diameter.—Some of the photographic measures given here seem to compare favorably with those made with the telescope and micrometer. Systematic error is, as has been noted, very much less. The mean of the most reliable telescopic measures made at night and unadjusted for systematic error is about 9″.68, or 0″.41 larger than the probable diameter. Photographic measures on Lantern Slide plates are as we have seen 0″.14 too small and on Seed 23 0″.16 too large. If we consider observations of Table 2 made on and previous to September 21, using only those made on Lantern Slide plates, the median residual is 0″.05. A complete photographic series of measurements at large and small apparent diameter might be expected to yield diameters comparable with the most trustworthy telescopic results. The determinateness of the diameters of the yellow images is undoubtedly greater because of the better defined edge that they present. For infra-red light of the wave-length here used the outlook is not so hopeful because of difficulties already discussed. It is probable, however, that photographs in this color more suited to measurements of diameter could be obtained on plates of less contrast. These might show the entire periphery of the disk without excessive density at any one part of it. The use of contrast plates for pictorial purposes should not be abandoned but it would seem advisable to try softer plates for diameter observations. In view of what has been said regarding the dependence of systematic error on the kind of plate used it is unnecessary to emphasize the caution that the plates on which a series of observations are made should be uniform in kind and quality.

Interpretations.—We may begin by referring to the fading of the infra-red images as the limb is approached. Edge effects of the sort are met with in the study of astronomical bodies known to have atmospheres, for example: *Jupiter* and the *Sun*, and are so well accounted for as the result of increased atmospheric absorption at the limb that it seems doubtful whether anyone would deny that explanation here. The absorption would appear to be due to opacity of the atmosphere and not to scattering since little or no infra-red light is returned by the planet's edge.

The violet pictures from their very uniformity offer a startling contrast to the lively presentation made by those taken in the infra-red. In seeking for an explanation of this great difference, two possible hypotheses naturally occur to one. The first is that the several materials of which the surface of *Mars* is composed, while they have very different reflectivities for infra-red light, may have equal reflectivities for light of the violet end of the spectrum. If that were true the violet images would, of course, present no contrasts whatever. The second is that the atmosphere, the existence of which we have inferred from the edge effect of the infra-red photographs, may be obliterating the details. With respect to the first of the proposed explanations we recall that differences in relative reflectivity of substances corresponding to the several parts of the spectrum are common enough, and furnish, in fact, the basis of color in ordinary objects. It would, therefore, be physically possible to have a number of materials that would stand out in contrast when viewed by light of one wave-length and could not be distinguished by that of another. Arrangements of the sort can be made artificially, but they are not to be expected in nature. We may photograph a landscape by red and by violet light and the relative values will, in general, be different in the two pictures, but there will be contrast in both pictures though it may be differently distributed. The chance that for a single region of the spectrum the intensities of all the elements of the picture would match exactly seems remote.

Nevertheless it has been urged in personal conversation that the possibility of the surface of *Mars* being composed of only two kinds of material should not be overlooked, and that if it were, the probability of a chance matching by violet light would not be negligible. With respect to such a contention it must be admitted that evidence of the kind we are considering is not absolute in the sense, let us say, that precise spectroscopic measurements are. Photographs in colors supply a versatile and elastic method of analysis, but one which is, after all, chiefly valuable for the suggestions which it offers. The evidence is, at best, seldom conclusive and its indications must in general be resolved on the basis of probability, or in an extremity, even of intuition. There is nothing in the observations to exclude the possibility implied in the above suggestion. On the other hand I can think of no good reason for assuming that there are exposed on the surface of *Mars* only two materials and that they match in ultra-violet light. Photographs of *Jupiter*, *Saturn*, the Moon, and the Earth's surface show contrast and detail quite as well by violet as by infra-red light of the wave length used in these observations. On the whole the first of the alternative explanations advanced to account for the quality of the violet images hardly seems a probable one.

Let us now consider the second suggestion: that the details are obliterated by a fairly dense planetary atmosphere which scatters light. In observing *Mars*, we should then have two superposed images, one, the atmospheric shell as seen by scattered light; and the other, the planet viewed by light which has twice traversed the atmosphere. If the atmosphere were relatively tenuous and possessed little scattering power, its image would be negligible, and only the body of the planet would be visible. This is assumed to be the case for infra-red light. If, on the other hand, we assume an atmosphere of such density and scattering ability that light would suffer practical extinction in the double vertical passage through the atmosphere and reflection at the planet, then the planet's image would not be present, and the atmospheric shell only would be visible. Furthermore, under the conditions of transparency just assumed, and the further one that the atmosphere is homogeneous, our image would be uniform in intensity for the reason that increase in atmospheric thickness due to inclination of the line of vision away from the center of the planet's disk could not contribute further to the amount of light sent to the observer. This uniformity is substantially what is found in the violet images, except with respect to the polar regions and to some comparatively minor peculiarities that have already been described. These considerations suggest that the detail of the planet's surface is obscured by its atmosphere and that the violet photographs depict the atmosphere alone.

However, the explanation just proposed encounters a grave difficulty if we assume that the Martian atmosphere has optical properties fairly similar to those of our own when at its best. The difficulty was mentioned to me by Professor Russell and had occurred to me in my own reflections on the matter. It is this: the amount of violet light sent back by the planet is much less than we should expect would be scattered by a pure atmosphere of sufficient density to obscure a reasonably bright planetary surface. The net reflectivity of the planet, atmosphere and all, for violet light is very small, probably not more than 10%. This is shown by the relatively low luminosity of the disk as compared with the polar cap, which is itself seen by returned sunlight, and also by the planet's low photographic albedo, given as nine-hundredths by Russell.[6]

If we assume, as is customarily done in regard to the atmosphere of the Earth, that, in the more refrangible portion of the spectrum, absorption of passing light is accomplished only through scattering, we should be led to believe, by the small amount of light—scattered or otherwise returned—coming from *Mars*, that very little absorption takes place in its atmosphere, that is to say, that the atmosphere must be relatively transparent, and we ought therefore to be able to photograph surface detail through it. In other words, the total amount of violet light actually reflected by *Mars* appears to be too small to satisfy the requirements of the hypothesis that the flattening out of detail in the violet images is due to ordinary atmospheric obscuration. We appear then to be driven to the assumption that *Mars* has a clear atmosphere and that the darkness of the planet by violet light is due to the low reflectivity of the permanent surface.

Let us carefully consider this hypothesis and direct our attention to the center of the planet's disk at the time of opposition. Of the light falling there a certain amount is sent back to us by molecular scattering, but in view of the clearness of the atmosphere a large percentage reaches the planet. This is, under our assumption, almost entirely absorbed, a small amount perhaps being sent back. If by any means we increase the atmospheric thickness, for example double it, we approximately double the amount sent back by the atmospheric scattering, but do not proportionally reduce the residual light reaching the planet. In other words, by thickening the atmosphere we increase the amount of light returned. Now we can obtain the necessary increase in atmospheric thickness by going out toward the edge of the disk; but that, as we have seen, does not alter the illumination. In order to suit the observed conditions it would be necessary that the atmosphere should be so tenuous as to scatter no light at all or so dense as to be almost opaque at vertical thickness. Accepting the first alternative we should conclude that the atmosphere would remain at all times invisible. By the second we might expect increased visibility of the atmosphere near the limb, with the development of phase, due to its superior illumination by a high Sun. That phenomenon is, in fact, shown strikingly on all photographs taken on or after September 16th, and the limb light is extremely strong on the later ones. The inference is that the Martian atmosphere is quite opaque to violet light. This deduction is not offered as a piece of close reasoning, nor is any assurance felt that its course could, even with care and pains, be paralleled by rigorous analysis. In reality what the subject needs is not speculation but more data of observation, but it is not always easy in reconnoitering to refrain from straying from the pavement of positive knowledge for an adventure on the stepping stones of conjecture. The suggestion that the Martian atmosphere exerts a true absorption, as distinguished from that which results from scattering, in light of short wave-length, is tentatively advanced pending further observation.

Whatever the opacity of the atmosphere to violet light there can not be much doubt that it is not entirely clear to the light used in making the infra-red photographs.[7] The deduction seems to be a fairly safe one to make from the phenomenon of edge fading. On the other hand the clearness of the image indicates that there is very little scattering. A comparison of the relative ease with which objects can be seen and photographed on *Mars* and on the Earth may not be devoid of interest. In our interpretation of the violet photographs we have assumed that the vertical thickness of the Martian atmosphere is sufficient to obliterate completely all traces of the planet's surface. One could not well make such an assumption regarding the atmosphere of the Earth. The vertical thickness of the Earth's atmosphere is approximately equivalent to a sea-level thickness of 5½ miles, and there is no difficulty in photographing at that distance with violet light. For example, the crest of the hill in the foreground of Plate I, figure 3, is 5.3 miles away and with suitable exposure the detail there could be well brought out. To the eye, on a hazy day, the valley floor beyond San Jose, at a distance of approximately 18 miles, impresses one as having somewhat the same visibility as the surface of *Mars*. Here the light path is approximately three times the vertical atmospheric thickness. Of course one would not be justified in assuming that the Santa Clara valley presents exactly the range of contrast provided by the surface of *Mars*. I wish only to make the point that the general aspect of the planet in the telescope suggests nothing contradictory to the assumption that the atmosphere of *Mars* is less clear than that of the Earth.

Adopting the view that the violet images are photographs of the atmosphere it seems necessary to interpret the mottlings and streaks as resulting from local unhomogeneities, which may for convenience be termed clouds. It is not intended to imply that they resemble terrestrial clouds. In fact a very important difference between the two is that our clouds are intensified when viewed by red light while the Martian objects are not seen at all in that color. They appear in fact to be transparent to it but show to advantage in violet and ultra violet light. Nor should they be confused with the bright temporary objects sometimes seen with the telescope near the terminator of *Mars*. The telescopic objects are much brighter and of comparatively rare occurrence, and they may be true clouds in the terrestrial sense, as they have usually been interpreted to be. The photographic objects are to be found on the disk at almost any time. They appear to be very tenuous. The streaky one of September 21 brightens at the limb, presumably because it is there observed end on. The rim light observed to occur rather persistently on the oncoming limb when the observations were begun, which was near the time of opposition, appears explicable as a morning cloud which had formed during the night. Such a cloud is shown in Plate II, figures 1 and 10, for August 21 and September 5, respectively.

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[6] *Astrophys. Jour.* **43**, 173, 1916.

[7] See comment on occurrence of the A band within the region of sensitivity (p. 50, foot of col. 1).

One of the most striking features of the series of photographs is the relatively great strength of the polar cap on those made by light of the shorter wave-lengths. Under our general interpretation of these photographs as pictures of the Martian atmosphere it seems necessary to draw the conclusion that the polar caps are largely atmospheric, but doubt as to the validity of that conclusion has been expressed by a number of astronomers with whom I have had the advantage of discussing some of these matters. The polar caps are sometimes seen divided and broken, when viewed in the telescope, and the divisions have been observed to occur repeatedly at the same places and at the same Martian season. There are really a number of questions at issue here. There is a south polar cap on both red and violet images and what may be called a north polar cap on some of the violet ones. The latter was not, that I am aware, seen in the telescope, and appears most readily explicable as a great cloud (using the word in the sense to which we have adapted it) which extends over from the hidden north pole. The south polar cap was plainly visible on both violet and infra-red photographs taken before September 21. On the infra-red plates of that day it was all but invisible, recording only as a minute speck, but it remained fairly strong on the violet photographs. The insignificance of the cap on the infra-red image for that day is due in part to the fact that, being eccentrically situated, it lay beyond the pole and was therefore unfavorably placed for observation. The continued visibility of the cap in the telescope when the object photographed by infra-red light—which it seems natural to assume was the surface cap—had all but disappeared, led me to believe that the object which records on the violet images, whatever may be its constitution, contributes substantially to the image of the polar cap as seen in the telescope. This seems reasonable in view of the fact that the limb light also recorded in the violet photographs is easily visible. A question that arises then is whether the bright object photographed by violet light is an atmospheric or a surface phenomenon. If it is a surface phenomenon, situated as it is near the limb, it must be seen by light that has traversed a comparatively great thickness of whatever Martian atmosphere there is. Now the cap photographs much more readily by violet light than by infra-red, and this is extremely difficult to reconcile with ordinary experience in terrestrial photography. From Mount Hamilton one can, on a clear day, and with infra-red sensitive plates such as those used in photographing *Mars*, take landscape views of forest, mountain, and snow, at distances as great as 185 miles, the equivalent of thirty times the thickness of the Earth's atmosphere; but with violet light one photographs only the intervening air with its burden of dust.

In presenting the case for the surface polar cap, it has been contended that the cap, presumably snow, is photographed through an admittedly obscuring atmosphere because of its high reflecting power for violet light, while the general surface of the planet is too dark to show. A test made by photographing a distant landscape partly covered with snow shows that at a distance of about twenty miles snow can be photographed while contiguous territory bare of it is not recorded. But this requires quite a delicate adjustment of distance. A little farther off the snow is invisible, at a closer view the rest of the landscape appears. Nor has the snow at that distance the brilliancy of the Martian cap. To the extent that such observations may be of significance in considerations relating to the atmosphere of another planet, it may be said that they admit the explanation as a possibility but do not offer it much encouragement. It is extremely difficult for me to set aside the idea that the polar light in the blue image is the result of diffusion by a fine haze of some kind. Whether this light dominates the image seen in the telescope and may therefore be regarded as the telescopic cap is another matter. Apparently on September 21 it did, but the condition was exceptional and at other times the surface cap may have been visible. Possibly we may have some answer to the question at the next opposition. Meanwhile any records of the telescopic visibility of the cap subsequent to September 21st would have an interest in connection with some of the later photographs made here, and the privilege of access to them would be appreciated.

The fact that the bright objects seen on the violet photographs are relatively rich in light of short wave-lengths suggests that they are clouds or aggregates of finely divided material—finer than the droplets or ice crystals in terrestrial clouds—which is, presumably, condensed out of the upper atmosphere. It is unfortunate that the angle subtended at *Mars* by the Earth and Sun never approaches 90° (the maximum value is approximately 42°) and it is therefore not possible to apply the polarization test for scattered light.

The foregoing discussion has doubtless suggested the desirability of supplementing the photographs by spectroscopic observations. They are made at widely separated points in the spectrum, and it is very important to know whether the differences among the several groups of images result from a progressive development through the spectrum, or are caused by spectral discontinuities such as absorption bands or, possibly, phenomena of emission. A matter of special interest is the polar cap whose richness in light of short wave-lengths, as shown by its startling strength in the violet and ultra-violet photographs, suggested that it might be shining by luminesence of some kind. A number of spectrograms of *Mars* were secured by Dr. Shane while these observations were in progress and he has kindly given me permission to refer to them. The more refrangible spectrum was photographed with the ultra-violet spectrograph and the blue to red region with a spectrograph of low dispersion. The spectrum of the polar cap is seen to extend with great strength into the ultra-violet and in the matter of absorption lines is a replica of the spectrum of the rest of the planet, which means, of course, that it is reflected sunlight. The spectrograms of the visible region are of low dispersion and are not complete, for they do not reach to 7600 A, the wave-length of the infra-red light employed; but the observations, taken together, leave little room for doubt that the changes are progressive and not the result of local irregularities in the spectrum of the planet.

Reference has been made to the growing sharpness at the limb with advancing phase shown by the images made with infra-red light. This is interpreted as being due to the better illumination of the limb by a high Sun.

Attention has been called to the apparent difference in the sizes of the images recorded by violet and infra-red rays. If we adopt the view that the violet images are representations of the planet's atmospheric surroundings and the others are photographs of its surface, a difference in size would naturally be expected, though one is surprised on first realizing that it might be large enough to measure. Half of the difference in diameter may be interpreted as the depth of the atmosphere or rather the depth of that part of it that is bright enough to photograph under the Sun's illumination. The discussion of diameters has provided an estimate of $0''.27$ for the difference. The equivalent in miles of half of this is, in round numbers, 60 which is offered as a crude estimate of the order of thickness of the Martian atmosphere. This estimate is about half that given in the earlier paper. I can only repeat the comment made there that "This may be regarded as an estimate of the order of the atmospheric thickness based on a single method which is not free from difficulty nor from the possibility of systematic error, and it is very desirable that the result be checked in any way that may be possible. The point of greatest weakness is the determination of the diameter of the body of the planet and a method that suggests itself is to derive the planet's diameter by timed measurements of suitable markings across the disk. If we assume the simple case of the motion of a point on the equator, we should, by measurements extending over the period of an evening's observation, taken in conjunction with the known period of rotation of the planet, be able to derive a close value of the planet's diameter." The markings known as the *Sabaeus Sinus* and the *Margaritifer Sinus* are distinct and are situated close to the equator; they appear to be well suited to the purpose. I have not felt that the photographs made at the past opposition are suitable for such an investigation but Dr. Trumpler has made use of a modification of the proposal to derive the true diameter of the planet and his result will probably be given in his account of his observations. The measurements are of very great delicacy and I believe the problem of determining the true diameters of the several images and of estimating the diameter of the solid planet by surface triangulation will have to go over to another opposition.

I wish, in concluding, to acknowledge assistance, in securing some of the photographs, rendered by Mr. Hugo Benioff, formerly a member of the staff of this observatory.

SUMMARY

This paper is a report on a series of photographic observations of *Mars*, largely experimental in character, which were made near the time of the planet's opposition in 1924. The observations consist of photographs made in light of several colors of the spectrum ranging from the near infra-red to the ultra-violet. They were made for the purpose of ascertaining whether observations of the kind would hold promise of usefulness in the study of *Mars* at the favorable opposition of 1926. The undertaking was therefore of a preliminary sort and the tentative interpretation of the somewhat fragmentary yield of the observations is offered more for the purpose of outlining or suggesting problems on which the observations appear to have an incidence than of contributing

to positive knowledge of *Mars*. More latitude in discussion and speculation has, therefore, been allowed than would have been the case in a formal investigation carried out by more familiar methods. Some of the facts derived from the observations and a number of provisional conclusions arrived at in this paper are summarized in the following paragraphs:

1. Photographs of different aspect have been secured by infra-red, yellow and violet light. The infra-red photographs are interpreted as portraying the planet proper; the violet ones as supplying a sort of representation of its atmosphere impressed by light which it scatters; while the yellow photographs, which show the planet somewhat as it is seen in the telescope, are regarded as combinations of these two images. Alternative interpretations are discussed.

2. The planet is surrounded by an atmosphere of appreciable absorbing and light-scattering power.

3. A provisional estimate of the thickness of the atmosphere to the height that it can be photographed by diffused light is 60 miles. This estimate is regarded as only approximate and is chiefly valuable in the indication it gives that the extent of the atmosphere is measurable. An independent method of checking the result is suggested.

4. The south polar cap, as photographed by violet light, is an atmospheric phenomenon. There is some evidence of the existence of a low cap which rests on the planet. The atmospheric cap is probably transparent to infra-red light. On the violet, and more especially on the ultra-violet images, there is at times a conspicuous white area over the north point of the disk. It does not occur on the infra-red photographs and is, at times, quite certainly transparent to that light. This is regarded as an area of condensation which extends over from the hidden north pole. Its apparent coming and going may be due to actual change in size or is equally explicable as the result of greater southerly extension in some longitudes than in others.

5. The violet and ultra-violet photographs exhibit impermanent white configurations which are interpreted as clouds of fine haze having relatively great light scattering power. One of these objects of unusual persistence was found close to the equator to the south of *Mare Sirenum* and *Mare Cimmerium*. The fact that they brighten at the limb, where they are seen edge on, indicates these objects to be of a high degree of tenuity. At opposition, when the morning terminator lies at the limb, they occur with persistence upon it, and are then possibly the product of night condensation.

6. After opposition, with the development of phase, a rim of light appeared on the western limb in the violet images. This increased in strength and spread inward on the disk as phase developed, finally becoming the most noticeable feature of the violet photographs. It is suggested that this is due to superior illumination of the atmosphere (assumed to be rather opaque) at the limb by high Sun, though in the later observations the effect is probably complicated by asymmetry of illumination at phase.

June 12, 1925.
Issued July 31, 1925.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 367

MEASURES OF COMETS AND OF PLANET 1924 TD (BAADE)

By

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The positions given below are all mean places for the beginning of the year, that is, the position of the object is that obtained by applying to the mean place of the comparison star the measured $\Delta\alpha$ and $\Delta\delta$, corrected for refraction. No corrections have been made for the effects of differential precession, nutation, and aberration. The $\Delta\alpha$'s between object and comparison star have usually been measured directly, with the clock running. Where transits have been employed the time records were made on a chronograph. The parallax factors have been taken from the tables in *Publications of the Lick Observatory*, Vol. I, except in the case of the three observations of Comet *c* (Orkisz) at the end of May. Thanks are due to Mrs. Jeffers for recording practically all the observations.

Planet 1924 TD (Baade)

1925	Gr. C. T.	Star	No. of Comp.	Obj.	Star	α (1925.0)	δ (1925.0)	Log $\rho\Delta$	
								α	δ
Feb. 25	$3^h\ 18^m\ 59^s$	1	10, 10	$+\ 9^s.35$	$-0'\ 18''.4$	$3^h\ 20^m\ 12^s.26$	$+0°\ 16'\ 46''.6$	9.401	0.725
25	3 51 30	1	8, 8	+11.96	−0 9.2	3 20 14.87	+0 16 55.8	9.486	0.725

Mean Place of Comparison Star, 1925.0

	α	δ	Authority
1	$3^h\ 20^m\ 02^s.91$	$+0°\ 17'\ 05''.0$	Algiers ph. 0°, $3^h\ 20^m$, 34.

The above measures were made with the 36-inch refractor. The object was easily visible; about the 13th magnitude. Stormy weather prevented further measures. The earlier observations of the series are in *Lick Observatory Bulletin* 360.

Comet *d* 1924 (Wolf)

Three Crossley plates were exposed in attempts to obtain positions of this object. One plate on January 18 had a doubtful image, and the first of two plates obtained on January 20, the only one of the three not interfered with by clouds, has a fairly good image. On the plate of January 20 there were but two comparison stars, very badly placed. The orientation, however, is from a trail.

The following position is necessarily provisional:

1925	Gr. C. T.	α (1925.0)	δ (1925.0)
Jan. 20	$5^h\ 0^m.6$	$4^h\ 06^m\ 15^s.9$	$+19°\ 12'\ 54''$

Stormy weather and moonlight prevented further observations. The object on the plate of January 18 is quite certainly a plate defect, as judged by the appearance of the image and the comparison with the ephemeris. Thanks are due Professors Crawford and Meyer for the ephemeris by which the object was located, and for the comparison with their most recent orbit.

Comet *a* 1925 (Shajn-Comas Solà)

1925	Gr. C. T.	Star	No. of Comp.	Comet-Star		α (1925.0)	δ (1925.0)	Log ρ Δ	
				Δα	Δδ			α	δ
Mar. 26	$4^h\ 55^m\ 48^s$	2	8, 8	$+25^s.53$	$-0'\ 42''.3$	$11^h\ 43^m\ 44^s.06$	$+1°\ 54'\ 58''.0$	9.484n	0.713
26	6 12 24	3	6, 8	+ 7.36	+0 37.8	11 43 38.29	+1 55 14.1	9.233n	0.708
Apr. 2	6 34 07	5	8, 9	+10.61	−0 18.5	11 31 6.83	+2 28 30.3	8.672n	0.702
13	4 25 01	7	8, 8	+25.26	−2 45.7	11 12 42.56	+3 14 37.8	9.260n	0.696
24	8 24 18	9	8, 8	−11.36	+0 18.0	10 56 0.82	+3 51 24.9	9.572	0.702
May 1	6 18 47	11	8, 8	−16.05	−0 48.3	10 47 3.55	+4 08 03.2	9.371	0.689
1	7 05 44	11	8, 8	−18.58	−0 42.5	10 47 1.02	+4 08 9.0	9.496	0.695
22	5 21 48	12	6, 7	−17.96	+4 38.8	10 26 43.49	+4 29 25.1	9.494	0.690
22	5 46 05	12	6, 6	−18.73	+4 37.6	10 26 42.72	+4 29 23.9	9.542	0.695
25	5 35 13	14	8, 8	− 6.87	−0 55.7	10 24 36.58	+4 29 7.7	9.546	0.696
June 17	5 06 51	16	16, 12	−17.38	+2 6.7	10 14 6.49	+4 02 46.5	9.566	0.700

Mean Places of Comparison Stars, 1925.0

	α	δ	Authority
2	$11^h\ 43^m\ 18^s.53$	$+1°\ 55'\ 40''.3$	A. G. Albany 4351.
3	11 43 30.93	+1 54 36.3	Referred to (2) $+12^s.40$, $-1'\ 4''.0$.
4	11 29 48.34	+2 32 1.7	Bonn 1900, 5114.
5	11 30 56.22	+2 28 48.8	Referred to (4) $+1^m\ 7^s.88$, $-3'\ 12''.9$.
6	11 14 48.14	+3 14 31.0	A. G. Albany 4241.
7	11 12 17.30	+3 17 23.5	Referred to (6) $-2^m\ 30^s.84$, $+2'\ 52''.5$.
8	10 56 29.49	+3 52 21.6	A. G. Albany 4180.
9	10 56 12.18	+3 51 6.9	Referred to (8) $-17^s.31$, $-1'\ 14''.7$.
10	10 48 23.26	+4 09 52.2	A. G. Albany 4155.
11	10 47 19.60	+4 08 51.5	Toul. ph. +5°, $10^h\ 44^m$, 141; referred to (10) $-1^m\ 3^s.61$, $-1'\ 0''.9$.
12	10 27 01.45	+4 24 46.3	Toul. ph. +5°, $10^h\ 28^m$, 94.
13	10 23 27.05	+4 25 38.2	A. G. Albany 4042.
14	10 24 43.45	+4 30 03.4	Referred to (13) $+1^m\ 16^s.40$, $+4'\ 25''.2$.
15	10 16 37.65	+4 00 2.2	A. G. Albany 4007.
16	10 14 23.87	+4 00 39.8	Referred to (15) $-2^m\ 13^s.81$, $+0'\ 38''.4$; Toul. ph. +5°, $10^h\ 12^m$, 118.

The measures of March 26, April 13, and May 1 were made with the 12-inch telescope; for the others the 36-inch was used. The measures are not so easily made with the 12-inch, because the comet is faint, between the 12th and 13th magnitude. A nucleus could sometimes be glimpsed, but not when there was illumination on the micrometer wires. The Δα of star (5) referred to (4) is weak, owing to interference by clouds. It depends on but three transits.

Comet *b* 1925 (Reid)

1925	Gr. C. T.	Star	No. of Comp.	Comet-Star		α (1925.0)	δ (1925.0)	Log ρ Δ	
				Δα	Δδ			α	δ
Apr. 6	$10^h\ 09^m\ 36^s$	1	5, 8	$-0^s.72$	$-9'\ 12''.4$	$13^h\ 16^m\ 39^s.90$	$-23°\ 31'\ 22''.2$	9.346	0.870
6	10 36 42	1	5, 9	− 2.23	−9 30.7	13 16 38.39	−23 31 40.5	9.439	0.862
7	10 55 09	2	5, 6	− 5.62	−3 52.4	13 15 21.92	−23 47 31.7	9.505	0.852
24	6 18 18	3	8, 6	−10.90	−0 9.8	12 51 01.41	−28 03 01.9	8.869n	0.903
24	6 33 49	3	10, 8	−11.94	−0 21.1	12 51 00.37	−28 03 13.2	8.580n	0.904
May 2	6 11 23	4	10, 10	−20.72	−3 27.9	12 38 44.41	−29 53 44.4	8.041	0.911
2	6 31 50	4	10, 10	−22.04	−3 39.0	12 38 43.09	−29 53 55.5	8.771	0.910
21	5 27 56	5	10, 10	+17.00	+5 30.6	12 14 59.20	−33 42 46.1	9.158	0.914
21	6 19 21	6	8, 8	−28.78	−1 34.8	12 14 57.07	−33 43 09.3	9.422	0.898
26	4 43 11	7	10, 10	−26.03	−1 55.0	12 11 03.94	−34 37 39.5	8.978	0.922
June 12	5 09 17	9	20, 15	+13.75	−2 49.0	12 07 17.79	−37 48 40.4	9.520	0.892

Mean Places of Comparison Stars, 1925.0

	α	δ	Authority
1	$13^h\ 16^m\ 40^s.62$	$-23°\ 22'\ 09''.8$	Cordoba A 9857.
2	13 15 27.54	−23 43 39.3	Cordoba A 9846.
3	12 51 12.31	−28 02 52.1	Cordoba B 8268.
4	12 39 05.13	−29 50 16.5	Cordoba B 8153.
5	12 14 42.20	−33 48 16.7	Perth ph. −33°, $12^h\ 13^m$, 55.
6	12 15 25.85	−33 41 34.5	Perth ph. −34°, $12^h\ 09^m$, 196; −34°, $12^h\ 18^m$, 216; −33°, $12^h\ 13^m$, 81.
7	12 11 29.97	−34 35 44.5	Perth Merid., III, 947.
8	12 06 28.90	−37 45 35.5	Perth Merid. VI 1027; Cordoba Zone, VIII, 12^h, 240.
9	12 07 4.04	−37 45 51.4	Perth ph. −37°, $12^h\ 05^m$, 47; referred to (8): $+35^s.16$, $-15''.7$.

Of the above observations of Reid's Comet only those of May 2 were made with the 36-inch telescope.

Comet *c* 1925 (Orkisz)

1925		Gr. C. T.	Star	No. of Comp.	Comet-Star $\Delta\alpha$	Comet-Star $\Delta\delta$	α (1925.0)	δ (1925.0)	Log $\rho\Delta$ α	Log $\rho\Delta$ δ
Apr.	6	$12^h\ 52^m\ 50^s$	1	4, 7	$-23^s.00$	$-1'\ 50''.5$	$22^h\ 28^m\ 09^s.15$	$+18^\circ\ 06'\ 04''.5$	9.670n	0.640
	7	12 04 58	2	8, 8	+17.33	−6 40.6	22 29 07.42	+19 07 29.3	9.692n	0.677
	7	12 19 05	2	6, 7	+18.01	−6 01.8	22 29 08.10	+19 08 08.1	9.688n	0.664
	8	12 27 29	3	8, 10	− 7.52	+5 59.4	22 30 09.96	+20 13 12.4	9.687n	0.646
	12	12 31 47	4	8, 8	− 5.39	+1 26.6	22 34 30.95	+24 41 32.0	9.701n	0.596
	13	12 12 00	5	10, 8	+ 2.78	+3 10.9	22 35 39.19	+25 50 01.3	9.705n	0.611
	13	12 29 32	6	10, 8	−20.27	+0 35.5	22 35 40.08	+25 50 55.3	9.696n	0.586
	24	10 56 33	7	8, 8	+31.59	−0 44.7	22 50 44.85	+39 30 00.3	9.784n	0.601
	24	11 19 29	8	10, 10	−22.44	−3 57.6	22 50 46.07	+39 31 17.7	9.781n	0.548
	28	11 17 44	9	10, 11	+43.21	−5 01.7	22 57 53.61	+44 53 17.2	9.816n	0.476
	28	11 43 25	9	8, 8	+45.26	−3 32.1	22 57 55.66	+44 54 46.8	9.806n	0.387
May	22	7 18 18	10	10, 10	− 7.17	+1 59.4	0 51 40.60	+76 04 27.4	0.040n	0.838
	24	9 27 28	11	10, 10	−39.88	+0 47.4	1 21 40.53	+78 10 10.4	0.300n	0.642
	25	6 34 25	12	8, 8	+116.35	−0 20.5	1 37 04.70	+78 57 56.3	9.800n	0.884
June	21	7 01 00	13	20, 15	−23.71	−2 19.5	9 29 45.27	+69 56 03.1	0.109	0.612

Mean Places of Comparison Stars, 1925.0

		δ	Authority
1	$22^h\ 28^m\ 32^s.15$	$+18^\circ\ 07'\ 55''.0$	A. G. Berlin A 9209.
2	22 28 50.09	+19 14 09.9	A. G. Berlin A 9211.
3	22 30 17.48	+20 07 13.0	A. G. Berlin A 9225.
4	22 34 36.34	+24 40 05.4	A. G. Berlin B 8696.
5	22 35 36.41	+25 46 50.4	Oxf. ph. 25.79493, 26.76683.
6	22 36 00.35	+25 50 19.8	Oxf. ph. 25.79512, A. G. Cam. E. 13620.
7	22 50 13.26	+39 30 45.0	A. G. Lund 10943.
8	22 51 08.51	+39 35 15.3	A. G. Lund 10951.
9	22 57 10.40	+44 58 18.9	A. G. Bonn 17339.
10	0 51 47.77	+76 02 28.0	Greenw. ph. +75.349.
11	1 22 20.41	+78 09 23.0	Greenw. ph. +78.510.
12	1 35 08.35	+78 58 16.8	Greenw. ph. III +78.643, A. G. Kasan 253.
13	9 30 08.98	+69 58 22.6	Greenw. ph. III +70.4261; A. G. Berlin C 1348.

The observations of April 13, May 22, and May 25 were made with the 36-inch refractor, the others with the 12-inch.

Comet *d* 1925 (Tempel II)

1925		Gr. C. T.	Star	No. of Comp.	Comet-Star $\Delta\alpha$	Comet-Star $\Delta\delta$	α (1925.0)	δ (1925.0)	Log $\rho\Delta$ α	Log $\rho\Delta$ δ
June	17	$7^h\ 24^m\ 19^s$	2	8, 8	$+11^s.16$	$+1'\ 22''.7$	$18^h\ 25^m\ 45^s.14$	$-0^\circ\ 39'\ 09''.2$	9.246n	0.733
	17	8 07 17	3	16t, 10	−28.02	+0 39.4	18 25 45.69	−0 39 31.6	8.968n	0.721
	18	7 47 37	4	10t, 10	+46.53	−0 55.8	18 26 9.94	−0 51 44.0	9.086n	0.736

Mean Places of Comparison Stars, 1925.0

	α	δ	Authority
1	$18^h\ 25^m\ 27^s.37$	$-0^\circ\ 43'\ 19''.2$	Algiers ph. 0°, $18^h\ 24^m$, 92; −1°, $18^h\ 28^m$, 4.
2	18 25 33.98	−0 40 31.9	Referred to (1) $+6^s.61$, $+2'\ 47''.3$.
3	18 26 13.71	−0 40 11.0	Algiers ph. 0°, $18^h\ 24^m$, 97; −1°, $18^h\ 28^m$, 9.
4	18 25 23.41	−0 50 48.2	A. G. Nicolajew 4591.

Tempel's II comet, in contrast to the others above, was very much condensed, though the nucleus was not quite stellar. The general appearance was that of an imperfectly defined star, with nebulosity around it. The magnitude was about the eleventh. The measures on June 17 were made with the 36-inch refractor. For that of June 18 the 12-inch was used. "t" following the number of comparisons in α indicates a measure by transits.

As it is planned to follow comets *a*, *c*, and *d* as long as possible, the above series of observations are not complete. It was thought best to publish at the present time the measures so far made.

Most of the measures given above were made especially for use in the orbit investigations carried out at the Berkeley Astronomical Department.

June 25, 1925.
Issued July 31, 1925.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 368

AZIMUTH CORRECTIONS FOR SUNSET AND SUNRISE TRANSITS

By

R. H. Tucker

There appears to be a direct and rather suggestive connection between the observed azimuths of the Lick Observatory meridian circle, and the difference between clock corrections observed near the epochs of sunset and sunrise. This difference, of about 0^{s}06, has been discussed in earlier Bulletins.*

There is a diurnal variation in the azimuth of the instrument, derived from transits of the circumpolar stars, which has been evident in all our fundamental work. Any real change in the position of the instrument would be eliminated in the computed clock corrections. If, however, there is an atmospheric effect, such as a deviation along the plane of the meridian, at sunset and sunrise, the azimuths computed from transits of the circumpolar stars would be affected, and the erroneous values of a would increase the direct effect upon computed clock corrections. If azimuths were derived from transits of stars south of the zenith in combination with transits of zenith stars, such an effect would be eliminated in the computed clock corrections. This is evidently true, whether we are dealing with an instrumental change or an atmospheric deviation.

The circumpolar stars are almost universally employed for the measurement of azimuth. The difference of the azimuth factors for a pair of stars at declination 75°, one at each culmination, is close to 6. A south star, at zenith distance equal to the mean of the circumpolar pair has a factor of 0.8. The combination of a south star and an upper culmination of a circumpolar star gives a satisfactory determination of the azimuth, the difference of the factors being above 3. The actual clock corrections are eliminated in all such computations. Since the accidental errors of observation in most cases are nearly in the proportion of the respective secants of declination, an azimuth from a pair of circumpolar stars at 75° has only weight two, as compared with that from a south star and pair of zenith stars. The accidental error of observation for *Polaris* is smaller than the ratio of its secant of declination. If it were in proportion to the secant, the weight of an azimuth determination would not much exceed that from a combination of a south star and zenith stars. The actual weight is about four to one. These figures show that azimuths from south stars, in combination with zenith stars, have considerable intrinsic value, if the effect of systematic errors can be well eliminated.

The comparison of azimuths from stars north and south of the zenith gives a test which is significant, in its bearing upon an atmospheric deviation.

Our fundamental work of the years 1905 and 1906 is well designed for such a comparison. Over one hundred azimuths were measured from transits of *Polaris*, nearly evenly distributed at both culminations. They were generally arranged in sets of three or more consecutive culminations, twelve hours apart. On three-quarters of the dates there are observations of south stars, some below $-30°$ declination, in sufficient numbers to provide determinations of a, in combination with a pair of zenith stars, on each date. On about one-half of the dates there are observations of south stars at the two epochs, which differ by twelve hours. There are usually from eight to ten south stars, with an average azimuth factor of 0.8. Azimuths from *Polaris*, near noon, have been derived from combinations with *a Andromedae*, or by interpolating clock corrections from the successive full dates of observation.

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* *L. O. Bull.* **10**, 47, 1919, and **10**, 93, 1921.

The individual results are tabulated in Table I to two places of decimals, as the figures are thus easier to follow; but the computations have been carried out to three places. The epochs of the day, for the transits of *Polaris*, and the corresponding intervals from sunset or sunrise have been given for each date. The azimuth from transits of south stars can be inferred, for each date, from the Δa (N − S), of the last column. Also the corresponding ΔM, for the observed position of the mire, from south stars, can be inferred.

The probable error of ΔM for a single date, from the transits of *Polaris* and zenith stars is closely $\pm 0^{s}03$. This is mainly the probable error of *a*. The probable error of ΔM for one date is closely $\pm 0^{s}04$, from the transits of south stars and zenith stars. The accidental error of *a*, from one south star and the pair of zenith stars, is $\pm 0^{s}05$. There would also be some systematic error in this method of deriving azimuth. The probable error of Δa (N − S), on one date, is $\pm 0^{s}040$. The readings on the mire do not affect this last figure.

TABLE I.—AZIMUTH AND MIRE

1905		Polaris L.C.	Sunset	*a*	ΔM	N—S
July	10	$6^{h}2$ P.M.	$-1^{h}1$	$+0^{s}20$	$0^{s}88$	$+0^{s}06$
	13	6.0	1.3	25	93	+ 14
	14	5.9	1.3	38	87	+ 17
	18	5.6	1.5	52	91	+ 07
	20	5.5	1.7	57	91	+ 10
	21	5.4	1.7	53	93	+ 08
	26	5.1	2.0	34	89	+ 07
Aug.	2	4.6	2.4	30	1.06	− 11
	8	4.3	2.6	52	89	− 01
	9	4.2	2.7	32	99	− 15
	16	3.7	3.0	44	97	+ 09
	23	3.3	3.3	40	92	+ 04
	25	3.2	3.4	40	88	+ 05
	31	2.8	3.7	45	91	
Sept.	6	2.5	3.9	49	93	
	14	2.0	4.1	61	93	
	20	1.6	4.4	62	88	
Oct.	3	0.8	5.0	74	82	
	10	0.4	5.3	71	84	
	16	0.1	5.5	30	88	
	17	0.0	5.5	59	66	

1905		Polaris U.C.	Sunrise	*a*	ΔM	N—S
July	11	$6^{h}1$ A.M.	$+1^{h}2$	$+0^{s}18$	$0^{s}92$	$+0^{s}04$
	13	5.9	1.1	21	88	− 01
	17	5.7	0.8	45	90	+ 11
	18	5.6	0.7	42	92	+ 09
	20	5.5	0.6	45	91	+ 06
	25	5.1	0.2	32	79	+ 23
	26	5.1	+0.1	31	78	+ 18
Aug.	1	4.7	−0.4	40	86	− 04
	2	4.6	0.5	43	84	+ 06
	8	4.2	1.0	39	84	+ 11
	15	3.8	1.5	34	93	+ 10
	16	3.7	1.6	40	89	+ 11
	23	3.3	2.1	29	92	+ 03
	30	2.9	2.7	44	84	+ 17
	31	2.8	2.8	34	90	+ 12
Sept.	5	2.5	3.2	38	89	+ 13
	6	2.4	3.3	47	93	+ 04
	13	2.0	3.8	49	92	− 01
	14	2.0	3.9	55	90	+ 11
	19	1.7	4.3	54	94	+ 03
	20	1.6	4.4	51	90	+ 06
Oct.	2	0.9	5.4	69	82	+ 03
	3	0.8	5.5	61	86	+ 03
	9	0.5	6.0	67	78	+ 14
	10	0.4	6.1	73	79	+ 13
	16	0.0	6.5	44	75	+ 09

Table I.—Azimuth and Mire—(Continued)

1905		Polaris L.C.		a	ΔM	N—S
			Sunrise			
Oct.	18	11^{h}9 A.M.	+5^{h}3	+0^{s}38	0^{s}94	
	23	11.6	5.0	50	76	
	24	11.5	4.9	46	87	
	25	11.5	4.8	51	86	
	31	11.1	4.3	46	78	
Nov.	1	11.0	4.2	40	87	
	2	11.0	4.1	40	82	
	3	10.9	4.1	43	82	
1906						
Jan.	4	6.4	−0.9	38	84	+ 09
	29	4.6	2.3	31	87	+ 04
	30	4.6	2.4	32	90	+ 07
	31	4.5	2.5	27	87	+ 10
Mar.	7	2.2	4.0	62	74	+ 15
	8	2.2	4.1	52	78	+ 12
	9	2.1	4.1	44	77	+ 09
			Sunset			
May	8	10.4 P.M.	+3.5	22	84	+ 08
	9	10.4	3.4	12	87	+ 11
	17	9.8	2.8	48	75	+ 04
	18	9.8	2.7	39	78	+ 09
June	7	8.4	1.2	39	81	− 07
	13	8.0	0.7	22	86	− 09
	14	7.9	0.6	26	89	− 03
	18	7.6	0.4	39	82	− 17
	19	7.6	0.3	35	86	− 02
	20	7.5	0.2	27	84	− 07
	21	7.4	+0.1	11	87	− 11
July	2	6.7	−0.5	33	91	+ 05
	3	6.6	0.6	24	91	+ 03
	5	6.5	0.7	17	93	+ 12

Sum	Epoch	Means		
50	1905.96	+0.400	0.866	
34	06.04	351	874	+0.036
24	05.90	355	896	+ 047

1905		Polaris U.C.	Sunrise	a	ΔM	N—S
Oct.	17	0^{h}0	−6^{h}6	+0^{s}50	0^{s}82	+0^{s}07
			Sunset			
	23	11.6 P.M.	+6.1	50	77	+ 16
	24	11.5	6.1	49	85	+ 03
Nov.	1	11.0	5.7	46	79	+ 12
	2	10.9	5.7	42	84	+ 10
1906						
Jan.	4	6.4	1.7	45	87	00
	5	6.4	+1.6	40	89	+ 06
	29	4.7	−0.4	38	87	+ 02
	30	4.6	0.5	44	83	+ 02
	31	4.5	0.6	35	88	+ 07
Mar.	9	2.1	3.7	54	79	− 03
			Sunrise			
May	7	10.5 A.M.	+5.3	28	82	
	8	10.4	5.3	29	76	
	16	9.9	4.9	40	78	
	17	9.8	4.9	37	81	
June	10	8.2	3.4	24	84	
	13	8.0	3.2	15	88	
	14	7.9	3.2	21	90	
	17	7.7	3.0	31	84	
	18	7.6	2.9	25	80	
	19	7.6	2.9	24	82	
	20	7.5	2.8	09	83	+ 15
	21	7.4	2.7	07	85	+ 03
July	2	6.7	2.0	28	82	+ 10
	4	6.5	1.9	21	84	+ 13
	5	6.4	1.8	20	81	+ 10

Sum	Epoch	Means		
52	1905.96	+0.385	0.850	
42	05.83	411	856	+0.078
24	05.89	335	861	+ 071

Of the mean values summed up at the foot of Table I, the Δa (N−S) is the really significant figure. The mean azimuths from the two culminations of *Polaris* differ by $0^{s}.015$. The two mean values of ΔM differ by $0^{s}.016$. The means of the mire readings differ by $0^{s}.03$. These small differences are due to the error in the adopted right ascension of *Polaris*, and to the systematic changes in the position of the meridian mark, not entirely eliminated in the summations of the means.

The mean azimuths from *Polaris*, and from south stars, differ by $0^{s}.06$, which is undoubtedly due to errors of the adopted right ascensions. Though there is some irregularity in the groups of south stars, owing to the difficulty of observing all of them in daylight, only small errors arise from this defect. The variation of the (N−S) can hardly be anything except an atmospheric effect. It is a differential factor, from which nearly all known sources of constant errors have been eliminated. Individual values of azimuth indicate the relatively large and irregular changes in the position of this instrument. The average change in a between consecutive transits of *Polaris* at the same culminations, 24 hours apart, is $\pm 0^{s}.07$, and the largest is $0^{s}.29$.

The total number of dates is given for three summations. Thus *Polaris* was observed on 102 dates, a group of south stars was observed on 76 of these dates, but there are only 48 dates on which the two groups of south stars were observed in sequence. The south group with *Polaris* L. C. is lacking, from the end of August until the end of the year 1905. The south group with *Polaris* U. C. is lacking for May, and part of June, 1906.

Combinations of the single values are given in Table II for four periods of the day. Values within two hours of sunset or sunrise are included in those groups. For *Polaris* L. C. the combinations at sunset, and by day are relatively strong. For *Polaris* U. C. those at sunrise, and at night are relatively strong.

TABLE II

FOUR PERIODS

							Polaris	
		No.	Epoch			a	ΔM	N—S
Polaris L.C. and South stars								
Night.	Mar. 7 to May 18	7	$11^{h}.8$ P.M.			$+^{s}.398$	$0^{s}.790$	$+0^{s}.100$
Sunset.	June 7 to July 26	17	6.6 P.M.	Sunset	$-0^{h}.5$	324	882	+ 019
Day.	Aug. 2 to Aug. 25	6	4.4 P.M.	Sunset	−2.9	396	952	− 016
Day.	Aug. 31 to Nov. 3	16	11.3 A.M.			503	848	
Sunrise.	Jan. 4 to Jan. 31	4	5.0 A.M.	Sunrise	−2.0	320	872	+ 077
Night—Sunset						+ 074	−092	+ 081
Night—Day						+ 002	−162	+ 116
Night—Day						− 105	−058	
Night—Sunrise						+ 078	−082	+ 023
Sunset—Sunrise						+ 004	+010	− 058
Polaris U.C. and South Stars								
Night.	Aug. 30 to Nov. 2	18	1.0 A.M.			+ 512	850	+ 085
Sunset.	Jan. 4 to Jan. 31	5	5.3 P.M.	Sunset	+0.4	406	868	+ 033
Day.	May 7 to June 19	10	8.8 A.M.	Sunrise	+3.9	275	825	
Day.	Mar. 9 and June 20 and 21	3	9.7 A.M.			237	825	+ 050
Sunrise.	July 2 to Aug. 23	16	5.2 A.M.	Sunrise	+0.2	331	866	+ 087
Night—Sunset						+ 106	−018	+ 052
Night—Day						+ 237	+025	
Night—Day						+ 275	+025	+ 035
Night—Sunrise						+ 181	−016	− 002
Sunset—Sunrise						+ 075	+002	− 054
Mean Δa (N—S) (Sunset—Sunrise) (42)								− 056
							Probable error	± 012

The systematic difference of azimuths from *Polaris* and south stars, which is $0^{s}.056$ between the epochs of sunset and sunrise, is the outstanding result of this comparison. One-half the difference probably represents the effect at each epoch. One-half should be due to each computation of azimuth.

Periodic variations have been computed, from all of the data, to represent the changes, if they are not mainly cumulative at the epochs of sunset and sunrise. The epochs of sunset, during a year, may vary from $4^{h}.7$ P.M. to $7^{h}.3$ P.M.; and the epochs of sunrise have a similar variation, from $7^{h}.3$ A.M. to $4^{h}.7$ A.M.

Ordinarily it is difficult to separate a diurnal term from an annual term, in meridian work carried on at nearly every hour of the day, and through a year. But the sequences for the same groups of stars, in this year of observation, are well adapted to distinguish between the two terms. A sequence of three culminations of *Polaris*, two upper and one lower, or the reverse, gives a mean value of a, a mean value of ΔM, and a mean value of (N−S). If there is a sequence of four culminations, there are two mean values of each factor. The sequence of six, at the end of January, gives four mean values, and the sequence of eight, in June, gives six mean values.

The half sums of upper and lower culminations eliminate any diurnal periodic terms, and are well adapted to the rigorous computation of annual terms. Similarly, the differences between upper and lower culminations eliminate any annual terms, and are adapted to the computation of diurnal terms.

The periodic terms in mire readings, and in the observations of *Polaris* include all 102 dates. The annual terms from south stars, with 48 dates, have only one-quarter of the weight of the full series of the year. This is due to the lack of data, in the especially unfavorable spring season. The diurnal terms, for the south stars, have three-quarters of the weight of the full series, since the lacking data have not as much significance, in a diurnal term.

Accordingly, the annual terms from south stars have been strenghtened, by correcting the values, by the diurnal periodic terms, on dates when but one group of south stars was observed. Thus 76 dates for south stars can be utilized, and the weights are three times the weights of the 48 dates, in the computation of annual terms, or three-quarters of the weight of the full series.

TABLE III

ANNUAL PERIODIC TERMS

					Wt.
Mire Reading		102	$1^{\mathrm{s}}.22$	$-0^{\mathrm{s}}.130 \sin t$	4
a	*Polaris*	102	$+37$	$-\ 090 \sin t$	4
a	South stars	48	$+30$	$+\ 047 \sin t$	1
	Corrected for Diurnal Term	76	$+32$	$-\ 091 \sin t$	3
ΔM	*Polaris*	102	0.85	$-\ 019 \sin t$	4
ΔM	South stars	48	0.90	$-\ 083 \sin t$	1
	Corrected for Diurnal Term	76	0.90	$-\ 042 \sin t$	3
Δa	(N—S)	48	$+05$	$-\ 007 \sin t$	1
	Corrected for Diurnal Term	76	$+05$	$+\ 009 \sin t$	3

DIURNAL PERIODIC TERMS

Mire Reading		102	$-0.101 \sin T$	4
a	*Polaris*	102	$-\ 061 \sin T$	4
a	South stars	48	$-\ 116 \sin T$	3
ΔM	*Polaris*	102	$-\ 036 \sin T$	4
ΔM	South stars	48	$+\ 021 \sin T$	3
Δa	(N—S)	48	$+\ 043 \sin T$	3

The following conclusions can be drawn from these computations. Both of the periodic terms in the mire readings are distinct. They include the variations of the azimuth of the instrument, and the variations in the position of the meridian mark. The epoch t, from which the annual terms are reckoned, is the first of January. The epoch T, for the diurnal terms, is midnight.

The periodic terms in azimuth are distinct. The annual term from south stars, when strengthened to include all 76 dates, changes sign. The annual term in a, from both sources, is $-0^{\mathrm{s}}.09 \sin t$. The diurnal terms differ by $0^{\mathrm{s}}.05$, between results from *Polaris* and from south stars, and their mean is $-0^{\mathrm{s}}.09 \sin T$. The annual term in ΔM is $-0^{\mathrm{s}}.03 \sin t$, mean of both, which agrees well with the results of many other years of observation. The diurnal term in ΔM is small, less than $0^{\mathrm{s}}.01 \sin T$, from both computations.

The periodic terms in Δa (N−S) are the essential features. The annual term is negligible. The diurnal term is distinct, and amounts to $+0^{\mathrm{s}}.04 \sin T$.

This periodic diurnal term of Table III has a larger amplitude than the difference given by the summation of means, in Table III, when the average epochs of sunset and sunrise are taken at six hours from midnight. The resulting correction to azimuths, determined from transits of *Polaris*, is close to $0^{\mathrm{s}}.02$, ($0''.3$), at each of the two epochs. With the azimuth factor for clock stars at the equator, the average correction to the computed clock corrections is $0^{\mathrm{s}}.012$. Since this increases the direct effect, ($0^{\mathrm{s}}.02$), upon transits of the clock stars, the total effect upon computed clock corrections is $0\overset{s}{.}03$; which is the difference given by the direct comparison of observations at the two epochs of the day.

Periodic variations, in the form of a trigonometric function, rarely give as large a double amplitude as the original results. The least squares solution balances the effects of plus and minus residuals, as well as is mathematically possible; while the full amplitude of the observed results is generally due to a plus residual near one extreme, and a minus residual near the other extreme.

In line with the results for a distinct diurnal term in Δa (N−S), there is the excess of rates in azimuth from mire readings, over the rates in azimuth from transits of the circumpolar stars, which has been a persistent feature of our meridian work throughout many years of observation. The diurnal term in the mire readings above is $0\overset{s}{.}04$ greater than the diurnal term in azimuth from *Polaris*. The mean of the diurnal terms in azimuth, from *Polaris* and south stars, agrees better with the diurnal term in the mire.

There is also the excess of negative clock rates at night, over the daily rate of the clock, which prevailed in our observations of many years. This appears to have been due to the periodic error of azimuth measures from circumpolar stars, and the consequent error in azimuth rates, as deduced in this present investigation.

These phenomena converge upon the conclusion here reached, that the observed transits of all stars are subject to a systematic deviation, near the epochs of sunset and sunrise.

One other feature that has received ample treatment is the discussion of our level errors, in fundamental work. There is a distinct, and very consistent annual variation, in the level of this instrument. But there is no diurnal variation, of any sensible amount. Had there been any such variation, the effect upon clock corrections would not have been traceable so directly to atmospheric sources, which are illustrated in the computed azimuths.

There is no material for a direct discussion of a diurnal term in the collimation, since the measures were made either in the late forenoon or the early afternoon. The correction has always been small, and the changes from date to date have been very slight. If there were a diurnal variation, it should appear in the level errors, derived by correcting the readings at the nadir with the same value of c for several consecutive dates, and at opposite epochs of the day.

In the fundamental right ascension observations at Greenwich, summarized in the recent catalogue for the epoch 1925.0, there is recorded a systematic correction of $-0\overset{s}{.}02$ to daytime transits. These were made, however, between 10 A.M. and 4 P.M. according to Sir Frank W. Dyson, Astronomer Royal, and thus do not contribute to the solution of the sunset and sunrise effect. An effect of this kind has been found, by Mr. M. L. Zimmer, at Cordoba; and it is distinct in the long series at San Luis, and at Albany. The observations at the U. S. Naval Observatory have been stated as lacking in confirmation of this difference.

There is an analogy in the change in vertical atmospheric refraction, which has been derived here, occurring mainly in the intervals near sunset and sunrise, and which amounts to $0\overset{''}{.}5$ tan Z. D. The effect upon transits appears to be about half as large. The recently published fundamental work of the Poulkova Observatory confirms a change in vertical refraction, between day and night, but of smaller amplitude, ($0\overset{''}{.}14$ tan Z. D.), than that derived here.

Lick Observatory,
September 28, 1925.
Issued November 12, 1925.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 369

THE RADIAL VELOCITIES OF TWENTY SOUTHERN VARIABLE STARS OF CLASS Me

By

LEAH B. ALLEN

A series of spectrograms of twenty of the brighter Me long period variable stars in the southern sky was made at the Chile Station by Dr. G. F. Paddock in 1922 and 1923, the faintest star being 7.5 magnitude at maximum. The plates were made with a one-prism spectrograph attached to the 37.5-inch reflecting telescope, and have a dispersion of 36 A per millimeter at Hγ. The iron comparison is in fair focus from about λ 3700 to λ 4957. The continuous stellar spectrum fades out rapidly in wave-lengths less than Hδ and the spectra on most of the plates are too faint to show absorption beyond λ 4200.

I have measured the radial velocities of these stars with a half-millimeter Toepfer engine, and have used the stellar wave-lengths published by Dr. P. W. Merrill in *Contributions from the Mount Wilson Observatory*, No. 265 in the reductions. Comparisons of my measures of T *Columbae* with results of other observers, of α *Orionis* with measures by Paddock of the same plate, and of L_2 *Puppis* with results from 2-prism spectrograms, show that there is probably no systematic error greater than one kilometer in my measures.

Table I gives the following data: Column 1) the designation of the stars; 2 and 3) the equatorial coördinates for 1900; 4) the velocity in kilometers per second from the absorption lines. The values for R *Reticuli* and X *Centauri* were estimated from the correlations between emission and absorption velocities shown by Merrill[1]; all other values are from my measures. Column 5 gives the total number of absorption lines measured; 6) the number of spectrograms with measurable absorption lines. Column 7 gives the velocity from the emission lines Hγ and Hδ; but since one or the other of the lines was missing on some of the plates, columns 8 and 9 give respectively the total number of the lines and plates measured. Thus the velocity +9 km/sec in line 1 of column 6 rests on 13 measures of the lines Hγ and (or) Hδ on 9 plates. Column 10 gives the mean velocity of all the emission lines; and column 11 the number of sources of the emission lines; in addition to Hγ and Hδ, the sources are Hβ, Hε, Hζ, Hη, λλ 3905, 4178, 4202, 4233, 4308, and 4571. Column 12 gives the difference between the absorption and emission velocities read from Merrill's curve[2] in which the difference is correlated with period.

Column 13 tabulates the residuals obtained by subtracting the values in column 12 from the observed difference between the absorption and emission velocities. Columns 14, 15, 16 give the period of light variation and the magnitudes at maximum and minimum taken from the *Vierteljahrschrift*. Column 17 gives the spectral class at maximum as estimated by me except in the cases indicated by the notes.

I have used the radial velocities of these southern variable stars to supplement the data from the 133 similar variable stars from which Merrill computed values of the solar motion, and have made a new solution. Assuming the apex at $A=270°$, $D=30°$ and solar velocities of 20 and 50 km, I computed the residual velocities of each star in Merrill's Table V (*Contrib. Mt. Wilson Obs.* No. 264), and in the above list. Whether computed with 20 km or 50, the only stars whose velocities lie conspicuously off of an error curve are S *Librae* and S *Carinae*. In the further computations, therefore, these two stars were omitted; five others are common to the two lists, leaving 146 as the basis for the solution.

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[1] *Contrib. Mt. Wilson Obs.* **12**, 125, 1924.

[2] *Contrib. Mt. Wilson Obs.* **12**, 161, 1924.

It can be seen from column 13 of table I that the value given by the curve for the difference between absorption and emission velocities agrees with the observed difference within a small fraction of the velocity except for the three stars R *Centauri*, S *Gruis*, and R *Phoenicis*. The first of these has a longer period than any from which the curve was derived and the result was obtained by extrapolation. The measured velocities of R *Phoenicis* and S *Gruis*, from few poorly defined lines, may have errors comparable with the differences between them and the computed values. Six of the stars of Merrill's list have periods longer than any for which he had measured velocities; but if the adopted velocity of each of the six should be 20 km. too large, the solar motion computed from the 146 stars would not be significantly changed.

TABLE I

Star	α1900	δ1900	Vel. abs.	No. lines	No. plates	Vel. Hγ, Hδ	No. lines	No. plates	Total vel. em.	No. sources	V.abs.-v.em. from curve	(O–C) v.abs.-v.em.	Period	Mag. Max.	Mag. Min.	Spectral class
S Sculptoris	0^h $10^m.3$	−32° 36′	+ 35	11	2	+ 9	13	9	+ 14	4	+14	+ 7	367*a*	6.3	9.8	M8e[1]
R Horologii	2 50.6	−50 18	+ 59	43	6	+ 46	12	6	+ 48	7	+18	− 7	405	4.0	10.2	M8e
R Reticuli	4 32.5	−63 14	(27)	0		+ 18	2	2	+ 18	1(δ)	+ 9		279	7.1	<13.5	Mbe[2]
R Pictoris	4 43.5	−49 26	+208	21	7	+204	10	6	+200	3	+ 7	+ 1	165	6.7	9.2	M0e
T Columbae	5 15.6	−33 49	+ 66	26	6	+ 54	12	6	+ 58	4	+13	− 5	225	7.0	12.4	M5e[1]
R Octantis	5 56.4	−86 26	+ 46	8	3	+ 30	6	3	+ 31	4	+18	− 3	408	6.8	<12?	M6e
L_2 Puppis	7 10.5	−44 29	+ 54	204	12	+ 48	24	12	+ 51	9	+ 3	0	140	3.3	6.3	M5e[1]
R Carinae	9 29.7	−62 21	+ 28	97	8	+ 14	30	15	+ 17	9	+11	0	309	4.7	<10.0	M5e
S Carinae	10 6.2	−61 4	+289	108	9	+278	18	9	+274	6	+ 5	+10	149	5.8	9.3	K9e[3]
X Centauri	11 44.2	−41 12	(38)	0		+ 26	8	4	+ 28	3	+11		314	7.5	11.8	M7e
T Centauri	13 36.0	−33 6	+ 31	120	28	+ 28	45	23	+ 30	6	+ 1	0	91	5.6	9.0	K2[4]
R Centauri	14 9.4	−59 27	− 20	51	4	− 25	8	4	− 28	5	+24	−16	568	5.3	13	M4
T Normae	15 36.4	−54 40	− 31	20	6	− 36	12	6	− 41	4	+10	0	244	7.0	12.0	M4e
RS Scorpii	16 48.4	−44 56	+ 3	45	6	− 5	18	9	− 1	5	+12	− 8	323	6.5	<12.0	M6e
RR Scorpii	16 50.3	−30 25	− 35	40	7	− 46	16	8	− 42	5	+ 9	− 2	281	6.2	12.2?	M7e
T Pavonis	19 39.5	−72 1	+ 68	14	5	+ 62	16	8	+ 63	4	+10	− 5	244	7.3	<12.5	M7e
S Pavonis	19 46.8	−59 27	− 23	48	6		0		− 27	3	+11	− 7	397?	7.2	9.6	M8e[5]
S Gruis	22 19.9	−48 57	− 19	7	3	− 24	6	3	− 21	4	+18	−16	402	7.4	12.0?	M7e
R Phoenicis	23 51.3	−50 21	+ 23	5	1	− 5	6	3	− 5	2	+ 8	+20	270	7.4	13.0?	M7e[6]
S Phoenicis	23 53.9	−57 8	+ 10	12	3	+ 6	6	3	+ 5	3	+ 5	0	151	7.4	8.2	M5e

[1] *Contrib. Mt. Wilson Obs.* 264, Table V.
[2] *Henry Draper Catalogue.*
[3] *Harv. Bull.* 798, 1924.
[4] *Harv. Circ.* 253, 1924, page 1.
[5] From spectrogram taken 3 months after maximum.
[6] Spectrum very faint.

Although the velocity derived from the measured emission lines seems sufficiently reliable for statistical purposes, it was of interest to compute the solar motion also from the 63 stars whose velocities were measured by absorption lines. A larger proportion of these stars are southern and there are fewer of the northern stars fainter than the southern. The results are:

From 146 stars

$V_o=57\pm\ 6$ km/sec; $A_o=278°\pm\ 7°$; $D_o=35°\pm 6°$; $K=2$ km

From 63 stars:

$V_o=72\pm 13$ km/sec; $A_o\pm 275\ \pm 14°$; $D_o=42°\pm 9°$; $K=9$ km

The first value is 5 km greater than that found by Merrill for 132 stars. The second is 7 km greater than that found by him for 65 stars having high residual velocities. The probable errors, indicating large dispersion in the stellar velocities rather than errors of measurement, show the somewhat indeterminate character of the problem.

In the course of measuring the spectrograms, I have found two stars of high velocity; R *Pictoris*, +208 km/sec, with respect to the Sun; and S *Carinae* +289 km/sec. Their spectra change with the light phase through the early subdivisions of class Me and their periods are shorter than the average of this type of variable. These results are in accord with the correlations of high velocity, short period, and spectral type described by Merrill.[3] To see if their apices lie in the same hemisphere, between galactic longitudes 131° and 322°, with those of the fast moving stars investigated by Adams and Joy,[4] and also, like them, near the galactic plane, I computed their space motions from the formulae given by Adams and Joy.[5] To get the parallaxes for this computation I assumed the absolute magnitude, −0.5, adopted by Merrill and Strömberg[6] for stars of this spectral type, for both stars and used their apparent magnitudes at maximum.[7] The resulting parallaxes are 0″.004 for R *Pictoris* and 0″.006 for S *Carinae.* The proper motions found by Dr. R. E. Wilson[8] and the solar motion derived from the 146 stars were adopted. The results, in harmony with those from other high velocity stars, are:

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[3] *Pub. A. S. P.* **35**, 171, 1923.
[4] *Ap. J.* **49**; 183, 1919.
[5] *Loc. cit.*

R *Pictoris:* $v=172$ km/sec; $L=197$; $B=-17°$

S *Carinae:* $v=254$ km/sec; $L=238$; $B=0°$

September, 1925.

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[6] *Ap. J.* **59**; 97, 1924.
[7] *V. J. S.*, **59**; 234, 1923.
[8] *A. J.* **34**; 183, 1923.

A STUDY OF THE CHANGES IN THE SPECTRUM OF T CENTAURI

By

LEAH B. ALLEN

Dr. Paddock[1] has called attention to several peculiarities in the spectrum of T *Centauri.* His observation of close agreement in absorption and emission velocities accords with the reading on Merrill's curve[2] for a period of 91 days and the mean of my measures shows a difference of only 3 km/sec. Paddock

TABLE I

Plate	J. D.	Phase	V. abs. selected lines	V. abs. all lines	V. em. Hδ & Hγ	Intensity of Emission Lines					Spectral class
						Hβ	Hγ	Hδ	Hε	Hζ	
9065	2423439	−21 m	+22 (5)	+19 (7)	+23	4	4	2			M2e
9084	72	+12 m	28 (2)	27 (3)	36	7	8	4			M2e
9093	79	+19 m	36 (3)	32 (4)	30	12	12	8	1	2	M1e
9099	83	+23 m	28 (5)	25 (6)	30	12	12	8	1	1	M1e
9106	89	+29 m	36 (4)	32 (5)	31	10	12	8	1	1	M1e
9112	94	−16 M	36 (5)	36 (6)	29	12	12	8	2	1	M1e
9117	498	−12 M	40 (1)	24 (4)	32	12	12	8	1		M1e
9123	504	− 6 M	27 (4)	25 (6)	18	12	12	8	2½		M0e
9128	8	− 2 M	32 (5)	23 (8)	25	12	12	10	4		M0e
9133	14	+ 4 M	28 (4)	30 (6)	30	6	4	2			M1e
9137	19	+ 9 M	31 (12)	34 (14)	36	¼	1	2			M1e
9142	24	+14 M	28 (11)	26 (15)		0	0	?			M1e
9150	30	+20 M	34 (12)	33 (11)		0	0	?			M3e
9157	36	−16 m	36 (2)	36 (2)							M4e
9163	47	− 5 m	12	12 (2)							M4e
9167	52	Minimum	45 (1)	45 (1)	31	2	4	1			M4e
9173	54	+ 2 m	39 (4)	39 (3)	34	8	8	4			M4e
9177	58	+ 6 m	35 (3)	36 (4)	35	16	16	12			M3e
9181	61	+ 9 m	36 (5)	37 (8)	34	18	18	14	½	½	M2e
9188	67	+15 m	36 (2)	32 (5)	39	12	12	6			M1e
9193	70	−22 M	36 (8)	32 (10)	28	12	12	8			M1e
9197	75	−17 M	40 (2)	30 (4)	22	12	12	10	1	1	M0e
9203	90	− 2 M	24 (4)	23 (5)	24	12	12	10	4		K9e
9210	95	+ 3 M	29 (3)	23 (4)	25	8	6	4	2		M0e
9213	598	+ 6 M	34 (2)	21 (5)	20	4	4	4	2		M0e
9214	601	+ 9 M	15 (4)	18 (6)	20	6	6	4	1		M0e
9219	604	+12 M	17 (4)	17 (4)	20	2	2	2	0		M0e
9221	611	+21 M	23 (3)	23 (3)		½	½	½	0		M1e
			—	—	—						
Mean			32	28	28						
2511	18396	+10 m		43 (3)	42 γ	7	8	5			M3e
2571	18436	+10 M		18 (9)	23 γ	7	7	5			M1e
6798	21002	+12 M	17 (2)	3 (4)	17 (2)	12	12	8			M0e
2511				38	36 γ	Measured by G. F. P., *Lick Obs. Bull.* **9**, 69, 1917					
2571				19	20 γ						

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[1] *Lick Obs. Bull.* **9**, 70, 1917.

[2] *Contrib. Mt. Wilson Obs.* **12**, 161, 1924.

has noted the lack of coincidence between maximum light and strong emission lines, the rapid fading of the emission lines and the possibility of variable radial velocity. In 1923 he took a series of one-prism spectrograms throughout two consecutive periods of the star's variation. The dispersion is 36 A per millimeter at Hγ.

Table I contains the results of my measures of these plates. The columns are as follows:

1) Plate number. 2) Julian Day. 3) Phase expressed in number of days before (−) and after (+) minimum (m) and maximum (M). 4) Velocity from absorption lines, omitting a few lines which differ consistently from the mean of all. 5) The number of lines included in the values in the preceding column. 6) The mean velocity from all the absorption lines not judged too poor for measuring. 7) The number of lines included in the values given in column 6. 8) The velocity given by the mean of the emission lines Hγ and Hδ. 9–13) The relative strength of the five hydrogen lines. 14) The subdivision of class Me to which each spectrum seems to belong. The estimates in columns 9–13 are an attempt to show both the relative strength of the hydrogen emission lines compared one with another on each plate and also each compared with itself on different plates. To lessen the effects of different exposures, Hγ was compared with the continuous spectrum about midway between lines λ 4225 and λ 4237 as well as with itself on different plates. The other lines were compared with Hγ. The number thus indicates relative density and length but not width of the lines. The comparisons are made uncertain by the different effects of contrast in the background and changing character of the whole spectrum. Although only rough, the estimates indicate the change from weak to strong soon after minimum light, greatest intensity shortly before maximum light, the rapid fading, weakness at maximum, and vanishing a few days later. This is strikingly different from the behavior of hydrogen in other Me stars. Usually the lines are strong at maximum and most intense soon afterward at the phase in which in T *Centauri* they are weakest. Hγ and Hδ are much alike, but on one plate Hγ is not visible, and there is a slight uncertain speck which may be Hδ. Hδ appears symmetrically placed with respect to absorption on its edges. Hγ is in an absorption line toward the violet from the center of the absorption. When Hβ is visible it seems as wide or wider than Hγ, but it disappears earlier. There is an absorption line on its edge toward the red. When hydrogen is faint near maximum light and no flutings are present, the lack of contrast makes Hβ inconspicuous although apparently as intense as Hγ. When the flutings appear, the outline of Hβ becomes irregular and it has a veiled appearance before vanishing. Hε is more prominent than Hζ and comes to its maximum later than the other hydrogen lines.

The absorption lines at λλ 4206, 4258, and 4347, low temperature lines noted by Adams[3] as strong in stars of high absolute magnitude, seem to fluctuate irregularly. On most of the spectrograms of T *Centauri* λ 4258 is very nearly equal to λ 4260; but on one taken six days before a maximum, it is greater; two days before the following maximum it appears less; eight days later, greater, and twenty-one days after maximum, less. The emission line at λ 4571 is not present in the spectrum of T *Centauri*, but there is a rather strong absorption line at that wave length, slightly displaced toward the red with respect to the emission line in other stars.

Table I indicates a change of velocity: but since the values given by different lines on the same plate differ so consistently from each other that a variation in velocity might appear by a different selection of lines from plate to plate, Table II was made, containing the velocities derived from individual lines. However much these differ from one another, each shows a decrease near maximum. The decrease is also shown by the emission lines Hγ and Hδ in Table I. The absorption and emission velocities have been arranged separately in two groups each; one for the dates within

TABLE II

Plate	Phase	λ4216	λ4315	λ4344	λ4347
9065	−21m	+11	+46	+10	+17
9084	+12m			25	
9093	+19m		36	24	
9099	+23m		42	14	31
9106	+29m	40	32	2	
9112	−16M	42	40		45
9117	−12M		41	8	40
9123	− 6M	30	42	0	
9128	− 2M	36	39	− 4	33
9133	+ 4M		46	+21	24
9137	+ 9M	25	32		31
9142	+14M	33	43		31
9150	+20M	25	40	3	51
9157	−16m		38	19	
9163	− 5m		33	10	
9167	Min.		45		37
9173	+ 2m	31	40		
9177	+ 6m	27			
9181	+ 9m	21	47		34
9188	+15m		38		
9193	−22M	40	40		42
9197	−17M		47	16	34
9203	− 2M	20			
9210	+ 3M	39	30		20
9213	+ 6M		44	− 5	
9214	+ 9M		34		
9219	+12M	23	20	+ 2	9
9221	+21M	15	28		

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[3] *Pub. A. S. P.* **35**, 328, 1923.

15 days before and after maximum, one for all other dates.

The means of these groups are:

	Absorption	Emission
Near maximum	+26 km/sec	+25 km/sec
Other dates	34 km/sec	32 km/sec

The 1909 and 1916 spectrograms show this difference of velocity, but it is not evident at the earlier of the 1923 maxima. The magnitudes[4] at maximum range from 5.5 to 6.5 and at minimum from 7.1 to 9.1; but the observations do not show any connection between the velocity at a maximum and the magnitude at that maximum or the following minimum. The magnitude at the minimum preceding the low velocity in 1923 was 7.1, the brightest observed at minimum. More observations both of velocities and of magnitudes at times of maximum and minimum are needed.

It is with pleasure that I acknowledge my indebtedness to Dr. Aitken for the Martin Kellogg Fellowship; to him and other members of the Lick Observatory Staff for advice and encouragement, especially Messrs. Moore, and van de Kamp; and to Miss Applegate who assisted in the computation; also to Dr. P. W. Merrill of the Mount Wilson Observatory for valuable counsel.

Mount Hamilton,
August, 1925.
Issued November 12, 1925.

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[4] *Harv. Bull.* No. 254.

Ackd.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 370

PRELIMINARY ELEMENTS AND EPHEMERIS OF COMET *j* 1925 (VAN BIESBROECK)

By

A. D. MAXWELL and L. C. DAMSGARD

Comet *j* 1925 was discovered at the Yerkes Observatory by Van Biesbroeck on November 17. Dr. Jeffers describes it as having a sharp nucleus and a distinct tail about 2′ in length. Its estimated magnirude on November 18 was eight. The following observations, received by telegraph and telephone at the Students' Observatory, were used to determine a preliminary orbit and ephemeris:

1925, U. T.	α (1925.0)	δ (1925.0)	Observer
Nov. 17.4805	11ʰ 56ᵐ 32ˢ.6	+34° 55′ 08″	Van Biesbroeck, Yerkes
Nov. 18.5163	11 57 25.6	+34 33 39	Jeffers, Lick
Nov. 19.5173	11 58 14.8	+34 13 08	Jeffers

ELEMENTS

T	1925, Sept. 28.5866 U. T.	
ω	85° 17′ 05″	1925.0
☊	335 36 04	
i	41 04 07	
$\log q$	0.08319	

THE CONSTANTS FOR THE EQUATOR 1925.0

$$x = [9.98339] \sin (156^\circ\ 24'\ 19'' + v)$$
$$y = [9.72310] \sin (\ 39\ \ 28\ \ 44 + v)$$
$$z = [9.94998] \sin (\ 74\ \ 39\ \ 21 + v)$$

The following residuals, obtained by Leuschner's Method without differential correction, are comparable with errors of observation:

O—C	I	III
$\cos \delta \Delta\alpha$	+4″	+3″
$\Delta\delta$	−2	−1

EPHEMERIS FOR GREENWICH MIDNIGHT

U. T.	True α	True δ	log Δ	Br.
Nov. 21.0	11ʰ 59ᵐ 24ˢ	+33° 42′.9	0.131	0.97
22.0	12 00 07	23.2		
23.0	00 47	+33 03.8		
24.0	01 26	+32 44.6		
25.0	02 02	25.7	0.131	0.93
26.0	02 35	+32 07.1		
27.0	03 07	+31 48.8		
28.0	03 36	30.8		
29.0	04 02	+31 13.0	0.130	0.89
Nov. 30.0	04 27	+30 55.5		
Dec. 1.0	04 49	38.3		
2.0	05 08	21.3		
3.0	26	+30 04.6	0.129	0.85
4.0	40	+29 48.2		
5.0	05 53	32.0		
6.0	06 03	16.1		
7.0	11	+29 00.4	0.127	0.82
8.0	16	+28 45.0		
9.0	20	29.9		
10.0	20	14.9		
11.0	19	+28 00.2	0.125	0.80
12.0	14	+27 45.8		
13.0	06 07	31.6		
14.0	05 57	17.6		
15.0	45	+27 03.9	0.123	0.77
16.0	30	+26 50.4		
17.0	05 13	37.0		
18.0	04 53	23.9		
Dec. 19.0	12 04 31	+26 11.0	0.119	0.74

The unit of brightness is that of November 18.

BERKELEY ASTRONOMICAL DEPARTMENT,
November 20, 1925.

PRELIMINARY ELEMENTS AND EPHEMERIS OF BROOKS' PERIODIC COMET *g* 1925

By

N. W. Storer and H. S. Mendenhall

Brooks' Periodic Comet was rediscovered at Simeis, Russia, by Albitzky and Shain on September 24. Its estimated magnitude at that time was twelve. In his most recent observation Dr. Jeffers describes it as faint, about 13.5 magnitude, diffuse, and rather difficult.

Prof. Dubiago's elements, as given in *A. N.* 5332, were corrected by Leuschner's Method to represent Nos. 3, 5 and 7 of the following observations. The last observation was received after the orbit was computed.

No.	U. T. 1925	α (1925.0)	δ (1925.0)	Observer	Place
1	Sept. 9.81468	23ʰ 23ᵐ 10ˢ.04	−1° 46′ 44″	Albitzky and Shain	Simeis
2	Sept. 24.79021	18 04.2	−4 03 11	Albitzky and Shain	
3	Sept. 25.99701	17 45.17	−4 12 22.3	W. Baade	Bergedorf
4	Sept. 28.33669	17 07.72	−4 30 12.7	Van Biesbroeck	Yerkes
5	Oct. 10.28919	15 43.04	−5 49 51.5	Jeffers	Lick
6	Oct. 15.32542	16 11.26	−6 14 16.7	Jeffers	
7	Nov. 5.12631	26 10.16	−6 43 29.5	Jeffers	
8	Nov. 18.29569	23 38 46.76	−6 03 34.4	Jeffers	

Elements

T	1925 Nov. 2.21196 U. T.	
ω	195° 54′ 07″.1	1925.0
☊	177 25 25.8	
i	5 32 52.1	
φ	29 13 03.4	
μ	511″.939	
P	6.93117 yrs.	
log a	0.560526	

Constants for the Equator 1925.0

$x=r[9.999996] \sin (103° \ 20′ \ 16″.3+v)$

$y=r[9.978435] \sin (13 \ \ 25 \ \ 05.9+v)$

$z=r[9.487842] \sin (12 \ \ 34 \ \ 02.6+v)$

The representation of the observations above is the following:

No.	Date		O—C cos $\delta\Delta\alpha$	O—C $\Delta\delta$
1	Sept. 9.815		−3′ 20″.9	+13′ 01″.8
2	Sept. 24.790		−0 36.0	− 0 26.8
3	Sept. 25.997	*	−0 00.1	− 0 00.2
4	Sept. 28.337		−0 03.7	+ 0 19.9
5	Oct. 10.289	*	0 00.0	0 00.0
6	Oct. 15.325		+0 02.4	− 0 03.0
7	Nov. 5.126	*	−0 00.6	+ 0 00.1
8	Nov. 18.296		−0 12.2	− 0 00.9

* The orbit is based upon these observations.

The orbit telegraphed to Harvard College Observatory on November 10 was based upon observations Nos. 2, 5, and 7. It was later found that the observations of September 25 and 28 were not well represented, showing a probable error in that of September 24, so a new solution was made using the observation of September 25 as first place.

Ephemeris for Greenwich Midnight

1925 U. T.	True α	True δ	log Δ	log r	Br.
Nov. 21.0	23ʰ41ᵐ57ˢ.7	−5°50′00″	0.0858	0.2714	0.56
22.0	43 09.4	44 44			
Nov. 23.0	23 44 22.3	−5 39 15			
Nov. 24.0	23ʰ45ᵐ36ˢ.5	−5°33′34″			
25.0	46 52.0	27 40	0.0990		
26.0	48 08.6	21 35			
27.0	49 26.4	15 18			
28.0	50 45.4	08 50			
29.0	52 05.5	−5 02 11	0.1123	0.2731	0.49
Nov. 30.0	53 26.7	−4 55 21			
Dec. 1.0	54 49.0	48 21			
2.0	56 12.3	41 10			
3.0	57 36.6	33 50	0.1255		
4.0	23 59 01.9	26 20			
5.0	0 00 28.2	18 40			
6.0	01 55.4	10 51			
7.0	03 23.6	−4 02 52	0.1388	0.2755	0.43
8.0	04 52.7	−3 54 44			
9.0	06 22.7	46 28			
10.0	07 53.6	38 03			
11.0	09 25.3	29 30	0.1520		
12.0	10 57.8	20 48			
13.0	12 31.2	11 59			
14.0	14 05.4	−3 03 02			
15.0	15 40.4	−2 53 58	0.1652	0.2784	0.38
16.0	17 16.2	44 47			
17.0	18 52.7	35 28			
18.0	20 29.9	26 02			
19.0	22 07.9	16 30	0.1782		
20.0	23 46.6	−2 06 52			
21.0	25 26.0	−1 57 07			
22.0	27 06.0	47 16			
23.0	28 46.7	37 20	0.1912	0.2818	0.33
24.0	30 28.0	27 18			
25.0	32 09.9	17 11			
26.0	33 52.4	−1 06 58			
27.0	35 35.4	−0 56 41	0.2040		
28.0	37 19.0	46 19			
29.0	39 03.1	35 53			
30.0	40 47.7	25 23			
Dec. 31.0	0 42 32.9	−0 14 48	0.2166	0.2857	0.29

The unit of brightness is that of September 25.

Berkeley Astronomical Department,
November 21, 1925.

SECOND ELEMENTS OF Y. O. 22 (1924 UA)

By

A. D. MAXWELL

The preliminary elements of this minor planet, published in *Lick Observatory Bulletin* **12,** 359, were corrected on the basis of later observations communicated by Van Biesbroeck. The observations received by the Students' Observatory are as follows:

	1924, Gr. M. T.		α (1925.0)	δ (1925.0)	Mag.
1.	Dec.	1.86105	$5^{h}\ 27^{m}\ 56^{s}.02$	+34° 26′ 06″.5	15.3
2.		15.59397	5 13 44.58	+34 32 33.5	
3.		20.81802	5 08 23.45	+34 28 10.2	
4.	Dec.	24.67366	5 04 38.09	+34 22 36.2	14.2
	1925, U. T.		α (1925.0)	δ (1925.0)	
5.	Jan.	30.24358	$4^{h}\ 46^{m}\ 59^{s}.96$	+32° 46′ 25″.3	
		30.26979	4 47 00.07	+32 46 19.3	
6.	Feb.	12.08456	4 50 01.48	+32 13 42.2	14.9
7.	Feb.	27.11609	4 59 05.80	+31 43 48.3	15.1
8.	Mar.	15.08073	5 14 06.33	+31 19 54.9	15.9
		15.09045	5 14 06.99	+31 19 54.4	16.1
9.	Mar.	29.10526	5 30 49.00	+31 01 33.1	16.8
		29.11741	5 30 49.81	+31 01 31.5	16.6

All of the foregoing observations were made by Van Biesbroeck at the Yerkes Observatory. From the run of the residuals, it was apparent that the first observation is less accurate than the others. Hence the present orbit is based on observations numbers two, five, and nine.

THE ELEMENTS

Epoch 1925, Jan. 30.24434 U. T.

M	25° 17′ 11″.8	
ω	51 16 16.8	1925.0
☊	11 27 58.5	
i	7 56 30.2	
φ	3 46 46.9	
μ	687″.93	
P	5.1578 years	
$\log a$	0.474975	

CONSTANTS FOR THE EQUATOR 1925.0

$$x = r[9.999837] \sin (152^\circ\ 37'\ 49''.9 + v)$$
$$y = r[9.932054] \sin (\ 63\ \ 35\ \ 05.2 + v)$$
$$z = r[9.715226] \sin (\ 60\ \ 02\ \ 14.3 + v)$$

The observations are represented as follows:

O—C			cos δΔα	Δδ
Dec.	1		−6″.4	+7″.3
Dec.	15	*	+0.2	−0.1
Dec.	20		+2.6	+2.0
Dec.	24		−0.7	−3.1
Jan.	30	*	0.0	0.0
Feb.	12		+2.1	−0.6
Feb.	27		−1.4	+0.3
Mar.	15		+1.9	+0.6
Mar.	29	*	0.0	+0.3

* Observations on which the orbit was based.

BERKELEY ASTRONOMICAL DEPARTMENT,
November 23, 1925.
Issued November 25, 1925.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 371

SECOND ELEMENTS AND EPHEMERIS OF COMET *j* 1925 (VAN BIESBROECK)

By

A. D. MAXWELL and L. C. DAMSGARD

The preliminary elements and ephemeris of Comet *j* 1925 are given in *Lick Observatory Bulletin* **12**, 370. The following observations were available for the determination of a second orbit:

	1925, U. T.	α (1925.0)	δ (1925.0)	Observer
(1)	Nov. 17.4805	11ʰ 56ᵐ 32ˢ.6	+34° 55′ 08″	Van Biesbroeck, Yerkes
(2)	18.5163	11 57 25.6	+34 33 39	Jeffers, Lick
(3)	19.5173	11 58 14.8	+34 13 08	Jeffers
(4)	21.5441	11 59 48.0	+33 32 02	Jeffers
(5)	Nov. 25.5398	12 02 27.2	+32 13 01	Jeffers

The first, fourth and fifth of these were made the basis of the second orbit, which was derived by differential correction of the preliminary orbit.

ELEMENTS

T 1925, Oct. 3.6500 U. T.

	1925.0	1926.0
ω	107° 23′ 19″.3	107° 23′ 19″.1
☊	334 30 16.4	334 31 06.8
i	49 37 33.4	49 37 33.8

$\log q$ 0.198995

CONSTANTS FOR THE EQUATOR 1925.0

$$x=r[9.975296]\sin(180^\circ\ 13'\ 20''.2+v)$$
$$y=r[9.661465]\sin(\ 47\ \ 57\ \ 27.2+v)$$
$$z=r[9.976442]\sin(\ 96\ \ 58\ \ 13.3+v)$$

CONSTANTS FOR THE EQUATOR 1926.0

$$x=r[9.975323]\sin(180^\circ\ 13'\ 56''.5+v)$$
$$y=r[9.661330]\sin(\ 47\ \ 58\ \ 37.3+v)$$
$$z=r[9.976447]\sin(\ 96\ \ 58\ \ 33.0+v)$$

The observations were represented as follows:

O—C	cos δΔα	Δδ
Nov. 17	−0″.6	−1″.7
18	−1.6	−2.7
19	−0.2	−0.3
21	0.0	0.0
Nov. 25	+0.5	−0.9

EPHEMERIS FOR GREENWICH MIDNIGHT

1925, U. T.	True α	True δ	log Δ	Br.
Dec. 9.0	12ʰ 07ᵐ 22ˢ	+28° 05′.0	0.198	0.99
10.0	07 28	+27 47.7		
11.0	07 32	30.5		
12.0	07 34	+27 13.4		
13.0	07 33	+26 56.5		
14.0	07 30	39.7		
15.0	07 25	23.0		
16.0	07 17	+26 06.5		
17.0	07 07	+25 50.0	0.185	0.98
18.0	06 55	33.7		
19.0	06 40	17.5		
20.0	06 23	+25 01.5		
21.0	06 03	+24 45.5		
22.0	05 41	29.6		
23.0	05 16	+24 13.8		
24.0	04 49	+23 58.1		
25.0	04 19	42.5	0.172	0.97
26.0	03 46	27.0		
27.0	03 11	+23 11.6		
28.0	02 34	+22 56.3		
29.0	01 53	41.0		
30.0	01 10	25.8		
Dec. 31.0	12 00 25	+22 10.7		
1926				
Jan. 1.0	11 59 37	+21 55.6		
2.0	58 46	40.6	0.159	0.96
3.0	57 52	25.7		
Jan. 4.0	11 56 56	+21 10.8		

1926, U. T.	True α	True δ	log Δ	Br.
Jan. 5.0	$11^h\ 55^m\ 57^s$	+20° 56′.0		
6.0	54 55	41.2		
7.0	53 51	26.5		
8.0	52 44	+20 11.8		
9.0	51 34	+19 57.1		
10.0	50 22	42.5	0.147	0.95
11.0	49 06	27.9		
12.0	47 49	+19 13.4		
13.0	46 28	+18 58.8		
14.0	45 05	44.3		
15.0	43 39	29.8		
16.0	42 11	15.4		
17.0	40 40	+18 00.9		
18.0	39 07	+17 46.4	0.137	0.94
19.0	37 32	32.0		
20.0	35 54	17.6		
21.0	34 13	+17 03.1		
22.0	32 31	+16 48.7		
23.0	30 46	34.3		
24.0	28 59	19.8		
25.0	27 11	+16 05.4		
26.0	25 20	+15 51.0	0.132	0.93
27.0	23 28	36.6		
28.0	21 34	22.2		
29.0	19 38	+15 07.8		
30.0	17 40	+14 53.4		
Jan. 31.0	11 15 42	+14 39.0		

1926, U. T.	True α	True δ	log Δ	Br.
Feb. 1.0	$11^h\ 13^m\ 42^s$	+14° 24′.6		
2.0	11 40	+14 10.2		
3.0	09 38	+13 55.9	0.132	0.87
4.0	07 35	41.6		
5.0	05 30	27.3		
6.0	03 25	+13 13.1		
7.0	11 01 20	+12 58.8		
8.0	10 59 14	44.7		
9.0	57 07	30.5		
10.0	55 00	16.4		
11.0	52 54	+12 02.4	0.138	0.79
12.0	50 47	+11 48.5		
13.0	48 40	34.6		
14.0	46 34	20.7		
15.0	44 28	+11 07.0		
16.0	42 23	+10 53.3		
17.0	40 19	39.7		
18.0	38 15	26.2		
Feb. 19.0	10 36 12	+10 12.8	0.152	0.66

The unit of brightness is that of November 21.

BERKELEY ASTRONOMICAL DEPARTMENT,
December 3, 1925.
Issued December 4, 1925.

A-6)

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 372

THE SPECTRUM OF *α CYGNI* BETWEEN WAVE-LENGTHS 3020 A AND 3000 A

By

W. H. Wright and Dorothy Applegate

In a paper published some years ago, one of the present authors[1] gave measurements of the spectrum of *α Cygni* in the ultra-violet region between 3250 and 4000 A. The measurements were made on plates taken with a quartz spectrograph attached to the Crossley reflecting telescope which record the stellar spectrum as far in the direction of shorter wave-lengths as is permitted by the reflectivity of a silver on glass mirror. The low reflectivity of such a mirror in the neighborhood of the lesser of the above limits is commonly known to be due to the relatively high transparency of silver to light of about 3250 A; and in giving the earlier measurements the suggestion was made that an extension toward 3000 A, the limit imposed by atmospheric absorption, might be effected through the elimination of the mirror. Shortly thereafter Mr. Wright secured photographs without a mirror, using a slitless spectrograph consisting of the two quartz prisms of the ultra-violet spectrograph and a quartz-fluorspar achromatic camera lens of 4.6 cm. aperture. These photographs provide records of the spectrum of *α Cygni* which extend in the direction of shorter wave-lengths to beyond 3020 A. They carry, of course, no comparison spectrum, and direct determinations of wave-length can consequently not be made from them, but the close correlation between the spectrum of this star and the spark spectra of certain of the metals makes it very easy to identify many of the stellar lines, and these serve very well as standards with respect to which the spectral positions of intermediate lines may be interpolated. Preliminary identification of these metallic lines was facilitated by the circumstance that the spectrograms cover a considerable part of the region described in the previous paper, so that, beginning on familiar ground, lines could be identified by a system of progressive extrapolation which was subject to continual check as the work progressed. In fact a Hartmann formula, calculated from lines in the known region of the spectrum, served admirably for the exploration of the entire unknown part, requiring only the aid of a graph of residuals derived from identified lines to provide wave-lengths of all of the measured positions.

In taking these photographs the spectrograph was mounted equatorially, together with a finder or following telescope, and so disposed on the mounting that the prism edge, and as a consequence the spectrum lines, were parallel to the diurnal motion. Furthermore the observations were made near the time of the star's diurnal elongation, so that the lines were also parallel to the refraction, and the effect of change in refraction during the exposure was minimized. The spectrum could, therefore, be broadened by "drifting" the star during the exposure without danger of injury to the definition. Data relating to the photographs are given in Table 1. Seed No. 23 plates were used except on October 31, when a Seed "Graflex" plate was exposed. The observations of October 28 were experimental in character and the plates were not measured.

TABLE 1

G. M. T.	*Exposure*
1921 Oct. 28.68	$0^h\ 30^m$
Oct. 28.71	1 00
Oct. 28.75	0 30
Oct. 30.69	2 40
Oct. 31.68	3 13
Nov. 1.70	4 00

A peculiarity of the spectrogram of October 30, remarked in a foot note to Table 2, is the exceptional strength of two lines, 3087 and 3110 A. The enhancement is especially noticeable in the line of greater wave-length because it is the stronger and also because it stands quite alone; the other line appears as a companion to a less refrangible stellar one of considerable strength and is found only on the plate referred to. There is nothing in the appearance of these lines which would indicate them to be spurious. Their variability might suggest that they owe their origin to an inconstant constituent of the Earth's atmosphere, but we do not know of anything that would serve to corroborate such a view. In this connection it may be remarked that the telluric bands due specifically to ozone which occur in this general region of the spectrum[2] are not found on our photographs. This is unquestionably due to the high altitude at which the star was observed, and also to the diffuseness of the bands themselves, which would contribute to their obliteration by the multitude of fine lines in the spectrum.

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[1] *Lick Obs. Bull.* **10**, 100, 1921.

The plates were measured by both of the authors and the wave-lengths were calculated by Miss Applegate. The lines used as standards were taken principally from Exner and Haschek's tables,[3] which are rendered in Rowland's system. The stellar lines are printed in Table 2 in both that and the International system.

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[2] Fowler and Strutt, *Proc. Roy. Soc. Lond.*, **A 93**, 577, 1917; also Strutt, **A 94**, 260, 1917.

[3] *Die Spektren der Elemente bei Normalen Druck.* Franz Deuticke, Leipzig u. Wien, 1912.

TABLE 2

THE SPECTRUM OF *a Cygni* BETWEEN 3020 A AND 3315 A

λ		Int.	No. Obs.	Description	Origin		Classification		Intensity	
Rowland	I. A.				Elem.	λ (I.A.)	Temp.	Series	Arc.	Spark
3020.3	3020.2	3	1	?	†	......	......	......	......	......
3026.7	3026.6	2½	1	?	Cr	26.69	......	......	–	8
3038.9	3038.8	3	1	......	*	......	......	......	......	......
3041.1	3041.0	3	3	......	Cr	40.91	......	......	–	10
3044.6	3044.5	2	2	......	......	......	......	......	......	......
3047.8	3047.7	2½	3	......	†	......	......	......	......	......
3049.2	3049.1	2	3	......	......	......	......	......	......	......
3050.3	3050.2	3+	3	......	Cr	50.11	VE	......	1	10
					Mn?	50.65	......	......	–	4
3057.0	3056.9	3	3	......	Fe?	57.45	II	......	15	3
3058.24	3058.12	3	3	......	Ti‡	58.11	V Eru	......	2	5
3059.64	3059.52	2½	3	......	†	......	......	......	......	......
3062.27	3062.15	4	3	*b*, prob. *d*	Fe‡	62.22	......	......	–	3
3066.44	3066.32	4	3	*b*	Ti$^+$	66.20	IV Er	$1f^4-1d'^4$	2	......
					Ti$^+$	66.36	IV Er	$1f^4-1d'^4$	2	3
3067.64	3067.52	1	2	......	Fe	67.12	IV	......	......	......
					Fe	67.25	II	......	15	3
					Fe	68.18	IV	......	2	1
3068.8	3068.7	2	2	......	......	......	......	......	......	......
3072.1	3072.0	4½	3	*b*, prob. *d*	Ti$^+$	72.10	III Er	$1f^4-1d'^4$	3	3
					Ti$^+$	72.97	III Er	$1f^4-1d'^4$	3	3
3075.26	3075.14	3	3	......	Ti$^+$	75.22	III Er	$1f^4-1d'^4$	4	4
3077.23	3077.11	3	3	......	Fe‡	77.13	......	......	1	3
3078.86	3078.74	3	3	......	Ti$^+$	78.64	III Er	$1f^4-1d'^4$	5	6
3083.0	3082.9	1+	2	‖	......	......	......	......	......	......
3087.07	3086.95	2	1	§	V‡	87.07	III	......	2	1
3088.07	3087.95	3+	3	......	Ti$^+$	88.03	III Er	$1f^4-1d'^4$	5	10
					V	88.10	III	......	2	1
3089.72	3089.60	2–	3	......	V‡	89.12	III	......	1	1
3093.32	3093.20	3½	3	......	V	93.14	III Er	......	4	20
3096.57	3096.45	2½	3	*b, n*	......	......	......	......	......	......
3097.44	3097.32	1½	1	......	Ti	97.18	V Er	......	2	4
3100.10	3099.98	1	1	?	Fe	99.97	II	......	5	4
					Fe	00.31	II	......	3	3
					Fe	00.67	II	......	3	3
3102.35	3102.23	2+	3	......	V	02.29	III Er	......	4	20
3103.7	3103.6	2–	3	......	Ti	03.80	V Er	......	2	5
3105.15	3105.03	3–	3	......	Mg$^+$	04.81	......	$2d^2-4f'^2$	......	......
3106.44	3106.32	2–	3	......	......	......	......	......	......	......
3110.78	3110.66	3–	3	§	V	10.71	III Er	......	4	20
3112.2	3112.1	1+	3	......	Ti	12.05	V E	......	1	3

TABLE 2—Continued

The Spectrum of α *Cygni* Between 3020 A and 3315 A

λ		Int.	No. Obs.	Description	Origin		Classification		Intensity	
Rowland	I. A.				Elem.	λ (I.A.)	Temp.	Series	Arc.	Spark
3114.61	3114.48	2	3	....	....	....	....	....	....	....
3116.8	3116.7	1+	3	....	....	....	....	....	....	....
3118.65	3118.53	3+	3	....	V	18.39	III Er	....	3	15
					Cr	18.65	V E	....	3	10
3120.40	3120.28	3+	3	....	Cr	20.38	V E	....	2	15
3122.88	3122.76	2	3	....	....	....	....	....	....	....
3125.13	3125.00	4−	3	....	Cr	24.98	V	....	3	20
3128.80	3128.67	3	3	....	Cr	28.60	V E	....	2	5
3130.61	3130.48	2−	3	....	V	30.26	III Er	....	3	10
3132.10	3131.97	3½	3	....	Cr	32.06	V E	....	4	20
3133.4	3133.3	1	2	....	V	33.31	III E	....	3	5
3135.53	3135.40	3−	3	....	V	34.97	....	....	2	8
					Fe	35.36	....	....	1	2
3136.84	3136.71	3−	3	....	V	36.48	V E	....	2	8
					Cr	36.64	V E	....	2	5
3140.18	3140.05	2−	3	....	V	39.70	V E	....	1	8
3144.0	3143.9	1	2	....	Ti	43.75	IV E	....	2	4
3145.06	3144.93	2	3	....	....	....	....	....	....	....
3147.34	3147.21	3	3	....	Cr	47.18	V E	....	2	5
3150.09	3149.96	2	3	....	....	....	....	....	....	....
3152.32	3152.19	2+	3	....	Ti^+	52.24	IV Eru	$2f^4-1d'^4$	2	5
3154.23	3154.10	3½	3	....	Ti^+	54.18	IV Eru	$2f^4-1d'^4$	2	4
					Fe	54.20	....	....	—	4
3155.8	3155.7	1½	3	....	Ti^+	55.65	IV Eru	$2f^4-1d'^4$	2	4
3157.7	3157.6	1−	1	?	....	....	....	....	....	....
3159.06	3158.93	3−	3	....	Ca^+	58.88	V	$1p^2-2d^2$	10	50
3162.30	3162.17	4	3	*b*, *n*, prob. *d*	Ti^+	61.19	III Eru	$2f^4-1d'^4$	4	4
					Ti^+	61.76	III Eru	$2f^4-1d'^4$	4	4
					Ti^+	62.56	III Eru	$2f^4-1d'^4$	4	6
3165.17	3165.04	1	2	....	V	64.83	V E	....	1	3
3166.72	3166.59	2−	2	....	....	....	....	....	....	....
3168.22	3168.09	3½	3	....	Fe	67.88	....	....	1	4
					V	68.12	V E	....	....	....
					Ti^+	68.52	III Eru	$2f^4-1d'^4$	5	15
3170.42	3170.29	2½	3	....	....	....	....	....	....	....
3172.17	3172.04	1	3	....	La?	71.63	....	....	1	20
3174.02	3173.89	1	1	?	....	....	....	....	....	....
3175.56	3175.43	1	3	....	....	....	....	....	....	....
3177.64	3177.51	3	3	....	....	....	....	....	....	....
3179.65	3179.52	3	3	....	Ca^+	79.34	V	$1p^2-2d^2$	15	50
3180.94	3180.81	3	3	....	Cr	80.72	III	....	3	10
					Ca^+	81.28	V	$1p^2-2d^2$	3	10
3183.23	3183.10	3	3	....	Fe	83.16	....	....	—	2
					Sn	83.60	....	....	—	20
3185.36	3185.23	2	3	....	....	....	....	....	....	....
3187.03	3186.90	3½	3	....	Fe‡	86.78	....	....	—	2
					Fe‡	87.32	....	....	—	2
3190.97	3190.84	3−	3	....	V	90.67	....	....	3	10
				....	Ti	90.87	IV Er	....	4	10
3193.51	3193.38	4	3	very *b*	Fe‡	93.87	....	....	—	3
3196.70	3196.57	4+	3	very *b*, *d*	Fe	96.12	....	....	1	2
					Cr	97.07	V E	....	2	15
3199.6	3199.5	1−	1	?	....	....	....	....	....	....
3202.8	3202.7	2	3	*b*, *n*	Ti	02.52	V Er	....	3	10
3205.4	3205.3	1	2	....	....	....	....	....	....	....
3209.12	3208.99	3	3	....	Cr	09.17	V E	....	2	10
3210.60	3210.47	3	3	....	Fe^+	10.46	....	$1p^4-1d'^4$	1	2
3212.3	3212.2	2	2	....	....	....	....	....	....	....
3213.44	3213.31	4	3	....	Fe^+	13.32	....	$1p^4-1d'^4$	1	3
3214.8	3214.7	2	1	?	....	....	....	....	....	....

TABLE 2—Continued

THE SPECTRUM OF *a Cygni* BETWEEN 3020 A AND 3315 A

λ		Int.	No. Obs.	Description	Origin		Classification		Intensity	
Rowland	I. A.				Elem.	λ (I.A.)	Temp.	Series	Arc.	Spark
3217.29	3217.16	4	3	....	Ti$^+$	17.04	III Er	$1f^4-1f'^4$	4	8
					V	17.15	....	....	2	10
					Cr	17.41	V E	....	2	8
3223.00	3222.87	2½	2	....	Ti$^+$	22.82	III Er	$1f^4-1f'^4$	5	8
3224.31	3224.18	1	2	¶	Ti$^+$	24.23	V Eru	$1h^2-2g'^2$	2	8
3226.0	3225.9	1	1	?	....	....	....	....	....	....
3227.90	3227.77	3+	2	....	Fe$^+$	27.76	....	$1p^4-1d'^4$	4	5
3229.40	3229.27	3	2	....	Ti$^+$	29.40	V Eru	$1g^2-2f'^2$	3	3
3232.27	3232.14	3	2	*b, n*	Ti$^+$	32.26	IV Eru	$1g^2-2f'^2$	2	5
3234.52	3234.39	3+	2	....	Cr‡	34.07	....	....	—	6
					Ti$^+$	34.52	III Er	$1f^4-1f'^4$	8	15
3236.69	3236.56	3	2	....	Ti$^+$	36.57	III Er	$1f^4-1f'^4$	5	6
3237.95	3237.82	2	2	....	Fe	37.82	....	....	—	2
					V	37.92	....	....	2	10
3239.26	3239.13	3	2	....	Cr	38.77	....	....	1	6
					Ti$^+$	39.03	III Er	$1f^4-1f'^4$	4	6
3242.08	3241.95	3	2	....	Ti$^+$	41.97	III Er	$1f^4-1f'^4$	4	10
3243.85	3243.72	2+	2	....	Fe	43.74	....	....	—	2
3247.42	3247.29	3	2	....	Fe‡	47.19	....	....	1	2
3249.02	3248.89	2	2	....	Ti	48.66	....	....	4	10
3251.98	3251.84	2	2	....	Ti$^+$	51.89	III Er	$1f^4-1f'^4$	3	5
3253.10	3252.96	2	2	....	Ti$^+$	52.85	III Er	$1f^4-1f'^4$	3	6
3254.50	3254.36	2	2	....	Ti$^+$	54.23	III Er	$1f^4-1f'^4$	3	5
3256.04	3255.90	3	2	....	Fe$^+$	55.89	....	$1d^4-1d'^6$	1	2
3259.09	3258.95	4−	2	....	Fe‡	58.67	....	....	—	2
					Fe‡	59.05	....	....	—	2
3261.66	3261.52	2½	2	....	Ti	61.59	V Er	....	4	15
3264.60	3264.46	1+	2	....	....	....	....	....	....	....
3267.40	3267.26	2	2	*n*	V‡	67.71	....	....	3	20
3271.0	3270.9	2+	2	*b*, irreg.	V	71.13	....	....	2	20
3277.14	3277.00	4	2	*b*	V	76.12	....	....	3	20
					Fe$^+$	77.36	....	$1d^4-1d'^6$	1	2
3279.25	3279.11	2	2	....	Ti?	78.96	III A	....	3	5
					Zr	79.31	....	....	—	8
3281.67	3281.53	3	2	*n*	Fe$^+$	81.30	....	$1d^4-1d'^6$	—	1
3283.6	3283.5	1	1	?	Sn	83.46	....	....	—	20
3285.5	3285.4	1½	1	....	....	....	....	....	....	....
3287.9	3287.8	2	2	....	Ti	87.67	V Er	....	2	8
3289.7	3289.6	2	2	....	....	....	....	....	....	....
3292.06	3291.88	1½	2	....	Cr?	91.76	....	....	—	3
3295.76	3295.62	4	2	....	Cr	95.43	V E	....	1	5
3298.0	3297.8	1	1	....	....	....	....	....	....	....
3299.4	3299.3	1	1	?	....	....	....	....	....	....
3303.20	3303.10	2	2	*n*	Na‡	02.38	....	....	100	20
					Na‡	02.96	....	....	100	10
3308.1	3308.0	2	2	*n, d*	Cr	07.07	....	....	—	8
					Ti$^+$	08.79	III Eru	$2f^4-1f'^4$	3	5
3312.4	3312.3	2−	2	....	Cr	10.72	....	....	—	4
					Cr	11.96	....	....	—	3
					Cr	12.23	....	....	1	3
3314.9	3314.8	2	2	....	Cr	14.63	....	....	1	3
					Ti	15.34	V E	....	2	5

---

* There is a group of three Mn spark lines near this position, but they are hardly strong enough to account more than partially for the stellar line.

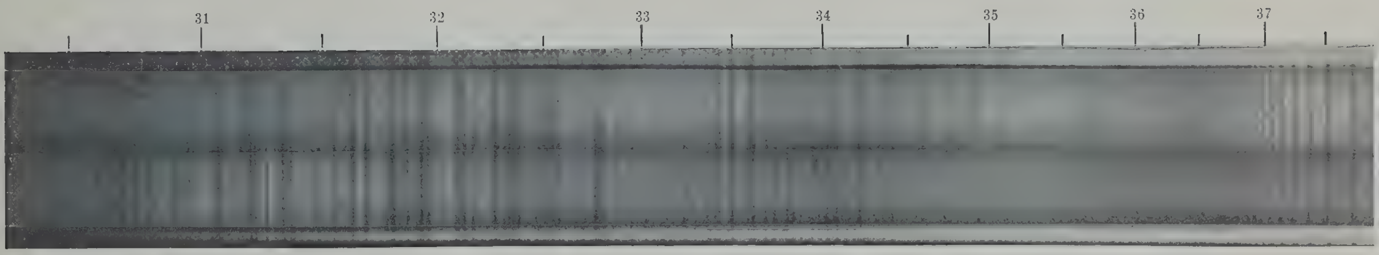
† There are strong lines in the arc spectrum of iron near these positions, but since they are assigned by King to Class I, it is doubtful whether the correspondences in wave-length which they supply are significant.

‡ Identifications regarded as doubtful, or at least, as incomplete; i.e., the suggested laboratory line is probably not wholly responsible for the stellar line.

‖ A hazy group of unresolved lines.

§ These two lines are much stronger on the plate of October 30 (No. 1099) than on other plates. The intensity estimates in column 3 are for the plate of that day. See comment in text.

¶ This line is one member of a Ti$^+$ pair. The other member, at 3218 A, is too close to the strong stellar line 3217 A to be distinguishable.



Spectrum of *α Cygni*

At the upper and lower edges are spectrograms of 1921 October 30, and November 1, respectively, the two central strips being astigmatically broadened copies of the same. There is a spurious line at 3109 A in the lower of the broad strips due to a small spot on the original negative; immediately to the right of it is one of the lines noted in the text as having altered in strength. In making the photographs for this illustration the exposure time was artifically graduated in order to bring out the more refrangible part of the spectrum, which is the fainter.

Explanation of Table 2: Column (1) gives the stellar wave-lengths on Rowland's, and column (2) on the International scale. The numbers are in most cases carried to two decimal places, though it is doubtful whether more than one is significant; however, the final digit serves as a guard figure, and its retention or omission in the table provides a convenient means of indicating the relative accuracy of the measures. Column (3) records the estimated intensity on a scale of 5, but a comment respecting the fainter lines is necessary. It seemed desirable to conform as far as possible to the intensity system of the former paper. The photographs used there were taken with a slit spectrograph, and, due to the superior defining power of that instrument, they record lines that are fainter than the faintest to be seen on the present plates. A comparison of the two sets of photographs in the region common to both of them shows that lines which are assigned an intensity of 2 in the earlier paper are close to the limit of visibility on the present plates. The fainter lines given in this paper are, therefore, generally assigned the intensity 2, though a few that are close to the limit of visibility are marked 1. This explains the scarcity in the present report of lines of intensity 1; lines of that strength are, no doubt, plentiful in the spectrum, but are not distinguishable on the photographs. Column (4) records the number of plates on which a line was measured. Column (5) contains descriptive comments and footnote references, *b*, signifying broad; *n*, nebulous or hazy; and *d*, double; "prob." and "irreg." are abbreviations for probably and irregular, respectively, while the mark of interrogation indicates that the existence of the stellar line is doubtful, as for example where it is "difficult" and was measured on only a single plate. Column (6) gives the element or elements to which the line is thought to be due. These assignments are regarded as provisional, as will be explained more fully in another paragraph. If the assignment is quite questionable an interrogation mark is added or reference is made to a footnote. The sign ($^+$) for ionization is used only when proof of ionization is provided by knowledge of the series to which the line belongs; many lines not so designated doubtless originate in ionized atoms. Column (7) gives the laboratory wave-length, taken, generally, from Exner and Haschek's tables (after reduction to the International scale); (8) lists King's temperature classification where it is known; (9) the series assignment according to Russell's paper;[4] (10) and (11) the arc and spark intensities recorded in Exner and Haschek's tables.

As has just been stated, the chemical origins given in the right hand part of Table 2 are assigned only provisionally. Though there can hardly be question of their general correctness it will doubtless be possible to improve the identification of elements in two ways: first, by a comprehensive survey of the entire observable spectrum, the material for which is now published and available; and second, through advancement in knowledge of line series, which there seems abundant reason to expect as a development of the near future. The most certain identifications made in the present work are, without question, those which are based on series relationships. For example in the case of Ti^+, Russell's list provides 28 lines, allocated to 4 groups, within the spectral limits of the present paper. Of these 27 were measured in the star, while the remaining member (3218.25 A) is within an angstrom of a strong stellar line and would not be distinguishable if it were there. Similarly all of the six Fe^+ series lines listed by Russell are present. Many of these lines are inconspicuous even in the spark spectrum of titanium and iron, and their occurrence as strong lines—and in complete groups—in the spectrum of the star is a most striking and significant phenomenon. It is to be noted that the temperature class of many of these lines is low, and as they are inconspicuous in both the arc and spark, it is very doubtful whether they could be identified in a stellar spectrum without the aid of series relationships.

Where series relations were not known, recourse was had to King's temperature classification where available, and to estimates by Exner and Haschek of relative intensity in arc and spark. The data from these two sources are not in every instance entirely consistent. For example, consider the vanadium lines:

Wave-length	King Temp. Class	E. and H.	
		Arc	Spark
3118.39	III Er	3	15
25.29	III Er	3	2
26.19	III Er	3	4
30.26	III Er	3	10

These four lines are assigned to the same temperature class, and are of substantially equal strength in the arc, but the spark intensities of the first and fourth members are relatively great, and these two appear to occur in the stellar spectrum, while the other two do not. This example might appear to indicate that the arc and spark intensities supply the safer guide, but that they cannot be entirely depended upon is shown by the relatively low spark intensities of some of the series lines that are quite certainly present in the star; for example the Ti^+ series line 3229.4 A with arc and spark intensities of only 3–3 is undoubtedly there, as are also several Fe^+ lines of intensity 2 or less in the spark. Neither of these items of information, the temperature class or the arc-spark intensity ratio, appears to be too reliable a guide in making identifications, and it will probably be necessary to await further development in knowledge of series relationships before the assignment of origins to lines in stellar spectra can be undertaken with an approach to complete confidence.

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[4] *Contrib. Mt. Wilson Obs.* No. 286; *Ap. J.* **61**, 223, 1925.

The upper limit reached in these observations is not far from that which is set by atmospheric absorption and is, in fact, beyond the end of the solar spectrum when photographed with normal exposure. By increasing the time of exposure a thousand fold Fabry and Buisson[5] were able to extend their record of the solar spectrum about 100 angstroms, but it is not probable that stellar spectra will be observed very much above the present limit, unless some exceptional need should justify an attempt to photograph a few angstroms further in the direction of shorter wavelengths. In the opposite direction the spectrum of α *Cygni* has been observed as far as 6800 A. A further extension into the infra-red might be made with plates sensitized with dicyanine, but between these widely separated limits the observations are quite complete.

We hope to bring together in another paper the somewhat scattered measurements of this spectrum in order to make reference to them more convenient.

December, 1925.

Issued January 28, 1926.

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[5] *Ap. J.* 54, 322, 1920.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 373

PRELIMINARY ELEMENTS AND EPHEMERIS OF COMET *b* 1926 (Blathwayt)

By

A. D. Maxwell and N. W. Storer

Comet *b* 1926 was discovered at Johannesburg by Blathwayt on January 16. Jeffers describes it as condensed, but having no stellar nucleus. Its estimated magnitude on January 20 was eleven. The following observations, received by telegraph and telephone at the Students' Observatory, were available for the determination of a preliminary orbit and ephemeris:

	1926, U.T.	α (1926.0)	δ (1926.0)	Observer
(1)	Jan. 16.9248	$12^h12^m14^s.50$	$-30^\circ11'23''.0$	Blathwayt, Cape
(2)	20.5576	11 53 43.61	−27 01 13.4	Jeffers, Lick
(3)	21.4524	48 27.90	−26 03 21.0	Van Biesbroeck, Yerkes
(4)	21.5672	47 46.08	−25 55 36.0	Jeffers
(5)	22.4038	42 32.50	−24 56 17.0	Van Biesbroeck
(6)	Jan. 22.5226	11 41 46.11	−24 47 24.9	Jeffers

The orbit was based upon the observations numbered one, two, and six. Due to the rapidity of the motion and the unequal intervals, a differential correction of the initial solution was necessary.

Elements

$$
\begin{aligned}
T &\quad \text{1926, Jan. } 3.57113\ \text{U.T.}\\
&\left.\begin{array}{ll}\omega & 328^\circ\ 54'\ 10''.5\\ \Omega & 136\ \ 02\ \ 48.3\\ i & 128\ \ 22\ \ 13.5\end{array}\right\}1926.0\\
\log q &\quad 0.129274
\end{aligned}
$$

Constants for the Equator 1926.0

$$x=r[9.923752]\sin(269^\circ\ 48'\ 07''.1+v)$$
$$y=r[9.809052]\sin(\ 50\ \ 09\ \ 15.5+v)$$
$$z=r[9.972502]\sin(346\ \ 00\ \ 54.7+v)$$

The observations were represented as follows (the differential correction concentrated the residuals into the first declination):

	O—C	$\cos\delta\Delta\alpha$	δ
(1)	Jan. 16.92	$+0''.7$	$+6''.7$
(2)	20.56	0.0	0.0
(3)	21.45	−1.0	+3.7
(4)	21.57	+1.6	+0.5
(5)	22.40	+4.1	−2.1
(6)	Jan. 22.52	+0.5	+0.5

A subsequent observation on January 23 by Van Biesbroeck was represented as follows: $\cos\delta\Delta\alpha=+10''$; $\Delta\delta=-14''$.

Ephemeris for Greenwich Midnight

1926 U.T.	True α	True δ	$\log\Delta$	Br.
Jan. 28.0	$10^h\ 59^m\ 42^s$	$-15^\circ\ 50'.8$	9.726	1.6
29.0	50 30	−13 41.2		
30.0	40 50	−11 21.0		
Jan. 31.0	30 43	− 8 50.3		
Feb. 1.0	20 09	− 6 09.7	9.675	1.9
2.0	10 09 10	− 3 20.3		
3.0	9 57 50	− 0 23.6		
4.0	46 10	+ 2 38.2		
5.0	34 16	+ 5 42.7	9.653	2.2
6.0	22 11	+ 8 47.1		
7.0	9 10 01	+11 48.5		
8.0	8 57 49	+14 44.5		
9.0	45 42	+17 32.8	9.670	2.0
10.0	33 45	+20 11.5		
11.0	22 01	+22 39.4		
12.0	8 10 35	+24 55.9		
13.0	7 59 31	+27 00.7	9.720	1.5
14.0	48 51	+28 53.9		
15.0	38 37	+30 36.1		
16.0	28 51	+32 08.0		
17.0	19 35	+33 30.2	9.784	1.1
18.0	10 47	+34 43.7		
19.0	7 02 29	+35 49.3		
20.0	6 54 39	+36 47.8		
21.0	47 18	+37 40.1	9.851	0.8
22.0	40 23	+38 26.7		
23.0	33 55	+39 08.5		
24.0	27 51	+39 45.8		
Feb. 25.0	6 22 11	+40 19.4	9.914	0.6

The unit of brightness is that of January 20.

Berkeley Astronomical Department,
January 27, 1926.
Issued January 29, 1926.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 374

A DETERMINATION OF THE SUN'S VELOCITY WITH RESPECT TO STARS OF MAGNITUDE 9 TO 10

BY

P. van de Kamp

Introduction

The desirability of determining the amount of the Sun's velocity with respect to faint stars is obvious. A knowledge of this value is necessary to convert the mean secular parallaxes of faint stars into annual ones. Using the value of about 20 km/sec derived from the brighter (naked eye) stars there is danger of introducing a systematic error in the annual parallaxes as derived from the secular ones. This was pointed out, for example, by Oort,[1] who made plausible a much higher value for the velocity of the solar motion with respect to the apparently fainter stars. Seares[2] finds from comparison of the secular parallaxes of stars of different apparent magnitude with their annual parallaxes as derived from the luminosity and density functions that the velocity of the solar system varies with the brightness (both absolute and apparent) of the reference stars; according to him the solar velocity with respect to 10th magnitude stars amounts to 25 km/sec.

Now it is recognized that it is hardly possible to determine the radial velocities of faint stars as accurately as those of the brighter ones. In order to get their spectra in a reasonable exposure time, a small dispersion and a short camera must be used and such a combination cannot be expected to give great accuracy to the radial velocities derived. However, it is also clear that for the purpose of determining the solar motion with respect to any group of stars great accuracy of the individual radial velocities is not essential, and indeed of no great concern. A great individual accuracy would be helpful in case the stars had no individual peculiar velocities; but they have, and in a problem concerning solar motion these velocities are of the same character as observational errors. In one respect they are more harmful; they cannot be cut down; we only know that in the case of the naked eye stars they are smaller for the early type stars than for those of the late type. Table 1 gives the average residual radial velocities and also the corresponding probable values of the "cosmical errors" for stars of different spectral types as derived from the Lick radial velocities of the naked eye stars.

TABLE 1

Spectral Class	Average residual radial velocity	Number of stars	Probable value of the cosmical error
B(B0−B5)	7.4 km/sec	243	6.3 km/sec
A(B8−A3)	9.6 "	441	8.1 "
F(A5−F2)	12.5 "	195	10.6 "
G(F5−G0)	15.8 "	242	13.4 "
K(G5−K2)	15.5 "	680	13.1 "
M(K5−Mb)	16.7 "	232	14.1 "
AF	10.5 km/sec	636	8.9 km/sec
GK	15.6 "	922	13.2 "
All	13.1 km/sec	2033	11.1 km/sec

In determining the solar motion from the radial velocities of a group of stars we may now look at the state of affairs in the following way. Every star presents to us the reflection of the Sun's motion, but this reflection is subject to two errors:

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[1] Some peculiarities in the motion of stars of high velocity, by J. H. Oort, *B. A. N.* **1**., 133, 1922.

[2] The Sun's motion and the mean parallaxes of stars of different apparent magnitudes, by F. H. Seares. *Ap. J.* **64**, 50; *M. W. C.* No. 281, 1924.

(1) A cosmical error (= the peculiar radial velocity of the star = total radial velocity freed from the Sun's motion), the probable value of which for the group will be designated by r_c.

(2) An observational error (plate error, error of measurement, etc.), the probable value of which will be designated by r.

The total error of a star in a solar motion determination is made up out of these two errors, and its probable value will be designated by R.

We thus have

$$R = \sqrt{r_c^2 + r^2}$$

For the brighter stars, down to visual magnitude $5^m.5$, the average peculiar radial velocity amounts to 13.1 km/sec corresponding to a probable cosmical error of 11.1 km/sec.

For the moment let us assume the position of the solar apex with respect to the brighter stars to be known; then we can determine the amount of the Sun's velocity simply by determining the radial velocities of a number of stars in the neighborhood of the apex and the antapex of the solar motion.

(1) If the observational error of these radial velocities is very small, as compared with the cosmical error, it is evident that, assuming the probable cosmical error to be 11.1 km/sec and the average distance from the apex or antapex about 20° to 30°, a hundred stars observed will give a value of the solar motion with a total probable error of about $\dfrac{11.1}{0.9\sqrt{100}} = 1.2$ km/sec.

(2) If the observational errors of the radial velocity are large, e.g., have a probable value of 10 km/sec, it is evident that the same number of stars will give a total probable error of

$$\frac{\sqrt{(11.1)^2 + 10^2}}{0.9\sqrt{100}} = \frac{15.0}{0.9 \times 10.0} = 1.7 \text{ km/sec}$$

in the value of the Sun's velocity.

As has been stated, it is hardly possible to get very accurate velocities of stars as faint as magnitude 10, but from the simple considerations given above it is evident that great accuracy in the individual stellar velocities plays only a minor role in the determination of solar motion. Naturally one will hesitate before starting a program of determining radial velocities with large errors of observation, as it is more satisfactory to get information at the same time about the individual velocities of the stars included in the investigation. But in view of the importance of knowing the Sun's velocity with respect to 10th magnitude stars, the significance of the result that can be obtained from individual velocities of low accuracy outweighs objections or hesitations that might exist.

With the 6-inch camera attached to the light one-prism spectrograph used in combination with the 36-inch refractor of the Lick Observatory it is possible to obtain measurable spectra of 10th magnitude stars in from 1 to 3 hours' exposure time, the exact time depending on the spectral type of the star, the zenith distance, the seeing and other circumstances under which the spectrograms are obtained.

The velocities obtained from such spectrograms have an average probable error of about 5 to 10 km/sec. Consequently they have quite sufficient accuracy for solar motion determinations provided there are no periodic systematic errors in the velocities.

Considering the limited period of time which I had available for such an investigation, it was clear that a complete determination of the solar motion with respect to, say 10th magnitude stars, including the position of the apex, could not be made. Fortunately however, the position of the solar apex with respect to 10th magnitude stars is pretty well known, from two earlier investigations:

(1) At the McCormick Observatory, Dr. H. L. Alden and I measured the proper motion in right ascension and declination of about 1900 10th magnitude stars, in 287 regions well distributed over the sky north of −30°. There were also available to us the proper motions in right ascension of about 3000 comparison stars of the 10th magnitude obtained as a by-product from the parallax solutions at the Allegheny, McCormick, Sproul, and Yerkes observatories. The investigation of the solar motion based upon these data is described[3] in *Astronomical Journal* No. 843, and the coördinates obtained for the solar apex are given in the second column of the table which follows. The third column gives the coördinates derived when a systematic correction of $+0''.006 \cos \delta$ is applied to the proper motions in declination. The proper motions used are all on the system of Boss, and several recent investigations indicate that a correction of approximately this size is required.[4]

TABLE 2

Values for the Coordinates of the Solar Apex with Respect to 10th Magnitude Stars as Derived at the McCormick Observatory.

	Uncorrected	Corrected	Adopted
Right ascension..........	279°.1±4°.5	279°.1±4°.5	270°
Declination..................	+45° ±3°	+25° ±4°	+30°

---

[3] The Solar motion derived from the proper motion of tenth magnitude stars, by H. L. Alden and P. van de Kamp. *A. J.* **36**, 17; *Pop. Astr.* **32**, 542; *Publ. Amer. Astr. Soc.*, 32nd meeting, **160**, 1924.

[4] On the determination of the systematic error in the Boss proper motions in declination, by P. van de Kamp. *L. O. B.* **13**, No. 364.

More recently I have made a solution for the solar motion on the basis of the relative proper motions of approximately 5300 stars between visual magnitude 10 and 14 obtained from measurements of photographs of 28 regions of the sky at the Kapteyn Laboratory, and of two regions at the McCormick Observatory, the latter being derived by Mr. A. Vyssotsky. The preliminary results[5] favor a position of the solar apex near right ascension 270° and declination +30°.

There seems to be no special reason to assume a position of the apex with respect to the 10th magnitude stars,[6] that differs very much from $\alpha=270°$, $\delta=+30°$, and these values were adopted in the present investigation.

Assuming the position of the apex to be known, the only unknown in the solar motion problem is the amount of the solar velocity (and perhaps the K term, a constant, common to all the radial velocities used). As has been shown before, a probable error of about 2 km/sec in the Sun's velocity is to be expected from the radial velocities of some 100 stars in the neighborhood of the apex and antapex, if those radial velocities are not very accurate. While this is not as small as could be desired, it does not seem too large for a preliminary result.

Stars near the antapex have to be observed mainly in wintertime. On account of the unfavorable conditions at that time of the year, it appeared advisable not to try to observe stars whose visual magnitude was about 10 but to set a somewhat brighter limit. A list was made containing all stars from the *Henry Draper Catalogue* fainter than $8^{\mathrm{m}}.99$ visual magnitude with well determined photometric magnitude (2 decimals given in the catalogue) in the two regions between 3^{h} and 9^{h} of right ascension, 0° to −30° declination and between 15^{h} and 21^{h} of right ascension, 0° to +60° declination. From this list stars were selected fairly equally distributed over the sky near the apex and antapex; their average visual magnitude amounted to $9^{\mathrm{m}}.2$.

Stars at −30° declination are not favorably placed for observation at Mount Hamilton and few at that declination and none farther south are included in this investigation. The distribution of the stars about the antapex therefore is not ideal, since they are all on one side of it. But this fact will not greatly affect the results though it will reduce to a slight degree the average weight of the stars in the solution. The distribution of the stars over 6 hours of right ascension is probably a sufficient guard against the danger of systematic error from the possible existence of clusters or local star streams.

A decided advantage of having used few stars below −10° of declination is that it diminishes the effect on the Sun's velocity of a possible dependence on the zenith distance of systematic corrections to the radial velocities. The rather large accidental errors of the radial velocities in this investigation make the determination of such a zenith distance effect somewhat uncertain and it is well to avoid the effect a priori, if possible. This actually has been done; the antapex stars were taken at an average declination of −7°, the apex stars at an average declination of +16°.

The radial velocities in this investigation may of course have been influenced by other causes, such as seasonal and secular changes in the instrument and in the method of measurement. No investigation of these possible sources of errors has been made. It may be stated that the greater part of the plates was measured within the period of half a year; in the case of the late type stars all plates were measured twice with an interval of only a few weeks between the measures. During the month of August many plates were obtained near both the antapex and the apex; those plates would probably be affected by the same systematic errors of instrument and of measurement and consequently would not affect the Sun's velocity in a systematic way.

One source of systematic error in the Sun's velocity remains to be mentioned. On account of the rather small number of stars involved in this investigation, I could not help getting acquainted with many of the plates and thus I often knew whether the spectrum that I measured belonged to an apex or to an antapex star. I do not think that this knowledge will have had much effect, however; if any, it probably will have increased the final value for the Sun's velocity.

As there may be possible differences in the Sun's motion as referred to stars of different spectral type it was decided to observe stars representative of all types. In *H. C. O. Circular* 226 is given the frequency of spectral divisions for successive magnitude intervals. Those frequencies are based on counts from the *Henry Draper Catalogue*. As this catalogue is however complete only to about magnitude 8.25 the frequencies given in their Table III., 9.26 to fainter, may very well be systematically wrong. It was therefore rather arbitrarily decided to observe the percentages of stars of different spectral class given below:

Class	A	F	G	K
Percentage	30%	10%	30%	30%

Stars of classes B, M, and exceptional types were not observed.

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[5] Preliminary results of a determination of the solar motion from the relative photographic proper motions of 5300 stars between magnitudes 10 and 14, by P. van de Kamp. *Publ. Astr. Soc. Pac.* **37**, 276.

[6] Preliminary results for mean secular and annual parallax of stars of magnitudes 10 to 14, by P. van de Kamp. *Publ. Astr. Soc. Pac.* **37**, 279.

INSTRUMENTS AND METHODS

As stated already, the light one-prism spectrograph, with a 6-inch camera, was used in combination with the 36-inch refractor. *All spectra were obtained with the telescope on the East side of the pier.* A slit of 0.1 mm and 0.7 mm and Seed 60 plates were used. The spectra thus obtained are very small. The distance from Hβ to the K line amounts to nearly 6 mm. The width of the spectra is about 0.16 mm, and the whole region appears in good definition.

For stars of type A and F hydrogen and helium comparison spectra were used. These spectra were measured on a Toepfer engine, one revolution of the screw of which amounted to ½ mm. In case of the stars of types G and K an iron spark was used for the comparison spectra. These spectra were measured on the Hartmann comparator, one revolution of the screw of which equals ½ mm. The G type spectra were measured against a sky plate, the K type spectra against an *Arcturus* plate. The spectrum of the late type stars was divided into 5 parts for each of which the displacement, star plate—standard plate, was measured. Details concerning these parts are given in Table 3, the lines given in the last column being the most prominent ones; many fainter lines are visible on the plates especially in the spectra of the K stars. Those marked with an asterisk are the ones which played the most important part in "lining up" star and standard spectra on the Hartmann comparator.

TABLE 3
DATA REGARDING THE SPECTRA OF G AND K TYPE STARS

No. of section	Central wavelength λ	Velocity in km/sec for mm 0.0005 displacement	Comparison lines λ	Principal star lines λ
1	4394 A	5.56	4415.1 4404.8 4383.5	*4384 Fe, blend
2	4316	5.28	4325.8 4307.9	4352 blend 4340 *4338 Hγ, blend *4326 Fe, blend 4315 4299 G band 4290 blend
3	4238	5.00	4271.5 4260.6 4250.5 4202.0 4198.7	4271 blend 4260 blend 4250 blend *4227 Ca, Fe, blend 4215 Sr$^+$
4	4118	4.56	4143.7 4132.1	*4144 blend *4102 H
			4071.7	
5	4054	4.34	4063.6 4045.8	*4078 Sr$^+$ *4072 Fe, blend 4067 blend *4064 blend 4057 blend *4046 Fe

All plates were measured from red to violet and then from violet to red. The velocities obtained from the individual lines were weighted according to the quality of the line, its purity and the dispersion at that part of the spectrum. In case of the A and F stars a curvature correction, amounting to −3 to −4 km/sec was applied to all the velocities.

The velocities thus obtained were corrected "to Sun" by means of the short method described by H. K. Palmer in *Lick Observatory Bulletin* No. 98; for this method, which gives the reduction to Sun with an average error of 0.3 km/sec is amply accurate, as the accidental errors of the velocities are so much larger.

The longitude and latitude of each star were read from a graph, which was constructed for the parts of the sky of the program, and which gives λ and β within a degree.

THE RADIAL VELOCITIES AND THEIR ACCURACY

Since the spectra of A and F stars were measured on a Toepfer engine, using hydrogen and helium comparison spectra, while those of stars of later type were measured on the Hartmann comparator, the class G stars against a sky plate, the class K stars against a plate of *Arcturus*, it is to be expected that the three sets of measures may be affected by different systematic errors. For this reason and because the Sun's velocity may depend on the spectral class of the reference stars, they have been treated separately.

A and F type stars.—On the average 6 lines (blends) were measured on the early type spectra. From the differences between the radial velocities as obtained from the different lines on a plate the probable error of a velocity of weight unity was found to be 22 km/sec. There may be uncertainties in the wavelength of some of the blends which would make this error too large; on the other hand there may be a plate error which would affect all starlines by the same amount and this would make the error too small. From the difference in velocities obtained from different plates taken on the same star a probable error of 20 km/sec for a velocity of unit weight was found. As part of these differences may be due to the binary character of some of the stars, this value is an upper limit.

It seems well to adopt 20 km/sec for the probable error of unit weight in case of the velocities of the A and F type stars.

The average weight of the radial velocity derived from a good spectrum of an A or F type star amounts to 2.9, which corresponds to a probable error of 12 km/sec. The probable error of a good plate amounts to 14 km/sec, a slightly larger value on account of the dispersion in the weights. Remeasuring twelve spectra led to a value of 4 km/sec for the probable error of measurement of an A or F type spectrum. The small size of this error compared with the plate error showed that it was not worth while to measure a plate twice as it does not improve the resulting radial velocity to any sensible extent; consequently no additional spectra of A and F type stars have been measured twice. In the case of plates where the comparison spectrum or the focus of the plate was poor, the adopted weights were half of the computed weights.

Since the average probable error of a good plate amounts to 14 km/sec it appears that the derived values have very little if any value as individual velocities. The large differences found between different plates taken of the same star are in nearly all cases satisfactorily explained by accidental errors. Perhaps in the case of stars H.D. 51308 and H.D. 136864 (see pages 93 and 94) the difference may be due to the binary character of the stars. H.D. 167163, having an observed velocity of −140 km/sec, may be a binary or else a high velocity star. On account of the interest connected with the possibility of a high velocity A type star, Dr. Trumpler will keep the star under observation.

The large accidental errors of the radial velocities of the early type stars account for the fact that no attempt has been made to determine the systematic errors of those velocities, or a possible dependence of such systematic errors on the zenith distance at which the plates were taken.

G and K type stars.—To get an estimate of the systematic and accidental errors of the radial velocities of the late type stars and a possible dependence of such systematic errors on the zenith distance at which the plates were taken, spectra were obtained of various bright stars whose radial velocities have been determined accurately at Mount Hamilton and Chile and which very probably are not binaries. Those velocities are reduced to one system which in what follows will be denoted as the Lick Observatory system of radial velocities. The accidental errors of the velocities of these bright stars are negligible in comparison with the errors in the light one-prism 6-inch camera velocities; in what follows they have been assumed to be zero.

As has been stated before, five parts of the spectrum of a late type star were compared with the corresponding parts of a sky (G) or *Arcturus* (K) spectrum. On many of the spectra of the faint stars only four, three or even two parts were compared, due to insufficient exposure or bad definition of the spectrum in the other parts. To insure a thorough reduction of each radial velocity to the Lick Observatory system of radial velocities, an investigation into the systematic and accidental errors of each spectral part appeared necessary. Sixteen plates of nine bright G type, and twenty-five plates of fourteen bright K type stars were taken and measured for radial velocity, each plate being measured twice. The resulting mean differences in velocity are given in Table 4.

TABLE 4

LICK OBSERVATORY VELOCITY MINUS LIGHT 1 PRISM 6-INCH CAMERA VELOCITY

Spectral type Part of spectrum	G	K
1	−18±6.5 km/sec	−32±3.8 km/sec
2	+ 7±4.7 "	+ 5±2.4 "
3	+17±3.2 "	− 2±2.2 "
4	+13±3.1 "	+ 2±2.0 "
5	+29+2.2 "	+ 8+1.2 "
Number of stars	16	25

There appears to be a systematic run of the correction with the wavelength. There may be various causes for this fact: (1) Coma effect of the camera lens, distortion of the film and accidental errors of the different parts of the standard plates. (2) The relatively deeper exposure of the standard spectra which may cause a displacement of the maximum of intensity of a line or blend. (3) The use of a narrower slit for the standard spectrum which might have a similar effect, also on the comparison lines. (4) The difference in color sensitivity of Seed 23 (standard) and Seed 60 (star) plates which would cause a difference in the relative intensities of the same lines on star- and standard plate. This might also cause a displacement of the maximum of intensity of those lines, the size of which would depend on the wavelength.

The differences in velocity, Lick Observatory − light one-prism 6-inch camera, as obtained for each of the bright stars, give the accuracy with which the radial velocity of a star can be determined from each part of the spectrum. The same part of the spectrum shows up differently on different plates; during the measurement of a plate each part was graded in accordance with its quality, the very best parts being called *fair*, while downward the scale extended to *very, very poor*. It may seem peculiar that no part of the spectrum ever was graded *good* or perhaps *fairly good*, but according to my idea of a good part of a spectrum, I never met such a part on any of the plates used in the present investigation. Perhaps other persons would have graded the poor parts as good and the very, very poor ones as poor-fair, for instance. In any event this is not a matter of any consequence, the important thing being that the same person does not change his system of grading appreciably throughout one investigation.

It was assumed a priori that the following relation between estimated quality and estimated weight existed:

TABLE 5

Estimated quality......	good	fair	poor-fair	poor	very poor	very, very poor
Weight..........	1.0	0.7	0.55	0.4	0.25	0.15

The dispersion in the estimates of the same parts of the spectrum on the plates of the bright stars is very small, and in forming the mean corrections as given in Table 4, equal weights were given to each difference.

The following table gives the probable errors of the radial velocity as derived for each part of the spectrum from the differences in velocity (Lick Observatory —light one-prism 6-inch camera) for the bright stars and corresponding to the average estimated weight of each spectral part.

TABLE 6

Spectral type	G		K	
Part of spectrum	P.E.	Estimated weight	P.E.	Estimated weight
1	26.1 km/sec	0.34	19.0 km/sec	0.40
2	18.8 "	0.31	12.0 "	0.36
3	12.7 "	0.35	11.2 "	0.45
4	12.5 "	0.32	10.2 "	0.36
5	9.0 "	0.37	5.9 "	0.51

In what follows 10 km/sec was adopted as the probable error corresponding to unit weight. In accordance with Tables 5 and 6 the following table was constructed giving the weight of each part of the spectrum in a radial velocity determination as a function of its estimated quality:

TABLE 7

Spectral type		G					K				
Quality		f	pf	p	vp	vvp	f	pf	p	vp	vvp
Part of spectrum	1	0.3	0.2	0.2	0.1	0.1	0.5	0.4	0.3	0.2	0.1
	2	0.6	0.5	0.4	0.2	0.1	1.4	1.1	0.8	0.5	0.3
	3	1.2	1.0	0.7	0.4	0.3	1.2	1.0	0.7	0.4	0.3
	4	1.4	1.1	0.8	0.5	0.3	1.9	1.5	1.1	0.7	0.4
	5	2.3	1.8	1.3	0.8	0.5	4.1	3.2	2.3	1.5	0.9

Using Tables 4 and 7 the final velocities on the Lick Observatory system of the bright G and K stars were formed; the results are given in Tables 8 and 9.

The average weight of the velocity of a bright G and K type star amounts to 2.9 and 5.6 with probable errors 5.9 km/sec and 4.2 km/sec, respectively. For the faint stars in our investigation which have good spectra the average weights are 2.1 (G) and 3.4 (K), with probable errors 6.9 km/sec and 5.4 km/sec. The *average* probable errors are 7.7 km/sec and 5.9 km/sec.

From measurements on different days of the same plates a probable error of measurement of a little over 7 km/sec on one spectral section and 4 km/sec on a whole plate has been found: a priori one might expect a somewhat smaller error of measurement on a bright

TABLE 8

Radial Velocities of Standard G Type Stars as Determined with the Light one-prism Spectrograph and 6-inch Camera and Reduced to the Lick Observatory System of the Bright Stars

Star	(1900)	(1900)	Vis. mag.	Sp. type	Plate No.	Radial velocity in km/sec		Velocity difference L.O.— lt. 1 pr.
						lt. 1 pr. 6″	L.O.	
α For	3^h 7^m8	−29° 23′	3.95	F8	PK 377	−32	−20	+12
					378	−22		+ 2
ψ Cap	20 40.2	−25 38	4.26	F8	332	+38	+26	−12
					333	+33		− 7
A_2 Aqr	23 36.6	−18 22	4.95	G0	334	+12	+ 5	− 7
					361	+ 5		0
α_1 Cap	20 12.1	−12 49	4.55	G0p	329	−31	−26	+ 5
					330	−35		+ 9
β Aqr	21 26.3	− 6 1	3.07	G0	327	+ 7	+ 7	0
					328	− 6		+13
α Sge	19 35.6	+17 47	4.37	G0	323	+ 4	+ 2	− 2
υ And	1 30.9	+40 54	4.18	G0	299	−12	−28	−16
					300	−18		−10
H.R. 7955	20 42.8	+57 13	4.63	G0	365	−38	−32	+ 6
					366	−31		− 1
η Cas	0 43.0	+57 17	3.64	F8	381	+ 2	+10	+ 8

TABLE 9

RADIAL VELOCITIES OF STANDARD K TYPE STARS AS DETERMINED WITH THE LIGHT ONE-PRISM SPECTROGRAPH AND 6-INCH CAMERA AND REDUCED TO THE LICK OBSERVATORY SYSTEM OF THE BRIGHT STARS

Star	(1900)	(1900)	Vis. mag.	Sp. type	Plate No.	Radial velocity in km/sec		Velocity difference L.O.— lt. 1 pr.
						lt. 1 pr. 6″	L.O.	
α Mic	20ʰ 43ᵐ7	−34° 9′	5.00	K0	PK 259	−20	−15	+5
π Hyi	14 0.7	−26 12	3.48	K0	201	+12	+27	+15
					224	+23		+4
41 Cap	21 36.4	−23 43	5.32	K0	263	−43	−46	−3
					264	−46		0
σ Cap	20 13.6	−19 26	5.46	K0	239	−9	−10	−1
					240	−13		+3
i Cap	21 16.7	−17 16	4.30	K0	260	+12	+11	−1
					262	+17		−6
τ Cet	1 39.4	−16 28	3.65	K0	280	−12	−16	−4
69 Aql	20 24.4	−3 13	5.11	K0	242	−23	−24	−1
1 Aqr	20 34.3	+0 8	5.39	K0	243	−46	−42	+4
					244	−33		−9
17 Del	20 50.9	+13 20	5.39	K0	245	−12	−11	+1
					246	−12		+1
ζ Cyg	21 8.7	+29 49	3.40	K0	267	+17	+17	0
					268	+20		−3
					269	+28		−11
52 Cyg	20 41.6	+30 21	4.34	K0	249	+3	−1	−4
					250	−17		+16
ρ Cyg	21 30.2	+45 9	4.22	K0	271	+3	+7	+4
					272	−9		+16
					273	+24		−17
α Cas	0 34.8	+55 59	2.47	K0	294	−11	−4	+7
ζ Cep	22 7.4	+57 42	3.62	K0	291	−18	−18	0

star spectrum on account of the better definition; however, there appears to be no sensible difference in error of measurement on bright or faint star spectra or on G or K type spectra.

As the error of measurement of G and K type stars is comparable in size with the plate error every plate has been measured twice; but this has hardly increased the accuracy of the velocities. The average probable errors of the radial velocity of a G and K type star amount to 7.7 and 5.9 km/sec respectively, and the error of measurement of those velocities is 2.8 km/sec since the plates have been measured twice. Thus the average plate error becomes $\sqrt{(7.7)^2-(2.8)^2}=7.2$ km/sec for the faint G type stars, and to $\sqrt{(5.9)^2-(2.8)^2}=5.2$ km/sec for the faint K type stars, and no amount of measurement can reduce this error. The average plate error can be reduced only by improving the average quality of the spectra.

It is well to point out that the weights and probable errors of the velocities of the faint G and K type stars are based on those obtained for the bright stars. Allowance for the better definition of the bright star spectra has been made in the grading of the spectral sections; and intercomparisons of the velocities as obtained from the various parts of the spectra of faint stars show that the weights and probable errors of the velocities of the faint G and K type stars in this investigation are sensibly correct.

In testing the effect of zenith distance on the radial velocities the differences in velocity (Lick Observatory —light one-prism 6-inch camera) for the bright stars were assumed to be proportional to

$$\sin(37°-\delta)-\text{average}\sin(37°-\delta)$$

since $(37°-\delta)$ is the approximate zenith distance of a star on the meridian at Mount Hamilton. For the present purpose this formula is sufficiently accurate as most of the bright stars were taken near the meridian.

The results are

(L. O.−lt. 1 pr. 6″) vel $=+1.6\,[\sin(37°-\delta)-0.46]$ km/sec from 16 bright G stars.
p.e. ±3.0

(L. O.−lt. 1 pr. 6″) vel $=-0.2\,[\sin(37°-\delta)-0.43]$ km/sec from 25 bright K stars.
p.e. ±2.2

or, combining the two

(L. O.−lt 1 pr. 6″) vel $=+0.4\,[\sin(37°-\delta)-0.44]$
p.e. ±1.7

We thus see that there is no sensible systematic error in the radial velocities depending on the zenith distance at which the spectrum has been obtained. Even if the true zenith distance effect were 6 km/sec instead of the derived value +0.4 km/sec, the difference in the systematic errors in the radial velocities at the average declination −7° and +16° of the stars used for the determination of the Sun's velocity would be 2.0 km/sec and the effect on the Sun's velocity only 1 km/sec.

THE CATALOGUE

The radial velocities of 105 stars between visual magnitudes 9 and 10 have been determined and are given in the Catalogue which follows; 53 of these are within 50° of the antapex, 52 within 50° of the apex of solar motion. The antapex stars are distributed over 6 hours of right ascension and 35° of declination, the apex stars over 6 hours of right ascension and 50° of declination.

Columns 1, 2, 3, 4, and 5 of the Catalogue have been taken from the *Henry Draper Catalogue.* Column 6 gives cos d, the cosine of the distance of the star to the antapex of solar motion; this quantity has been read from a graph. Column 7 gives the date and Greenwich Mean Time at which the spectrogram was obtained. Column 8 gives the radial velocity. For the A and F type stars, where the individual velocities have little or no significance, column 9 gives the weight of the radial velocities (unit weight corresponds to a probable error of 20 km/sec); for the G and K type stars this column gives the probable errors of the velocities. Column 10, finally, gives remarks concerning the spectra. Here c.o. means comparison spectrum overexposed, c.u. comparison spectrum underexposed, i.c. imperfect comparison spectrum, i.f. imperfect focus, s.u. star spectrum underexposed. As these defects may affect the accuracy (both systematic and accidental) of the radial velocities, such plates have been given half the computed weights. For many stars two, and for one star three plates were obtained. As the binary character of a star could not be established with any degree of certainty for any of these stars, the weighted mean of these velocities is given in the catalogue.

RADIAL VELOCITIES OF 45 A AND F TYPE STARS BETWEEN VISUAL MAGNITUDES 9 AND 10

H.D. No.	(1900)	(1900)	Vis. mag.	Sp. type	cos d	Date G. M. T.	Radial velocity	Weight	Remarks
20647	3h 14m3	− 9° 51′	9.46	A2	+.73	1925 Jan. 8.75	+ 7	1	c.o.
21336	21.4	+ 5 9	9.26	A2	+.61	1925 Aug. 3.95	−26	9	
22130	28.8	− 9 42	9.41	F0	+.75	1925 Jan. 11.70	−28	1	c.u.
22341	30.7	− 4 42	9.10	F0	+.73	1925 Sept. 9.90	− 8	4	
24017	44.5	− 4 50	9.05	A5	+.76	1925 Jan. 15.63	−28	1	
26932	4 10.3	− 9 54	9.21	A2	+.84	1925 Sept. 9.98	+66	5	
29563	34.2	− 4 53	9.10	A0	+.85	1925 Jan. 11.75	−27	1	c.u.
30838	45.9	− 0 4	9.28	A0	+.82	1925 Sept. 9.94	+27	4	
33778	5 7.5	+ 5 9	9.16	A0	+.77	1925 Sept. 10.00	− 5	2	
33902	8.4	− 5 6	9.40	A0	+.88	1924 Nov. 28.95	+15	2	i.c.
						1925 Jan. 11.78	−17	2	c.u.
						MEAN	− 1	4	
35020	16.6	− 5 0	9.60	A0	+.89	1925 Jan. 8.81	+ 7	3	
35052	16.8	− 4 59	9.30	F	+.89	1924 Nov. 12.94	+24	4	
39205	45.8	−20 7	9.38	A0	+.98	1925 Mar. 1.72	−56	2	
40169	52.1	−10 7	9.01	A2	+.94	1925 Sept. 10.03	+74	2	
40761	55.8	− 9 52	9.66	B9	+.94	1925 Jan. 5.86	−31	1	
42788	6 7.6	− 0 10	9.36	A2	+.86	1924 Dec. 8.94	−24	1	
46739	29.9	−20 4	9.08	A0	+.98	1925 Mar. 1.76	−24	1	
51308	51.7	−14 57	9.26	A0	+.94	1924 Nov. 28.98	+ 2	2	
						1925 Jan. 11.82	+100	1	c.u.
						MEAN	+38	3	
52565	56.6	−10 9	9.51	A0	+.91	1924 Nov. 23.92	+17	1	
						1925 Jan. 11.87	+13	1	c.u.
						MEAN	+15	2	

RADIAL VELOCITIES OF 45 A AND F TYPE STARS BETWEEN VISUAL MAGNITUDES 9 AND 10—(Continued)

H.D. No.	(1900)	(1900)	Vis. mag.	Sp. type	cos d	Date G. M. T.	Radial velocity	Weight	Remarks
62326	7 38.3	− 4 56	9.30	A0	+.83	1924 Dec. 8.98	−44	1	i.c.
62588	39.6	−10 1	9.11	A5	+.86	1924 Nov. 29.01	+45	2	
						1925 Jan. 5.90	− 6	2	
						MEAN	+20	4	
64888	51.0	− 5 15	9.15	A5	+.80	1924 Dec. 9.01	+32	2	i.c.
66759	59.9	−30 13	9.20	B9	+.90	1924 Nov. 18.00	+ 4	1	
						1924 Nov. 23.97	−13	3	
						MEAN	− 9	4	
67467	8 3.0	−10 7	9.36	A2	+.82	1924 Dec. 9.06	+ 9	6	
74488	39.2	−10 12	9.01	B9	+.74	1925 Jan. 30.98	+ 2	1	i.f.
136595	15 16.5	+49 44	9.32	A2	−.80	1925 Sept. 2.66	−11	2	
136864	17.9	+24 45	9.61	A2	−.81	1925 May 5.94	−62	2	
						1925 May 7.89	+22	5	
						1925 June 17.83	−78	3	
						MEAN	−25	10	
146067	16 9.2	+ 0 45	9.34	A2	−.77	1925 May 26.90	−25	1	i.f.
						1925 May 28.75	−27	3	
						MEAN	−26	4	
151370	41.9	+10 0	9.27	A0	−.88	1925 May 26.94	−45	1	
						1925 June 13.85	−51	4	i.c.
						MEAN	−50	5	
154467	17 0.5	+ 0 48	9.04	A3	−.84	1925 May 28.78	−40	3	
159582	30.4	+20 5	9.00	A2	−.97	1925 May 5.98	−30	1	
						1925 May 8.01	+18	1	
						MEAN	− 6	2	
161077	38.4	+ 1 7	9.24	A0	−.87	1925 May 21.94	−45	2	
164842	58.0	+35 3	9.17	A5	−.99	1925 May 6.01	−27	3	
166843	18 7.4	+10 2	9.24	F0	−.93	1925 June 10.89	− 6	7	
167163	8.9	+ 0 11	9.23	A0	−.86	1925 Sept. 2.70	−140	2	
171068	27.6	+ 9 57	9.47	A0	−.93	1925 June 16.89	−46	5	
171656	31.0	+20 7	9.10	A	−.97	1925 June 16.92	+13	6	
175467	50.4	+35 9	9.40	A0	−.97	1925 July 24.85	+ 4	2	i.c.
178162	19 2.4	+ 8 12	9.09	B9	−.89	1925 Sept. 2.73	− 6	2	
179076	6.0	+ 5 5	9.26	A0	−.87	1925 June 16.95	−45	1	
184726	30.7	+10 10	9.02	A0	−.86	1925 July 24.82	+ 8	2	i.c.
189708	56.3	+10 18	9.37	A2	−.83	1925 June 16.98	−46	5	
192360	20 9.4	+45 7	9.17	A0	−.88	1925 Sept. 2.79	−34	1	
192608	10.6	+25 1	9.06	A0	−.87	1925 Aug. 3.88	− 1	1	i.c.
196483	32.6	+19 59	9.25	A0	−.81	1925 Sept. 2.75	−56	3	

RADIAL VELOCITIES OF 32 G TYPE STARS BETWEEN VISUAL MAGNITUDES 9 AND 10

H.D. No.	(1900)	(1900)	Vis. mag.	Sp. type	cos *d*	Date G. M. T.	Radial velocity	P.E.	Remarks
19594	3^h $4^m.1$	+ 1° 4′	9.16	F5	+.61	1925 Aug. 16.95	−44	7	
23111	37.4	+ 5 10	9.11	G0	+.65	1925 Aug. 3.99	+53	5	
23705	42.1	− 9 41	9.41	G0	+.79	1925 Aug. 19.00	−10	6	
23842	43.1	− 0 12	9.41	G	+.72	1925 Aug. 28.92	+51	10	
27486	4 15.2	− 4 44	9.05	G0	+.82	1925 Aug. 15.00	+59	7	
28323	22.9	+ 0 9	9.48	G	+.79	1925 Aug. 31.93	+17	11	
29070	29.5	+ 4 52	9.30	F5	+.75	1925 Aug. 26.94	+26	14	
32941	5 1.6	− 9 49	9.01	F8	+.91	1925 Jan. 2.81	+ 6	10	c.o.
34957	16.1	+10 14	9.02	G	+.75	1925 Aug. 26.98	+10	7	
36202	25.0	− 5 1	9.35	F8	+.90	1924 Dec. 14.91	+45	7	
40936	56.9	− 5 8	9.15	F5	+.90	1924 Dec. 25.87	−28	16	c.o.
						1925 Aug. 27.00	− 2	8	
						MEAN	− 7	7	
46618	6 29.3	− 9 58	9.66	G0	+.93	1924 Dec. 14.98	−28	12	
55691	7 8.8	−15 0	9.41	F8	+.92	1925 Jan. 2.93	−20	16	c.o.
57136	14.8	− 5 6	9.20	G5	+.85	1925 Mar. 12.74	−11	13	i.f.
65545	54.1	− 4 59	9.05	F8	+.80	1924 Dec. 24.96	+53	20	c.o.
66856	8 0.3	−15 6	9.31	F5	+.86	1924 Dec. 15.06	+31	16	c.o.
134152	15 3.2	+14 44	9.54	G0	−.72	1925 Mar. 16.04	−29	16	i.c.
						1925 Mar. 25.95	−11	13	s.u.
						MEAN	−18	10	
142638	50.6	+ 9 56	9.12	G0	−.81	1925 May 28.70	−45	7	
146814	16 13.1	+44 54	9.02	G0	−.90	1925 May 7.94	−11	5	
152599	49.2	+15 15	9.04	F8	−.92	1925 May 7.98	− 8	7	
						1925 June 13.82	−25	6	
						MEAN	−18	5	
156799	17 14.5	+10 11	9.52	G0	−.92	1925 June 12.81	− 2	9	
163074	49.2	+49 55	9.12	G0	−.94	1925 May 21.99	−12	10	
167628	18 11.0	+ 4 59	9.01	G0	−.90	1925 June 10.93	−49	13	
						1925 June 28.84	−21	6	
						MEAN	−26	5	
171551	30.4	+ 9 58	9.32	F8	−.92	1925 June 19.97	+15	7	
172828	37.2	+ 1 7	9.04	K5	−.86	1925 June 13.92	−33	6	
176211	54.1	+25 13	9.16	G0	−.97	1925 Aug. 3.85	− 3	8	
178216	19 2.6	+ 8 2	9.07	G0	−.88	1925 Aug. 16.68	−71	8	i.c.
178890	5.3	+20 6	9.20	G5	−.95	1925 Aug. 19.78	+ 1	5	
179894	9.2	+ 0 7	9.28	G0	−.82	1925 Aug. 19.69	−10	6	
183285	23.7	+ 5 14	9.56	G0	−.84	1925 Aug. 19.74	+ 6	5	
189678	56.1	+ 1 3	9.09	F8	−.76	1925 July 11.95	− 3	7	
200272	20 57.4	+15 5	9.29	F5	−.72	1925 Aug. 19.83	−18	6	

Radial Velocities of 28 K Type Stars Between Visual Magnitudes 9 and 10

H.D. No.	(1900)	(1900)	Vis. mag.	Sp. type	cos d	Date G. M. T.	Radial velocity	P.E.	Remarks
20126	3^h 9^m0	+ 0° 2′	9.03	K0	+.63	1925 July 28.98	− 9	4	
20989	18.1	− 1 50	9.07	K0	+.67	1925 Aug. 1.98	+30	5	
22018	27.8	+ 5 7	9.21	G5	+.63	1925 Aug. 18.92	+33	4	
23033	36.7	−19 43	9.03	K0	+.83	1925 Sept. 1.00	+23	6	
24412	47.9	− 4 45	9.20	K0	+.76	1925 Aug. 10.98	+66	16	s.u.
30126	4 39.7	+ 1 10	9.14	G5	+.80	1925 Aug. 17.00	+ 7	5	
33156	5 3.2	+ 5 10	9.01	K0	+.79	1925 Aug. 27.02	+50	9	
33930	8.6	−15 1	9.21	G5	+.94	1925 Mar. 12.66	−49	16	s.u. c.o.
50326	6 47.6	−15 5	9.26	K0	+.94	1924 Dec. 30.94	0	8	
57774	7 17.6	−10 1	9.31	K0	+.89	1925 Jan. 4.82	+26	14	c.o.
73163	8 31.8	−10 0	9.11	K2	+.76	1925 Jan. 3.01	−10	14	c.o.
77534	58.0	−15 15	9.11	G5	+.72	1925 Jan. 30.91	+40	16	c.o.
138247	15 25.8	+20 1	9.00	K2	−.80	1925 Mar. 19.02	−42	8	
						May 28.80	−55	13	s.u.
						Mean	−45	7	
141267	43.1	+34 55	9.02	G5	−.87	1925 May 21.90	+ 2	5	
142178	48.0	+27 54	9.16	K0	−.87	1925 Mar. 26.02	−13	9	
147622	16 17.9	+50 2	9.02	G5	−.88	1925 May 26.99	−10	6	
147852	19.2	+24 53	9.21	G5	−.92	1925 June 28.79	−66	18	s.u.
151734	44.1	+10 7	9.07	K0	−.89	1925 June 12.74	−29	4	
153599	55.4	+19 59	9.00	K0	−.95	1925 June 13.86	−45	16	s.u.
154674	17 1.8	+20 4	9.15	G5	−.95	1925 June 7.86	+ 1	6	
155075	4.3	+ 9 59	9.07	K0	−.91	1925 June 17.72	−41	8	
160968	37.8	+ 4 51	9.10	K2	−.89	1925 June 28.73	−14	4	
164927	58.4	+ 4 48	9.05	G5	−.90	1925 June 17.78	−37	5	
171049	18 27.5	+ 9 56	9.07	K2	−.93	1925 June 7.92	+19	5	
177226	58.7	+15 2	9.14	K0	−.93	1925 June 19.91	+30	8	
188361	19 49.7	+45 16	9.57	G5	−.90	1925 Sept. 10.82	−15	6	
192048	20 7.8	+10 22	9.02	K0	−.81	1925 July 2.90	− 5	7	
200274	57.4	5 10	9.01	K2	−.65	1925 July 2.96	−14	4	

Solution for the Sun's Velocity

The average residual velocity of the bright A and F type stars is 10.5 km/sec, that of the bright G and K type stars 15.6 km/sec, which values correspond to probable values of the peculiar velocity or cosmical errors of 9 and 13 km/sec, respectively. Assuming these values to hold for the apparent faint stars and making allowance for the possible binary character of some of the stars considered here, it seems well to adopt probable cosmical errors (r_c) of 15 and 20 km/sec for the AF and GK stars, respectively.

If p is the weight of the radial velocity of a star, and p_c is the cosmical weight of the star, the weight P of a star velocity in the solution for Sun's velocity is given by

$$P=\frac{pp_c}{p+p_c}$$

Table 10 gives the value of P for different values of p for the AF stars.

TABLE 10

Weights of Radial Velocities of A and F Type Stars

(Unit weight corresponds to a probable error of 20 km/sec)

Weight of measured velocity	1	2	3	4	5	6	7	8	9	10	. . . ∞
Weight of velocity in solution for Sun's velocity	0.6	0.9	1.1	1.2	1.3	1.4	1.4	1.5	1.5	1.5	. . . 1.8
Adopted weight	1	2			3						

It seemed best to adopt the weights given in the last line of the table instead of those computed rigorously.

For the G and K type stars a value of 10 km/sec was adopted for the probable error of unit weight. Hence we have Table 11.

TABLE 11

Weights of Radial Velocities of G and K Type Stars

(Unit weight corresponds to a probable error of 10 km/sec)

Weight of measured velocity	0.5	1.0	2.0	3.0	4.0	5.0	6.0	.....	∞
Weight of velocity in solution for Sun's velocity	0.17	0.20	0.22	0.23	0.24	0.24	0.24	.....	0.25

The table shows that since the accidental errors of the radial velocities are small as compared with the cosmical error, all stars of these classes have practically the same weight in a solar motion solution. Equal weights have therefore been assigned to them.

For each star we have an equation of condition of the form

$$V_{\odot} \cos d - K = V$$

where $V_{\odot}$ is the Sun's velocity with respect to the group of stars.

d is the distance of the star to the antapex of the solar motion.

K is a constant term common to all velocities.

V is the velocity of the star.

A least squares solution made for the AF, G and K type stars separately gave the following results:

Type	$V_{\odot}$	p.e.	K	p.e.	Number of stars			Average value of			
					Ant-apex	Apex	All	Visual magnitude	Spectral type	cos d	cos ^2d
AF	+17.0 km/sec	±3.4 km/sec	−8.4 km/sec	±3.3 km/sec	25	19	44	9.25	A2	0.86	0.75
G	+16.9 "	±3.9 "	0.0 "	±3.3 "	16	16	32	9.23	G0	0.84	0.71
K	+20.3 "	±4.4 "	+0.7 "	±3.7 "	12	16	28	9.10	G9	0.84	0.70
					53	51	104	9.2		0.85	0.72

The A type star H.D. 161763, which has an observed velocity of 140 km/sec, has been excluded from this solution, as the star possibly may be a binary with a large range in velocity. Including this star, $V_{\odot}$ would become 20.0 km/sec for the AF type stars.

It has been stated that in case of the A and F type stars no attempt has been made to determine systematic errors of the velocities, as the accidental errors are too large to warrant the labor of the investigation. It thus is impossible to decide whether the large K term obtained for the A and F type stars has some cosmical significance, whether it is due to systematic errors in the velocities or whether its value is wholly or partly accidental. More data will be required to decide this point.

The small K-term derived for the stars of types G and K indicates that there probably is no appreciable systematic difference in the motions of the antapex and apex stars of those classes.

For the present we shall neglect possible local star streams and assume that the stars observed are representative of all stars between magnitudes 9 and 10.

(1) Assuming the percentage of AF, G and K type stars among the stars of this magnitude to be 40, 30 and 30, the resulting value for the Sun's velocity is

18.0±2.2 km/sec.

(2) Assuming the Sun's velocity with respect to stars of magnitude 9 to 10 to be independent of the spectral type, the weighted mean is

17.8±2.2 km/sec.

The final outcome of this investigation may be briefly stated as follows:

The Sun's velocity as derived from the radial velocities of 104 stars of mean visual magnitude 9.2 within 50° from either the antapex and apex of solar motion amounts to

18±2.2 km/sec.

Any error in the assumed position of the solar apex will make the Sun's velocity too small. Still, even an error of 20° in the position of the solar apex would have a percentage error of only $1.00 - \cos 20° = 0.06$ on the Sun's velocity, and the velocity computed would be only 1.1 km/sec too small. It seems safe to say that the computed velocity is systematically correct within 1 or 2 km/sec. The accidental error is still rather large but could not be expected to be any smaller from the small amount of material used. It is clear that the value here derived must be considered as preliminary and that a similar but complete determination of the solar velocity with respect to faint stars, including stars all over the sky is very desirable.

EARLIER ESTIMATES OF THE SUN'S VELOCITY WITH RESPECT TO 9^m—10^m STARS

Oort finds that the direction of motion of stars with a total peculiar velocity greater than 66 km/sec is limited to one hemisphere, the center of which lies at galactic longitude 230°±3°, galactic latitude $-5°\pm3°$ ($\alpha=8^h\ 20^m$; $\delta=-44°$). As the percentage of high velocity stars will be higher among the fainter stars on account of the correlation between high velocity and low luminosity, the Sun's velocity with respect to the fainter stars may be expected to be higher than the value usually assumed for the brighter ones. According to Oort 45 km/sec is an *upper limit* for the Sun's velocity with respect to stars of magnitude 10.

Seares finds from a comparison of the secular parallaxes of stars of different apparent magnitudes with their annual parallaxes as derived from the luminosity and density functions that the Sun's velocity changes with the brightness (both absolute and apparent) of the reference stars; this is in agreement with the results of Strömberg[7] who finds a value of 19 km/sec for the Sun's velocity with respect to 800 giants and 32 km/sec with respect to 415 dwarfs. According to Seares the solar velocity with respect to 10th magnitude stars amounts to 25 km/sec.

Van Rhyn[8] finds that in case of the K stars the Sun's velocity does not increase with decreasing brightness. He also points out that a selective effect is likely to enter which will make the Sun's motion relative to such a group of stars too large; for dwarfs having peculiar velocities in the direction of the parallactic motion are more apt to be discovered on account of their large resulting proper motion than those which have just as large peculiar velocities but which are moving in a direction opposite to the parallactic motion. It is clear that the Sun's velocity determined from material thus selected must come out too large. It seems that at present we have no adequate grounds for assuming a different solar velocity with respect to giant and dwarf stars. Any difference that may exist cannot be detected until we consider a group of non-selected dwarfs.

OTHER METHODS FOR DETERMINING THE SUN'S VELOCITY WITH RESPECT TO FAINT STARS

Another method for determining the Sun's velocity with respect to faint stars is given by the relation

$$\frac{\bar{\pi}}{\left(\dfrac{\bar{h}}{\rho}\right)}=\frac{4.74}{V_\odot}$$

where $\bar{\pi}$ and $\left(\dfrac{\bar{h}}{\rho}\right)$ are the mean annual and secular parallax of a group of stars, and $V_\odot$ is the Sun's velocity with respect to the group.

Thus if we can obtain values of $\left(\dfrac{\bar{h}}{\rho}\right)$ and $\bar{\pi}$ for faint stars, a value for $V_\odot$ will result.

(*a*) Now we have fairly reliable values for the mean secular parallax of the 10th magnitude stars. From the proper motions of 1900 stars of mean visual magnitude 10.1, measured on McCormick plates combined with results obtained from the proper motions of comparison stars used in parallax determinations, H. L. Alden and I derived a value of $0''.0137\pm0''.0007$ for the mean secular parallax of stars of visual magnitude 10 at galactic latitude 30°. A determination of the mean annual parallax of faint stars (magnitude 7 to 13) has been made by Lee in the selected areas at +45° of declination. The mean relative parallax of 205 stars between visual magnitude 9 and 11 reduced to magnitude 10.1 amounts to $0''.0081$ with an estimated probable error of $0''.001$. Neglecting the reduction to absolute parallax and any galactic latitude effect, this value in combination with the value of the mean secular parallax of stars of mean visual magnitude 10.1 gives 8±1 km/sec as an upper limit for the Sun's velocity with respect to 10th magnitude stars.

(*b*) Assuming the logarithm of the mean secular and annual parallax to be a linear function of the visual magnitude between magnitudes 10 and 14, a value of $-0''.0015\pm0''.0006$ for the derivative of the mean annual parallax with respect to the magnitude follows from the 979 stars between visual magnitude 9.9 and 13.2 for which Lee derived parallaxes. This value of $\dfrac{d\bar{\pi}}{dm}$ holds for visual magnitude 11.7. From the relative photographic proper motions of 5300 stars between visual magnitudes 10 and 14 I have derived a value of $-0''.0040^5\pm0''.0004$ for the derivative of the mean secular parallax with respect to the magnitude holding for magnitude $11^m.5$. This value of $\dfrac{d\dfrac{h}{\rho}}{dm}$ reduced to magnitude $11^{m}.7$ amounts to $-0''.0037\pm0''.0004$. Combining the values of $\frac{d\frac{h}{\rho}}{dm}$ and $\frac{d\bar{\pi}}{dm}$ a value of 12±5 km/sec is found for the Sun's velocity.

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[7] The distribution of the velocities of stars of spectral types F to M. *Ap. J.* **56**, 265; *M. W. Contr.* No. 245, 1922.

[8] Comparison between trigonometric, spectroscopic and mean statistical parallaxes, by P. J. van Rhyn. *Gron. Publ.* No. **37**, 11–13, 1925.

(*c*) In his determinations of stellar parallaxes van Maanen[9] has derived the relative parallaxes of the comparison stars. Taking the mean of those parallaxes as they are given in his series 1 to 7 the following values are obtained:

Group	Visual magnitude	Mean relative parallax	Number of stars
1	(*bb*) 11.6	$+0''.0102$	16
2	(*b*) 12.2	$+0''.0026$	94
3	(*bf*) 12.7	$+0''.0008$	365
4	(*f*) 13.4	$-0''.0009$	619
5	(*ff*) 14.1	$-0''.0033$	88
			1182

These mean values of the parallax are relative to the mean value of the parallax of all the stars used in the seven series, that is, relative to stars of magnitude 13.

Magnitude groups 2 to 5 give $\frac{d\bar{\pi}}{dm}=-0''.0028\pm0''.0002$ holding for magnitude 13.1.

The value of $\frac{d\frac{\bar{h}}{\rho}}{dm}$ corresponding to that magnitude amounts to $-0''.0024\pm0''.0002^{4}$. This leads to a value of 4 ± 0^{5} km/sec for the Sun's velocity.

The various investigations just described are summarized in the following table:

TABLE 12

VALUES OF THE SUN'S VELOCITY WITH RESPECT TO FAINT STARS

	$V_{\odot}$	Magnitude of stars	Data
a	<8±1	10.1	From Lee's relative parallaxes of stars in selected areas at +45° of declination.
b	12±5	11.7	
c	4 ± 0^{5}	13.1	From van Maanen's relative parallaxes of comparison stars in parallax determinations.

Although these values show a remarkable agreement and have small probable errors it must be remembered that magnitude errors in the parallaxes may have affected the results seriously. A direct determination of the Sun's velocity from the radial velocities of faint stars remains desirable provided large systematic errors can be avoided. If so, the value obtained for the Sun's velocity will show whether or not the parallax determinations just given are sensibly free from magnitude error. However this may be, these results may be regarded as contradictory to the view that the value of the solar motion is larger for the fainter stars.

COMPARISON WITH SEARES'S FORMULA FOR THE SUN'S VELOCITY

It is of interest to make a comparison between the values for the Sun's velocity given in this paper with those derived from Seares's formula[2]

$$V_{\odot}=12.9+1.21\,m$$

where m is the visual magnitude of the stars considered. Such a comparison, in tabular form follows:

TABLE 13

Data	Average visual magnitude	Sun's velocity	Seares's formula	$\frac{V_{\odot}\text{ Seares}}{V_{\odot}\text{ observed}}$
104 radial velocities	9.2	18±2.2	24.0	1.3
Lee's relative parallaxes	10.1	8±1	25.1	3.1
	11.7	12±5	27.1	2.3
van Maanen's relative parallaxes	13.1	4 ± 0^{5}	28.7	7

Seares's formula is not supported. The same conclusion has been reached recently by Oort and Doorn,[10] who derived the velocity of the Sun with respect to the stars of absolute magnitude brighter than 0.0 listed in Volume 3, No. 1, of the *Publications of the Dominion Astrophysical Observatory.*

PECULIAR VELOCITIES OF 9^{m} TO 10^{m} STARS

Assuming the peculiar velocities of a group of stars to be distributed according to an error law, the frequency of the observed peculiar velocities will also follow an error law, the average error η_0 of which is related to the average value of the peculiar velocity η_c and the average error η of the velocities by the formula

$$\eta_0^2=\eta_c^2+\eta^2$$

If η is small compared with η_c the latter quantity can be determined fairly accurately from the observed quantity η_0.

It will be of interest to see whether it is possible to get any information about the peculiar velocities of the stars included in this investigation. Since the accidental errors of the velocities of the early type stars are very large as compared with the individual velocities of those stars, there is little hope of getting any reliable information concerning their average peculiar velocity. For the late type stars, the velocities of which have much smaller errors, conditions are more favorable.

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[9] *M. W. Contrib.* Nos. 111, 136, 158, 182, 204, 237, 270, 290.

[10] The solar motion from radial velocities of the absolutely brightest stars of spectra F, G, K, M, by J. H. Oort and N. W. Doorn. *B. A. N.* **111**, 71, 1925.

Table 14 gives the values of η_0, η, and η_c as derived from η_0, η for the stars in this investigation which had good spectra.

TABLE 14

Type	AF	G	K
Number of stars	31	25	20
Observed average residual η_0	23.4 km/sec	20.2 km/sec	16.6 km/sec
Average error η	14.9 "	8.7 "	7.0 "
Average peculiar velocity η_c	18.0 km/sec	18.2 km/sec	15.1 km/sec
Bright stars (L.O.)	10.5 "	15.8 "	15.5 "

For all three groups of stars the observed residual velocities follow a normal error distribution very closely, and it is probable that the true distribution of the peculiar velocities of the late type stars does not differ sensibly from an error distribution. In case of the early type stars there is more uncertainty as regards this conclusion. The values obtained for the average peculiar velocities are upper limits as there may be binaries among the stars considered.

The average peculiar velocities of the 9^m to 10^m G and K type stars agree well with the corresponding values for the brighter stars; for the early type stars the values found for the fainter stars is much larger than that for the brighter stars; it is possible that this larger value has some real significance, but the data are insufficient to establish this definitely.

As the difference between the average velocities for spectral types G and K is small, it seems well to combine the two values. Thus the final and only reliable result of this discussion of the peculiar velocities of the 9^m to 10^m stars near the antapex and apex of solar motion is an upper limit of 16.8 km/sec for the average peculiar velocity of a star of type G or K.

ABSOLUTE MAGNITUDES

An attempt was made to determine the absolute magnitudes of the faint K stars included in this investigation, for their spectra show fairly distinct lines. The star lines and blends in the spectra of the stars of earlier type are rather too faint for this purpose. Seventeen plates of nine K type stars of known absolute magnitudes ranging from $-1\overset{m}{.}6$ to $+6\overset{m}{.}1$ were obtained, and estimates of the line pairs $\frac{4271}{4290}$, $\frac{4215}{4250}$, and $\frac{4072}{4078}$ were made. For each line pair the difference in intensity was estimated in steps. It appeared that the first two line pairs showed hardly any change in intensity difference with absolute magnitude. The dependence of the relation $\frac{4072}{4078}$ on the spectral type was very pronounced, and this line pair was used in the further investigation. In order to have a better elimination of systematic and accidental errors, estimates of the line pair $\frac{4072}{4078}$ were made by Miss Dorothy Applegate as well as by myself on the 17 standard plates and on 9 faint star plates which had sufficiently good spectra. The relation found between the difference $\frac{4072}{4078}$ and the absolute magnitude is practically linear and is given by the following formulae:

$$\text{Estimated difference in steps} = -2.7+(M-2.15)\times 1.04 \text{ P.v.d.K.}$$
$$= -2.0+(M-2.15)\times 0.71 \text{ D.W.A.}$$
$$\text{p.e.}\quad \pm 0.2 \qquad\qquad \pm .08$$

where M is the absolute magnitude.

The estimates on the 9 faint star spectra show a systematic difference of about one magnitude between the results of the two observers, from which we conclude that the quality of the faint star spectra is too poor to allow any safe conclusion as to absolute magnitude of the stars. The mean of the absolute magnitudes as found for the 9 stars amounts to +2.0, but for the reason given, this result should be accepted with reservation. The faintest absolute magnitude occurring among those 9 K stars was $+3\overset{m}{.}1$ the brightest $-1\overset{m}{.}6$; it seems safe to conclude that none of these stars is a faint dwarf. The systematic difference between the results of the two observers and the fact that only one line pair was available led us to abandon any further attempts at determination of absolute magnitude of the faint stars. We may conclude that more dispersion and better spectra are essential for the accurate determination of absolute magnitudes.

SUMMARY

The radial velocities of 105 stars between visual magnitudes 9 and 10 and within 50° either of the apex or antapex of solar motion have been determined with the light one-prism spectrograph and 6-inch camera attached to the 36-inch Lick Observatory refractor, in order to derive a preliminary value for the Sun's velocity with respect to those stars. The average probable error of a radial velocity derived from a good plate amounts to 14, 8, and 6 km/sec for the A and F, G, and K type stars, respectively.

The discussion of the radial velocities gives a value of 18±2.2 km/sec for the Sun's velocity; for the AF, G, and K type stars the values amount to 17.0±3.4, 16.9±3.9, and 20.3±4.4 km/sec, respectively. These values are believed to be affected by no systematic error larger than 1 km/sec.

The value of the solar motion here obtained from stars of visual magnitude 9.2 and the results derived from other considerations do not support the formula of Seares, which expresses the Sun's velocity as a function of the apparent magnitude of the reference stars.

An additional numerical result of the investigation is an upper limit of 17 km/sec for the average peculiar velocity of a late type (G, K) star between visual magnitudes 9 and 10.

I wish to express my sincere thanks to Director Dr. W. W. Campbell and Associate Director Dr. R. G. Aitken for putting the telescope at my disposal for this investigation and for continued interest in the work and to Dr. J. H. Moore and Dr. R. J. Trumpler for the valuable advice which I have received from them.

January, 1926

Issued March 6, 1926

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 375

ON THE CONTINUOUS HYDROGEN ABSORPTION IN SPECTRA OF CLASS *A* STARS

By

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INTRODUCTION

During the past quarter of a century instances have been found of continuous spectra associated with line series, the nature of the association being that the continuous spectrum begins near the limit of the line spectrum and extends in the direction of shorter wave-lengths. This curious fact early led to the belief that the two spectra have a common chemical origin, though the spectral relationship was not at first understood.

It was as early as 1876 that Huggins[1] first photographed a peculiar absorption in celestial spectra. He observed in *Vega* and other Class A stars a rather sudden fall of intensity of the "background" spectrum at about the place of the end of the series of Balmer lines. Evershed,[2] in his account of the eclipse of January 22, 1898, described an analogous emission spectrum in the solar chromosphere and prominences. This continuous spectrum begins near the limit of the Balmer series and extends with gradual decreasing intensity towards the region of greater refrangibility. Evershed, referring to the observations of Huggins, remarked the similarity of the two spectra and advanced the opinion that the spectrum like the Balmer series is due to hydrogen.

The first discovery of a continuous spectrum of this character in the laboratory was made by Wood,[3] who observed a continuous absorption on the violet side of the sodium series of dark lines. This discovery further substantiated the view put forward by Evershed that the continuous spectrum and its associated series spectrum have a common origin. Attempts at a theoretical explanation of the continuous spectrum, and for that matter of line spectrum also, were, however, not successful until 1913, when Bohr presented his epoch-making theory of spectra which has explained with great success the mechanism and behavior of the hydrogen atom. Bohr[4] applied his theory to account for the absorption of sodium observed by Wood. He did not, however, consider the case of hydrogen, although the phenomenon both of emission and absorption by that element was cited in Wood's paper. Wright,[5] in his work on the planetary nebulae, detected the continuous emission in their spectra and has provided a theoretical discussion of it based on a modification of the Bohr theory. We shall treat these theories more in detail later in the paper.

Among other examples found in the laboratory of the continuous spectrum in question may be mentioned the well known K and L absorptions in the X-ray region, which link up sharply with the K- and L-limits on the side of the short wave-lengths and extend with decreasing intensity toward the more refrangible region. Besides the continuous absorption of sodium observed by Wood, Holtzmark[6] found a similar phenomenon to exist also in the case of potassium. It seems natural then that we should expect to find, under favorable conditions, a continuous spectrum attending any optical series. It appears, however, that this spectrum for hydrogen has been observed in the laboratory only in a single instance, although no doubt other efforts have been directed towards its development. The instance refers to an observation of Stark,[7] who evidently found this peculiar hydrogen spectrum in canal ray tubes. It is true that an ultra-violet continuous spectrum of some kind was observed by Dufour[8] and another discovered by Lemon,[9] but the latter spectrum at least seems to bear no relation to the one in question. The excitation of the phenomenon for hydrogen is probably very difficult, and its production in the laboratory, even if with ease, will not, in the opinion of Nicholson,[10] prove of value so far as the quantitative determination of the law of intensity distribution along the spectrum is concerned, because of the overlapping of the spectra of water vapor, and the complication of the secondary hydrogen spectrum. The need for an accurate determination of this law is apparent in the light of the Bohr and the Quantum Theory.

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[1] *An Atlas of Representative Stellar Spectra*, p. 85 and Plate II.

[2] *Phil. Trans.*, **197***A*, 389 and 399, 1901.

[3] *Astrophys. Journ.*, **29**, 100, 1909.

[4] *Phil. Mag.*, **26**, 17, 1913.

[5] *Proc. Amer. Phil. Soc.*, **49**, 530, 1920; also *Nature*, **109**, 810, 1922.

[6] *Physikal. Zeitschr.*, **20**, 88, 1919.

It is evident then that the attack on this problem can be more hopefully undertaken in the observatory than in the laboratory. Fortunately this continuous spectrum is of common occurrence in many of the celestial spectra, and therefore it is extremely desirable that some effort be devoted to making an extensive and accurate investigation of its character.

An investigation of the absorption in several hydrogen stars was undertaken by Hartmann,[11] who recorded a general absorption from the limit of the Balmer series to about λ 3400. Wright[12] made an extensive study of the planetary nebulae and found an emission spectrum like the one in question to be a characteristic feature of this class of objects. It commences quite abruptly at about the limit of the Balmer series and extends with great uniformity far into the ultraviolet. He also investigated the intensity curves of *α Lyrae* and *α Cygni*,[13] and verified the observations of Huggins. In addition several remarkable and important facts relating to the absorption spectrum were brought out. The Balmer series near its limit is lost, to quote, "not through blending of the lines, since the last one is sufficiently separated from its less refrangible neighbor for easy resolution, but through faintness, or lack of contrast between the center of the line and the continuous spectrum." In other words, there is a merging of intensity between the series lines and the continuous absorption. Wright also confirmed Huggins's observations that the absorption does not begin exactly at the limit of the series, but some distance to the redward of it. The commencement of absorption for *α Lyrae* he estimated to be at λ 3800±, and for *α Cygni* λ 3700±. The significance of these facts will be discussed later.

All these results, as derived from astronomical observations, seem to lend strong support to the view that the spectrum in question belongs to the Balmer series. In the light of the Bohr theory this is substantially what it should do. The foregoing investigations may, however, be regarded as only qualitative in nature, especially in view of the difficulties which attend the study of astronomical spectra in the ultraviolet. Among these difficulties are the increase of absorption, both instrumental and atmospheric, in that region, and the falling off in reflectivity of the silver mirror. These naturally tend to render the determination of intensity difficult and uncertain, especially at the farther, or more refrangible limit of the spectrum in question. That the beginning of the continuous absorption as observed by the various investigators, is probably near the truth, may be accepted without much question, but the estimated positions of the farther end in any of the sources should be accepted with caution. There is, moreover, fundamental need for a determination of the law of intensity distribution along the spectrum, which has until now not been attempted.

With these ends in view, the writer made an extensive spectrophotometric study of the continuous absorption spectrum in class A stars, from which an attempt was made to determine the limits of the absorption and its law of intensity distribution along the spectrum on an absolute scale, so far as the accuracy of the method permits. Spectra of stars of other types were also observed for purpose of studying the variation and behavior of the absorption from class to class. Effective temperatures of the stars were determined in a somewhat approximate manner. The method and results are discussed in the following pages.

OBSERVATIONS

Stars observed.—For my observational material a list of stars was selected from Volume 50 of the *Harvard Annals*. It includes all the Class A stars (B5 to A9) brighter than the 4th photographic magnitude and north of Declination −18°. It is not desirable for photometric purposes to include stars whose zenith distances at the meridian are greater than 60°, owing to the uncertainty of the law of atmospheric transmission and the magnitude of the atmospheric dispersion at great zenith distances. One or more bright stars were also chosen from each of the other types beginning at Oe5 and extending to as far as K5, for the purpose already mentioned. This also provides material for obtaining a general scale of stellar temperatures. The list of stars thus selected totals a little over 100, but only 91 were suitably located for observation from December, 1924, to June, 1925.

Spectrograms of these 91 stars were taken with the two-prism quartz spectrograph attached to the Crossley reflecting telescope in both its slit and slitless form,[14] as described in the paper here referred to, except that a different camera and collimator with ultra-violet glass lenses were used in the slit arrangement. Table I gives the general list of the stars and the number of plates taken. Seed 23 plates were used throughout. The plates secured with the slit spectrograph are marked with an asterisk (*), whereas the others were obtained without the slit. In the course of observation the Crossley mirror was silvered twice. On March 12 the old silver surface was given a new coating but it was found necessary to obtain a more satisfactory one. Only 10 plates were taken with the old coat. They were numbered in the 100 series. Those secured with the intermediate silver surface have numbers in the 200 series while plates marked from 300 upward were taken with the final good coat. It will be seen from Table I that the great bulk of observations was obtained after the final silvering. As it is evident that observations in order to be of value in our present investigation must be homogeneous, i.e., taken as far as possible under identical conditions, only the last set was made the basis of our discussion in this paper, although the results reduced from the other two sets are found to be in general accord with the rest.

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[7] *Ann. d. Phys.*, **52**, 255, 1917.

[8] *Ann. Chem. et Phys.*, **9**, 361, 1906.

[9] *Nature*, **113**, 127, 1924.

[10] *Monthly Notices*, **85**, 253, 1925.

[11] *Physikal. Zeitschr.*, **18**, 429, 1917.

[12] *Lick Obs. Bull.*, **9**, 54, 1917; also *Publ. Lick Obs.*, **13**, 256, 1918.

[13] *Lick Obs. Bull.* **10**, 101, 1921.

In Table I the columns under Class and Photographic Mag. are taken from the *Henry Draper Catalogue.* The column headed by Z.D. denotes the zenith distances at which the observations were made. The rest of the table is self-explanatory.

TABLE I.—OBSERVATIONAL DATA

Star	α (1900)	δ (1900)	Class	Ptg. Mag.	Plate	Exp.	Z.D.	Date
β *Trianguli*	2^h $3^m.6$	+34° 31′	A5	3.22	Y155E*	12m	3°	1924 Dec. 30
ζ *Tauri*	3 21.7	+ 9 23	B8	3.70	Y156F*	18m	28	Dec. 30
17 *Tauri*	3 39.0	+23 48	B5p	3.69	Y157H*	18m	14	Dec. 30
η *Tauri*	3 41.5	+23 48	B5p	2.84	Y158B*	9m	14	Dec. 30
β *Eridani*	5 22.9	− 5 13	A3	3.00	Y165B*	14m	42	1925 Jan. 9
					Y210E*	8m	51	Mar. 14
μ *Leporis*	5 8.4	−16 19	A0p	3.30	Y166E*	18m	53	Jan. 9
β *Orionis*	5 9.7	− 8 19	B8p	0.29	Y167F*	2m	45	Jan. 9
γ *Orionis*	5 19.8	+ 6 16	B2	1.51	Y168H*	4m	31	Jan. 9
β *Tauri*	5 20.0	+28 31	B8	1.73	Y169R*	4m	9	Jan. 9
λ *Orionis*	5 29.6	+ 9 52	Oe5	3.66	Y170Q*	24m	27	Jan. 9
ι *Orionis*	5 30.5	− 5 59	Oe5	2.87	Y209B*	13m	52	Mar. 14
ε *Orionis*	5 31.1	− 1 16	B0	1.51	Y201B*	5m	41	Mar. 13
β *Aurigae*	5 52.2	+44 56	A0p	2.07	Y242B*	8m	48	Apr. 26
θ *Aurigae*	5 52.9	+37 12	A0p	2.71	Y243E*	15m	54	Apr. 26
β *C. Majoris*	6 18.3	−17 54	B1	1.77	Y203F*	5m	56	Mar. 13
γ *Geminorum*	6 31.9	−16 29	A0	1.93	Y204H*	5m	23	Mar. 13
α *C. Majoris*	6 40.7	−16 35	A0	−1.58	Y205Q*	40s	55	Mar. 13
					Y221B*	30s	60	Apr. 24
θ *Geminorum*	6 46.2	+34 5	A2	3.70	Y244F*	14m	51	Apr. 26
λ *Geminorum*	7 12.3	+16 43	A2	3.71	Y301B*	25m	49	May 4
δ *Geminorum*	7 14.2	+22 10	F0	3.79	Y302E*	30m	53	May 4
β *C. Minoris*	7 21.7	+ 8 29	B8	3.04	Y207H*	22m	29	Mar. 13
α *C. Minoris*	7 34.1	+ 5 29	F5	0.90	Y206R*	5m	32	Mar. 13
					Y214I*	3m	39	Apr. 9
ι *U. Majoris*	8 52.4	+48 26	A5	3.26	Y246I*	10m	38	Apr. 26
					Y413B	2m	54	June 16
κ *U. Majoris*	8 56.8	+47 33	A0	3.68	Y247U*	15m	38	Apr. 26
					Y414E	3m	57	June 16
η *Leonis*	10 1.9	+17 15	A0p	3.58	Y216F*	17m	20	Apr. 9
α *Leonis*	10 3.0	+12 27	B8	1.29	Y125B*	2m	25	Apr. 9
λ *U. Majoris*	10 11.1	+43 25	A2	3.58	Y248X*	15m	28	Apr. 26
					Y415F	3m	48	June 15
β *U. Majoris*	10 55.8	+56 55	A0	2.44	Y217H*	5m	20	Apr. 9
					Y305Q*	6m	31	May 4
					Y401B	32s	43	June 15
δ *Leonis*	11 8.8	+21 4	A3	2.66	Y309B*	10m	24	May 23
θ *Leonis*	11 9.0	+15 59	A0	3.41	Y321B*	19m	28	May 24
					Y424H	3m	48	June 17
β *Leonis*	11 44.0	+15 8	A2	2.29	Y228H*	7m	23	Apr. 13
					Y423F	30s	46	June 17

* Plates taken with the slit spectrograph.

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[14] *Lick Obs. Bull.*, **9**, 50, 1917.

Table I.—Observational Data—(Continued)

Star	α (1900)	δ (1900)	Class	Ptg. Mag.	Plate	Exp.	Z.D.	Date
β *Virginis*	$11^{h}\ 45^{m}5$	+ 2° 20′	F8	4.30	Y422E	7m	54°	1925 June 17
γ *U. Majoris*	11 48.6	+54 15	A0	2.54	Y218I*	6m	18	Apr. 9
					Y306R*	9m	32	May 4
					Y402E	32s	30	June 15
δ *U. Majoris*	12 10.5	+57 35	A2	3.50	Y322E*	21m	24	May 24
					Y403F	2m	33	June 15
γ *Corvi*	12 10.7	−16 59	B8	2.73	Y310E*	11m	55	May 23
δ *Corvi*	12 24.7	−15 58	A0	3.11	Y311F*	15m	54	May 23
κ *Draconis*	12 29.2	+70 20	B5p	3.76	Y237X*	10m	33	Apr. 25
					Y404H	2m	40	June 15
ε *U. Majoris*	12 49.6	+56 30	A0p	1.68	Y238B*	4m	19	Apr. 25
					Y405B	32s	36	June 15
α *Can. Venat.*	12 51.4	+38 52	A0p	2.90	Y323F*	12m	12	May 24
					Y331B*	8m	12	May 27
					Y406E	1m	39	June 15
ζ *U. Majoris*	13 19.9	+55 27	A2p	2.40	Y324H*	8m	21	May 24
α *Virginis*	13 19.9	−10 38	B2	1.02	Y332E*	2m	48	May 27
					Y425B	10s	51	June 17
ζ *Virginis*	13 29.6	− 0 5	A2	3.50	Y333F*	18m	37	May 27
					Y362E	3m	38	June 12
η *U. Majoris*	13 43.6	+49 49	B3	1.74	Y241H*	5m	13	Apr. 25
					Y408H	32s	34	June 15
η *Boötis*	13 49.9	+18 54	G0	3.36	Y426E	3m	25	June 17
α *Draconis*	14 1.7	+64 51	A0p	3.64	Y409B	2m	38	June 15
α *Boötis*	14 11.1	+19 42	K0	1.24	Y416H	20s	17	June 16
γ *Boötis*	14 28.1	+38 45	F0	3.28	Y427F	2m	13	June 17
μ *Virginis*	14 37.8	− 5 13	F5	4.37	Y428H	3m	45	June 17
107 *Virginis*	14 41.2	+ 2 19	A0	3.76	Y363F	3m	36	June 12
α^2 *Librae*	14 45.3	+15 38	A3	2.98	Y313B*	13m	53	May 23
					Y364H	2m	53	June 12
β *Librae*	15 11.6	− 9 1	B8	2.69	Y314E*	10m	47	May 23
					Y365B	1m	46	June 12
γ *U. Minoris*	15 20.9	+72 11	A2	3.20	Y334H*	12m	36	May 27
					Y410E	1m	38	June 15
α *Cor. Bor.*	15 30.5	+27 3	A0	2.31	Y315F*	7m	11	May 23
					Y366E	40s	10	June 12
β *Serpentis*	15 41.6	+15 44	A2	3.80	Y336E*	22m	22	May 27
					Y367F	3m	22	June 12
μ *Serpentis*	15 44.4	− 3 7	A0	3.63	Y368H	3m	40	June 12
ε *Serpentis*	15 45.8	+ 4 47	A2	3.81	Y369B	3m	33	June 12
τ *Herculis*	16 16.7	+46 33	B5p	3.79	Y325I*	27m	18	May 24
					Y411F	4m	18	June 15
χ *Ophiuchi*	16 21.2	−18 14	B3p	4.68	Y429B	3m	56	June 17
η *Draconis*	16 22.6	+61 44	G5	3.67	Y412H	4m	27	June 15
λ *Ophiuchi*	16 25.9	+ 2 12	A0	3.85	Y370E	3m	36	June 12
ζ *Ophiuchi*	16 31.7	−10 22	B0	2.46	Y316H*	7m	50	May 23
					Y338H*	5m	47	May 27
					Y371F	40s	48	June 12
					Y430E	40s	48	June 17
ε *Herculis*	16 56.5	+31 4	A0	3.92	Y337F*	30m	11	May 27
					Y373B	8m	8	June 12
η *Ophiuchi*	17 4.6	−15 36	A2	2.69	Y341E*	7m	53	June 5
					Y372H	2m	53	June 12
ζ *Draconis*	17 8.5	+65 50	B5	3.10	Y335B*	15m	35	May 27
					Y431F	48s	29	June 17
δ *Herculis*	17 10.9	+24 57	A2	3.22	Y342B*	16m	13	June 5
					Y374E	2m	15	June 12
α *Ophiuchi*	17 30.3	+12 38	A5	2.28	Y343F*	5m	26	June 5
					Y375F	40s	26	June 12
ξ *Serpentis*	17 31.9	−15 20	A5	3.28	Y344H*	17m	53	June 5
					Y381B	2m	54	June 14

*Plates taken with the slit spectrograph.

TABLE I.—OBSERVATIONAL DATA—(Continued)

Star	α (1900)	δ (1900)	Class	Ptg. Mag.	Plate	Exp.	Z.D.	Date
ι *Herculis*..........	$17^{\rm h}$ $36^{\rm m}.6$	+46° 4′	B3	3.62	Y432H	4m	11°	1925 June 17
γ *Ophiuchi*..........	17 42.9	+ 2 45	A0	3.74	Y376H	3m	36	June 12
γ *Draconis*..........	17 54.3	+51 30	K5	3.60	Y433B	4m	14	June 17
67 *Ophiuchi*..........	17 55.6	+ 2 56	B5p	3.80	Y382E	3m	19	June 14
72 *Ophiuchi*..........	18 2.6	+ 9 23	A3	3.81	Y383F	3m	29	June 14
ο *Herculis*..........	18 3.6	+28 45	A0	3.83	Y384H	3m	8	June 14
α *Lyrae*..........	18 33.6	+38 41	A0	0.14	Y345B*	40s	1	June 5
					Y385B	10s	1	June 14
γ *Lyrae*..........	18 55.2	+32 33	A0p	3.30	Y386E	2m	6	June 14
ζ *Aquilae*..........	19 0.8	+13 43	A0	3.02	Y347F*	12m	24	June 5
					Y387F	2m	24	June 14
λ *Aquilae*..........	19 0.9	− 5 2	B9	3.53	Y388H	2m	42	June 14
ι *Cygni*..........	19 27.2	+51 31	A2	4.00	Y435F	4m	15	June 17
δ *Cygni*..........	19 41.9	+44 53	A0	2.97	Y348H*	11m	10	June 5
					Y436H	2m	15	June 17
α *Aquilae*..........	19 45.9	+ 8 36	A5	1.03	Y349B*	3m	29	June 5
					Y389E	10s	30	June 14
9 *Sagittarii*..........	19 47.9	+18 25	Oe5	6.29	Y390F	15m	20	June 14
θ *Aquilae*..........	20 6.1	− 1 7	A0	3.37	Y350E*	16m	38	June 5
					Y391H	2m	38	June 14
ε *Delphini*..........	20 28.4	+10 58	B5	3.86	Y392B	3m	27	June 14
α *Delphini*..........	20 35.0	+15 34	B8	3.81	Y393B	3m	22	June 14
α *Cygni*..........	20 38.0	+44 55	A2p	1.39	Y353B*	4m	8	June 5
					Y437B	16s	17	June 17
ε *Aquarii*..........	20 42.3	− 9 52	A0	3.83	Y394E	3m	47	June 14
α *Cephei*..........	21 16.2	+62 10	A5	2.74	Y439F	2m	27	June 17
β *Cephei*..........	21 27.4	+70 7	B1	3.10	Y440H	48s	34	June 17
7 *Lacertae*..........	22 27.2	+49 46	A0	3.85	Y441B	4m	20	June 17
10 *Lacertae*..........	22 34.8	+38 32	Oe5	4.91	Y442E	8m	19	June 17

* Plates taken with the slit spectrograph.

Method of observation.—I shall now proceed to describe briefly how the observations were taken, without attempting to go into detailed reasons for each procedure, as these are familiar to investigators in this field of research and will become more evident when we come to discuss the requirements of photometry in the next section.

Since the spectral region with which we are chiefly concerned in this investigation lies in the ultra-violet, it is imperative that the spectra should suffer as little as possible from the diminution of intensity of the ultra-violet light by the moisture in the air which is a potent absorber of the short wave lengths. With this in view the plates were taken only on exceptionally clear and dry nights. They were also taken, with a few exceptions, near the meridian, where the star's zenith distance is the smallest and the atmospheric absorption is, consequently, at a minimum. The seeing was usually good during the periods of observation.

For the intensity study in the ultra-violet the slit spectrograph is not so well adapted as the slitless one, for there is a general loss of light at the slit and relatively more in the short wave region as will be explained later. However, the slit instrument was first employed, as it was originally planned to use the Sun as a standard of comparison, and of course the slitless spectrograph cannot be very well applied to such an extended and bright object as the Sun. Later, on account of some uncertainty in our knowledge of the Sun's intensity curve in the ultra-violet, it was given up as a standard of comparison. Several spectrograms of the Sun were nevertheless obtained and made to serve as a check in our reductions and deductions. The method by which the Sun's plates were taken will be described presently. In spite of the disadvantage just mentioned the slit spectrograph was used to secure a number of stars, especially those too bright to be conveniently photographed with the slitless.

The broadening of the spectrograms secured without the slit was effected for faint stars by the ordinary method of slowly trailing the stellar image, and for bright stars by moving the telescope back and forth along the declination circle with the aid of the electrically controlled right ascension slow motion. In the majority of cases the spectrograms thus obtained present a very uniform appearance suitable for measurement. In some cases, however, especially with the brighter stars when one or two rapid trails are sufficient for the exposure, streaks appear, due no doubt to the irregularity of the slow motion. In general a record free from streaks can be found among the three exposures impressed on each plate taken with the slitless spectrograph. The times of these three exposures are approximately in the ratio 1 : 2 : 4. The middle spectrogram being usually normally exposed was utilized most frequently. The exposure times which appear in Table I refer to the middle exposure.

It will be evident that this method of trailing the star images cannot be applied with advantage to the case in which the slit spectrograph is used except for making observations of stars near the zenith. In making these photographs an observing practice developed by Mr. Wright in the use of this instrument was adopted. In this procedure the slit instead of being set parallel to the equator, as is usually the case, is turned (by turning the spectrograph of course) so as to coincide with the vertical circle. The effect of this adjustment is to make the slit parallel to the atmospheric spectrum so that light of every wave-length can pass through the slit. This is an extremely important requirement which is often overlooked in the investigation of the intensities in the ultra-violet region; for under extreme conditions when the visible image of a star is held on the slit in the act of guiding, the whole ultra-violet end of the spectrum may be occulted. The estimate of relative intensities along the spectrum cannot, in these cases, be of value. With the slit in the vertical plane, the only way to produce a widening of the spectrum is to strain the telescope up and down, keeping the slit in view and the star upon it all the time, in order to insure uniform exposure of the spectrogram and freedom from streaks. As only very narrow slits were used the time required amounted at times to as much as 20 minutes. Guiding such as this is a task at once tiresome and unpleasant. However, the writer feels that his efforts are more than repaid by the greater reliability of the results obtained. It may be suggested that a delicate slow motion in declination, with which the Crossley telescope is not at present equipped, would partially solve the difficulty. Stars near the zenith were observed with the slit in the usual position-angle.

The Standardizing Photometer.—In order to make a study of the continuous absorption spectrum, its location and its intensity distribution, it is necessary first to know the normal or undisturbed intensity distribution of the stellar radiations. The photometric method involved in determining the intensities is very simple. It consists of comparing photographically the stellar spectra with a standard source whose intensity distribution along the spectrum is known or assumed to be known. This is essentially the method followed by Rosenberg,[15] Sampson,[16] and others, in their determination of stellar temperatures. Our method, however, differs from theirs in the respect that the comparison is made, not directly on the same photographic plate, but indirectly via a set of standard exposures impressed on each plate by means of a standardizing photometer now to be described.

Immediately before or after the photographing of the stellar spectra, 6 standard exposures with intensities increasing by the ratio 1 : 2 were impressed simultaneously on the plates with the photometer, a view of which is shown in Figure 1 below.

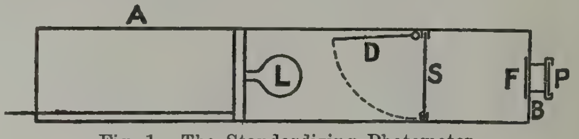

Fig. 1. The Standardizing Photometer.

The photometer is in all essential respects equivament to the one devised by Wright[17] for use in observing the solar eclipse of 1923, but was in part rebuilt for this study in order to allow the employment of a lamp of greater candle-power. It consists of a light tight 48″ x 12″ x 9″ wooden box, *A*, blackened inside. Figure 1 shows the box with the front side removed. *L* is a 200-watt frosted Mazda lamp supported by a wood block, whose distance from the ground glass, *S*, can be varied at will. *B* is the sensitometer box. There is a series of six accurately drilled holes in the side that faces the lamp, the areas of which increase progressively in the ratio 1 : 2. These holes serve as windows to permit light from the diffusing screen, *S*, to fall, in areas of graded intensity, on the photographic plate, *P*. Each chamber is adequately diaphragmed to eliminate stray light. *D* is a swinging diaphragm for making the exposures.

It will be noticed that the blackening of the standard exposures is caused by light composed of all colors, whereas the blackening at any point along the spectrum is effected by monochromatic light. Therefore the blackening of the spectrum, whose intensity distribution is to be determined, cannot be strictly comparable with the blackening of the standard exposures thus produced. To partially remedy this defect, an ultra-violet filter, *F* (Wratten No. 18), which transmits the light of the region we are chiefly interested in, was placed behind the holes. This, in fact, satisfies the requirements to a high degree of approximation; furthermore, as will be shown later, by using some known or assumed spectrum whose blackening is also compared with these standard exposures, any existing error will finally eliminate itself, or at least remain as only a second order quantity which may be neglected.

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[15] *Ast. Nach.*, Nr. 4628, 1912.

[16] *Monthly Notices*, **83**, 174, 1923; **85**, 212, 1925.

[17] *Publ. Astron. Soc. Pac.*, **35**, 281, 1923.

The distance of the lamp from the diffusing screen is made variable so as to alter the time of exposure according to the time used in exposing the star plates. The equality of time of exposure, as between standard and star, is a desirable feature because it eliminates some uncertainties in the photographic process. But it was soon realized that due to the trailing of the star the time of exposure is not the true time. Therefore it was finally decided to use one position of the lamp throughout. The time adopted to impress the standard exposures for the plates taken with the slit spectrograph was 4 minutes, and 2 minutes for those secured with the slitless. It may be added that the lamp draws current from a 220-volt D.C. generator. The current consumed was 0.8 amp. and kept constant by the control of a rheostat.

To insure as far as possible identical conditions in the process of development, all the plates were developed for 5 minutes in "Rytol" Universal Developer at 60° F. Care was also taken to fix and wash them under similar conditions. Figures 1 and 2, Plate IV, show the general appearance of the plates taken with the slit and the slitless spectrograph respectively. They were enlarged about one and one-third times from the originals.

Radiation standards of comparison.—The relative blackening of the plate along the spectrum does not, of course, indicate the relative intensities of the energy of different wave lengths as actually possessed by the star. The true intensities before producing the corresponding blackenings in the photographic plate are affected by the following factors: (1) Absorption within our atmosphere; (2) The reflectivity of the silver mirror; (3) Losses in the spectrograph; (4) The sensitivity of the photographic plate; and (5) The lack of normality of dispersion. All these factors are functions of the wave lengths and it is obvious that to obtain the real intensity distribution from the blackening of the plate, these factors have to be known. The first factor has been extensively observed and determined by Abbot,[18] and verified by other investigators, notably Fabry and Buisson[19] and Plaskett.[20] The last correction can be obtained from the constants of the instrument. The determination of the others, while not impossible, would involve errors which in the aggregate would render the corrections very uncertain. If, however, some standard source, whose energy distribution is known, could be chosen and its radiations be made to follow the identical path to the photographic plate that is traversed by the source under study, the energy distribution of the latter would be immediately secured by comparing the blackenings of the two sources, since the variable factors would be eliminated. This led us to adopt for our investigation a primary standard of comparison.

The ones which presented themselves to mind and which were considered are the acetylene flame and the positive crater of the carbon arc. These two standards have been employed with more or less success by various investigators. However, the relatively weak radiations of these two sources in the ultra-violet region, and numerous other difficulties both theoretical and experimental made us hesitate and finally abandon their adoption. It seems clear that no terrestial source at our command at present can satisfy our requirements.

The Sun was next considered. In view of the numerous and rather complete researches that have been undertaken on its energy spectrum, it would seem to be a suitable standard. But, unfortunately, its energy in the ultra-violet is only imperfectly known. It was therefore likewise given up as a comparison for our purpose. It may be emphasized here that a celestial standard, other things being equal, possesses a distinct advantage over any terrestial one in work of this kind, in that errors of atmospheric absorption, will be effective only in differential amounts.

It thus remained for us to pick out a star and use it as a standard of comparison. In following this course we are, in a sense, deferring the final solution of our problem until such a time as the spectral energy curve of our standard star shall be determined in some positive and dependable way. However, we may, meanwhile, derive a tentative law of distribution on the basis of two assumptions: (1), that the temperature of the star is known, and (2), that the star radiates as a black body, obeying Planck's law. The star *ζ Ophiuchi* of Class B0 was accordingly selected, on account of its relatively strong radiation in the ultra-violet, its freedom from numerous absorption lines, and, most important of all, the absence of the continuous absorption which is the subject of our investigation. The temperature assumed for the star is 22,000° K. In view of the recent researches on stellar temperatures by Rosenberg, Sampson and others, this figure cannot be regarded as extravagant. As a matter of fact a deviation of a few thousand degrees will not appreciably alter our results since the shapes of the Planckian curves for those high temperatures in the region from λ 3300 to λ 5000 covered by our observations are not very different. The second assumption seems highly probable, and is borne out to some extent by the work of Plaskett.[21] Furthermore the validity of these assumptions, as we shall see, is strongly substantiated by the concordant results yielded in this paper.

Method of taking the Sun's spectrum.—It may be of interest to describe here how the Sun's spectrum was photographed. The observations were made in order to see in what measure the results reduced by our method agree with those of Abbot obtained from direct bolometric measures. As is well known, Abbot's intensity measures of the Sun's energy spectrum include all the absorption lines. To conform to this condition our spectra were taken out of focus. The solar intensity was reduced by means of a screen with 12 evenly distributed holes 0.04 inches in diameter, placed over the top of the telescope. But it was found that, due to simple optical considerations, the spectrum was spread out to cover a wide area when the plate was put out of focus. To secure a narrow spectrum it was necessary to place all the holes in a straight line perpendicular to the slit. Assuming the silver mirror to have uniform reflecting surface throughout its disk, the question that next arose was where to put this line of holes so that the radiations as transmitted by the holes would be of the same character as if they had come from all parts of the mirror, as is the case in taking the stellar spectrum, with which that of the Sun is to be compared. This consideration will not be necessary if the camera lens is of sufficient diameter to transmit all the beams which pass the collimator lens. A simple though approximate calculation due to Mr. Wright shows that the line should so lie that $\alpha = \cos^{-1}\frac{\pi}{4}$, as shown in Figure 2. Through a slight error in interpretation the line was actually placed at $\alpha = 45°$. The difference in the photographic record caused by this error can, however, hardly be appreciable. In the figure, S is the position of the slit.

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[18] *Ann. Astrophys. Obs. Wash.*, **2**, 50, 1908.

[19] *Astrophys. Journ.*, **54**, 297, 1921.

[20] *Publ. Dom. Astrophys. Obs.*, **2**, 248, 1923.

[21] *Publ. Dom. Astrophys. Obs.*, **2**, 257, 1923.

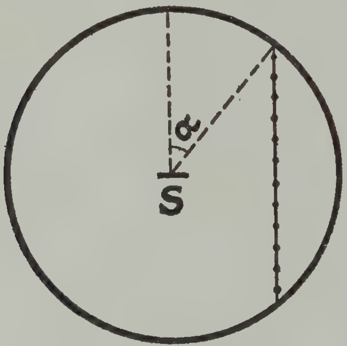

Fig. 2. Arrangement of Holes in Screen for taking the Sun's Spectrum.

The spectrum refers to the center of the Sun. Care was taken to see that no sunspot existed there when the observations were obtained. Standard exposures were also impressed on the solar plates.

Brief description of the spectrograms.—Before passing to the next section in which the method of measuring and reducing the plates is presented, a word or two about the plates may be desirable. Two typical spectrograms are reproduced in Figure 3 and Figure 4, Plate IV. They are identical with Figure 1 and Figure 2 of the same plate, respectively, except that they are positives and enlarged about seven times from the original negatives. With but few exceptions, where streaks are seen, the spectrograms present a very homogeneous and uniform appearance, an important requirement in the process of measurement. The Balmer series of hydrogen makes a remarkable exhibit in all the Class A stars. The first line on the right of the figures shown is Hβ. The series can generally be followed with ease as far as the 15th or 16th line and in some cases even farther, especially when the lines are sharp. The sudden fall of intensity of the continuous background near the limit of the series is readily perceived. The dispersion of the spectra taken with the slit and the slitless spectrograph is not the same. For the former it amounts to 1 mm = 51A at λ 3600 and for the latter 1 mm = 59A at the same place. The total spectral region covered by our observations extends from λ 3300 to λ 5100.

REDUCTION AND RESULTS

Photographic theory.—It is well established by the researches of Hurter and Driffield, Schwarzschild and others, that over the region of normal exposures of a photographic plate, the relation between the density or blackening D_λ, exposure time t, and incident intensity J'_λ, for any wave length λ, can be represented by the formula

$$D_\lambda = \gamma_\lambda \log_{10} \left(\frac{J'_\lambda t^p}{i_\lambda}\right) \qquad (1)$$

where p is Schwarzschild's constant, i_λ, the inertia of the plate and γ_λ, the photographic development constant.

Equation (1) may, for one purpose, be conveniently put in the exponential form

$$J'_\lambda = \left(\frac{i_\lambda}{t^p}\right) 10^{\frac{D_\lambda}{\gamma_\lambda}} \qquad (2)$$

Let equation (2) represent the relation existing in the photographic spectrum of a star, whose intensity distribution J'_λ is to be determined. In order to distinguish it from the true intensity outside the atmosphere I shall call J'_λ henceforth the *apparent* intensity. For the six standard exposures equation (2) becomes

$$K_{\lambda'} = \left(\frac{i_{\lambda'}}{t_1{}^p}\right) 10^{\frac{D_s}{\gamma_{\lambda'}}} \qquad (3)$$

where $K_{\lambda'} = 1, 2, 4, 8, 16, 32$; (see page 109) and $\lambda' = 3600\text{A} \pm$.

Dividing (2) by (3), we get

$$J'_\lambda = K_{\lambda'}\left(\frac{i_\lambda}{i_{\lambda'}}\right)\left(\frac{t_1}{t}\right)^p 10^{\left(\frac{D_\lambda}{\gamma_\lambda}-\frac{D_s}{\gamma_{\lambda'}}\right)} \qquad (4)$$

assuming p to be constant for all wave lengths, and the same for t and t_1. The assumption[22] is permissible if t and t_1 do not differ by more than 10 to 1. $t_1=4$ minutes for the slit plates and 2 minutes for the others. We see from Table I that t, the exposure time for a star, is within these limits without a single exception. As a matter of fact the times recorded there are not the actual times of exposure, due to the trailing of the star, a fact already noted. In order to be comparable with the standard exposure time, they have to be reduced by some such factor as 4 or 5, since the width of the spectrograms is about 4 or 5 times the width of one taken with the star image stationary. In view of this the ratio is even smaller than 10 to 1 especially in the case of slit observations. Since only the relative intensity is required, we shall discard the factor $\left(\frac{t_1}{t}\right)^p$, then assuming $i_\lambda = i_{\lambda'}$, and $\gamma_\lambda = \gamma_{\lambda'}$, equation (4) reduces to

$$J'_\lambda = K_{\lambda'} 10^{\left(\frac{D_\lambda - D_s}{\gamma_\lambda}\right)} \qquad (5)$$

or $$\gamma_\lambda(\log_{10} J'_\lambda - \log_{10} K_{\lambda'}) = D_\lambda - D_s \qquad (6)$$

This is the fundamental relation for obtaining J'_λ from the measured densities D_λ and D_s. For if $D_\lambda = D_s$, then from (6) $J'_\lambda = K_{\lambda'}$. D_s has six values corresponding to the six known intensities of $K_{\lambda'}$. For intermediate figures interpolation can be resorted to. In actual practice, however, the six values of D_s are plotted against the six intensities on logarithmic paper, and a smooth curve is drawn through them. This is shown in Figure 3 and serves as a converting curve, from which the apparent intensity J'_λ can be read off to correspond to any density measured.

It should be pointed out here that an error was introduced in equation (6) when we assumed that $i_\lambda = i_{\lambda'}$ and $\gamma_\lambda = \gamma_{\lambda'}$, which is not strictly true, for λ' only refers to a small spectral region centering around λ 3600, whereas λ varies within wide limits. However, the error is only apparent, for when a comparison star is chosen to calibrate the real intensity, it will eliminate itself in the following manner:

For a comparison star equation (4) becomes

$$[J'_\lambda] = K_{\lambda'}\left(\frac{[i_\lambda]}{i_{\lambda'}}\right)\left(\frac{t_1}{[t]}\right)^p 10^{\left(\frac{[D_\lambda]}{[\gamma_\lambda]}-\frac{D_s}{\gamma_{\lambda'}}\right)} \qquad (7)$$

where [ ] refers to the comparison spectrum.

Dividing (4) by (7), we obtain

$$\frac{J'_\lambda}{[J'_\lambda]} = \frac{i_\lambda}{[i_\lambda]}\left(\frac{[t]}{t}\right)^p 10^{\left(\frac{D_\lambda}{\gamma_\lambda}-\frac{[D_\lambda]}{[\gamma_\lambda]}\right)} \qquad (8)$$

It is seen that in (8) the quantities relating to the standard exposures have disappeared. Furthermore $i_\lambda = [i_\lambda]$ and $\gamma_\lambda = [\gamma_\lambda]$ since the sensitivity and development factors are assumed to remain unchanged. Thus in reality the apparent assumptions made above to reduce equation (4) to a simple form are not involved in the final results.

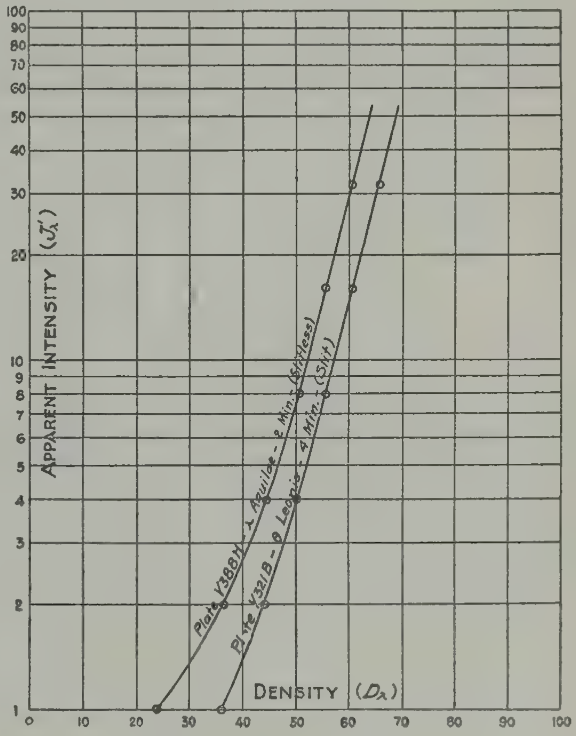

Fig. 3. Converting Curves.

Method of measurement.—The densities of the spectra (D_λ), and of the standard exposures (D_s), were measured on the Hartmann equalizing wedge microphotometer. A small narrow slit of silver coating in the interface of the double prism renders a narrow region of the spectrum visible in the center of field of view. A 100-watt lamp illuminates both the portion of the spectrum under examination and the surrounding field, whose brightness is regulated by a photographic wedge made also from a Seed 23 plate. The wedge was set at the moment when the entire field appeared uniformly illuminated, or when the boundary of the slit vanished due to the perfect matching of the silver grains. Its reading thus gives a measure of the density. The table on which the plate is mounted can be moved so that different parts of the spectrum are successively brought to view, and the amount of motion can be read on a graduated scale attached to the slide. This furnishes a means for locating the regions measured. In general a measure was made at every 0.5 mm along the spectrum except near the end of the Balmer series where the lines are close together. There settings were made between the lines. In addition to the continuous spectrum, the hydrogen lines, and the H and K lines if present, were measured. About 70 settings were made for each plate and approximately three quarters of an hour was required in measuring it. Considering the fairly large amount of work involved it may be surmised that considerable time would have been saved, and perhaps more accurate results obtained, if a Koch microphotometer of some form had been used in the reduction of the plates. However, in view of the extra labor entailed in taking another set of plates to obtain the density curves, which would in turn have had to be calibrated, it is questionable whether much would have been gained for our purpose. Of course it must be admitted that, while the measures were made only at intervals, a density curve obtained from a Koch microphotometer is continuous and therefore permits a study of finer details, like the structure and shape of the lines, the serrations of the continuous spectrum due to presence of faint lines, etc., but with these we are not especially concerned.

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[22] *Publ. Dom. Astrophys. Obs.*, **2**, 219, 1923.

The apparent intensity graphs.—With the aid of converting curves such as shown in Figure 3, the densities measured were transformed into apparent intensities (J'_λ), which were then plotted against the linear distance along the spectrum. These J'_λ graphs for a set of typical stars can be seen in Figure 6 and Figure 8, the first set being reduced from the slit plates, and the second from the slitless. As great accuracy is not required in identifying the wave-lengths the dispersion was evaluated by means of a *Hartmannsches Dispersonsnetz* using for the purpose only the Balmer series lines, although an iron arc comparison spectrum is available in all of the slit plates.

A glance at Figures 6 and 8 will give some idea of the accuracy of the measures. Each circle (o) represents a measure and a cross (+) indicates a spectral line. It will be seen that all points fall very close to a smooth curve. Three such curves were drawn in each figure; the heavy one which represents the apparent intensity variation of the spectrum, and two light curves, one traced through the maximum intensities and one through the bottom of the series lines. The last two curves and that part of the first one which extends from the end of the observed series lines to the ultra-violet will be utilized later in translating these apparent intensities into real ones. The curve through the bottom of the lines was omitted in early B stars and stars later than F0, as the intensities of the lines there are somewhat irregular and do not lie closely on a smooth curve. The point marked "L" in the graphs indicates the position of the theoretical limit of the Balmer series.

While these graphs do not yet represent the real intensity distribution along the spectrum, one cannot, in looking over them, help being impressed by the marked drop in intensity near the end of the Balmer series in all Class A stars. The phenomenon is most striking when a direct comparison is made between Figure 6a and say Figure 6c or between Figure 8a and Figure 8d. It may not be premature here to call attention also to several other features in these graphs, the significance of which will be treated later from a theoretical standpoint. One is a general continuation of the intensity of the Balmer lines with the intensity of the continuous spectrum at the observed limit. Another is a slight increase in the intrinsic intensity of the lines occurring near the end of the observed series. The last feature is exhibited strongly in graphs reduced from the slit plates and to a less degree in the others.

The true intensity graphs.—As has already been stated the apparent intensity differs from the true intensity for a number of reasons, namely, absorption by the atmosphere, imperfect reflectivity at the mirror, losses in the spectrograph, sensitivity of the plate and variable dispersion. All these quantities are functions of the wavelength.

Let

J_λ = true intensity* of emission from stars
J'_λ = apparent intensity as registered by Seed 23 plates
a_λ = atmospheric transmission coefficient
r_λ = reflectivity coefficient of the silver mirror
b_λ = transmisssion coefficient of the spectrograph
d_λ = coefficient of dispersion
s_λ = sensitivity coefficient of the Seed 23 plate,

Then

$$J'_\lambda = J_\lambda a_\lambda r_\lambda b_\lambda d_\lambda s_\lambda \tag{9}$$

The last four quantities are identical for all stars observed with one spectrograph, assuming that all the plates used have the same sensitivity and that the silver mirror did not change appreciably in its reflective power within the short period during which most of the observations discussed in this paper were made.

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* The term "true intensity" as used here means the actual relative intensity in the spectrum of the star, that is to say, the true or absolute intensity multiplied by an undetermined quantity which is constant for the plate. In other words the graphs are without absolute energy scale.

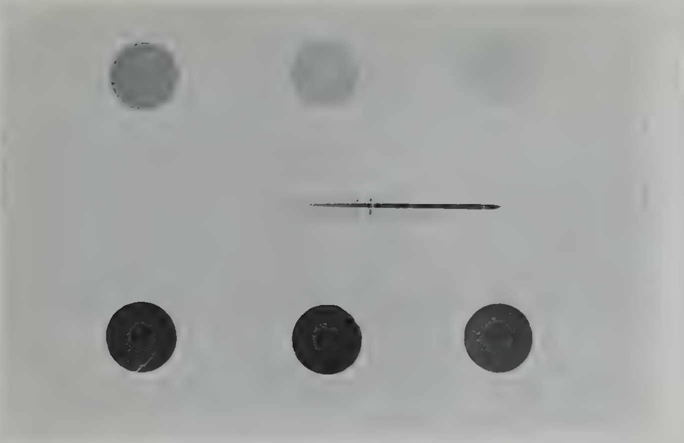


Fig. 1. Plate Y321B. *θ Leonis* A0.
(Slit)

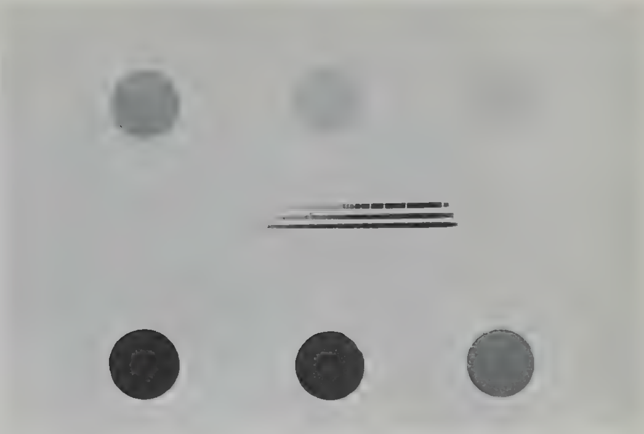


Fig. 2. Plate Y401B. *β U. Majoris* A0.
(Slitless)

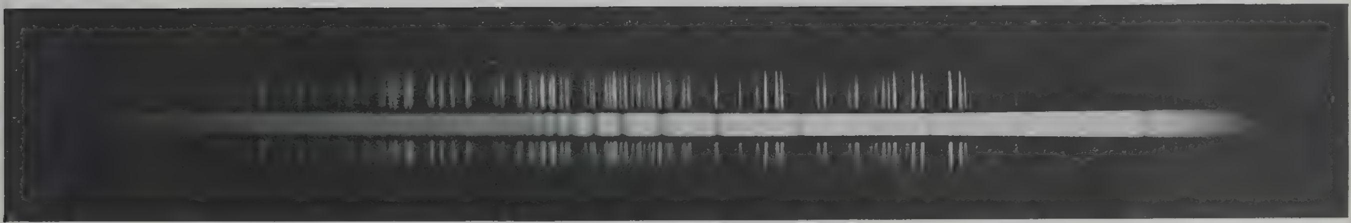


Fig. 3. *θ Leonis* A0 (Slit).

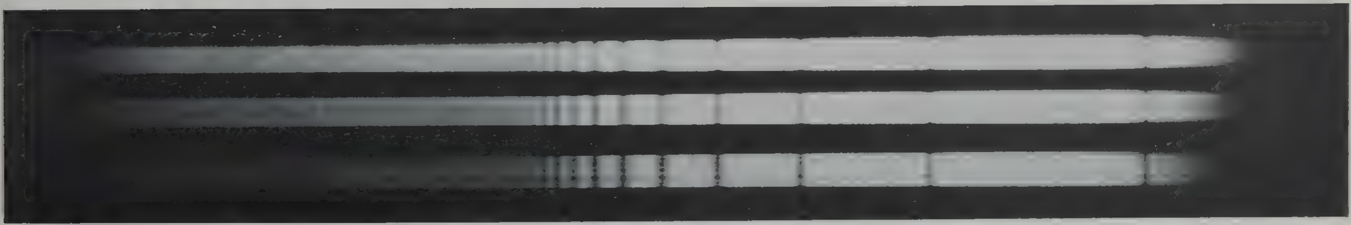
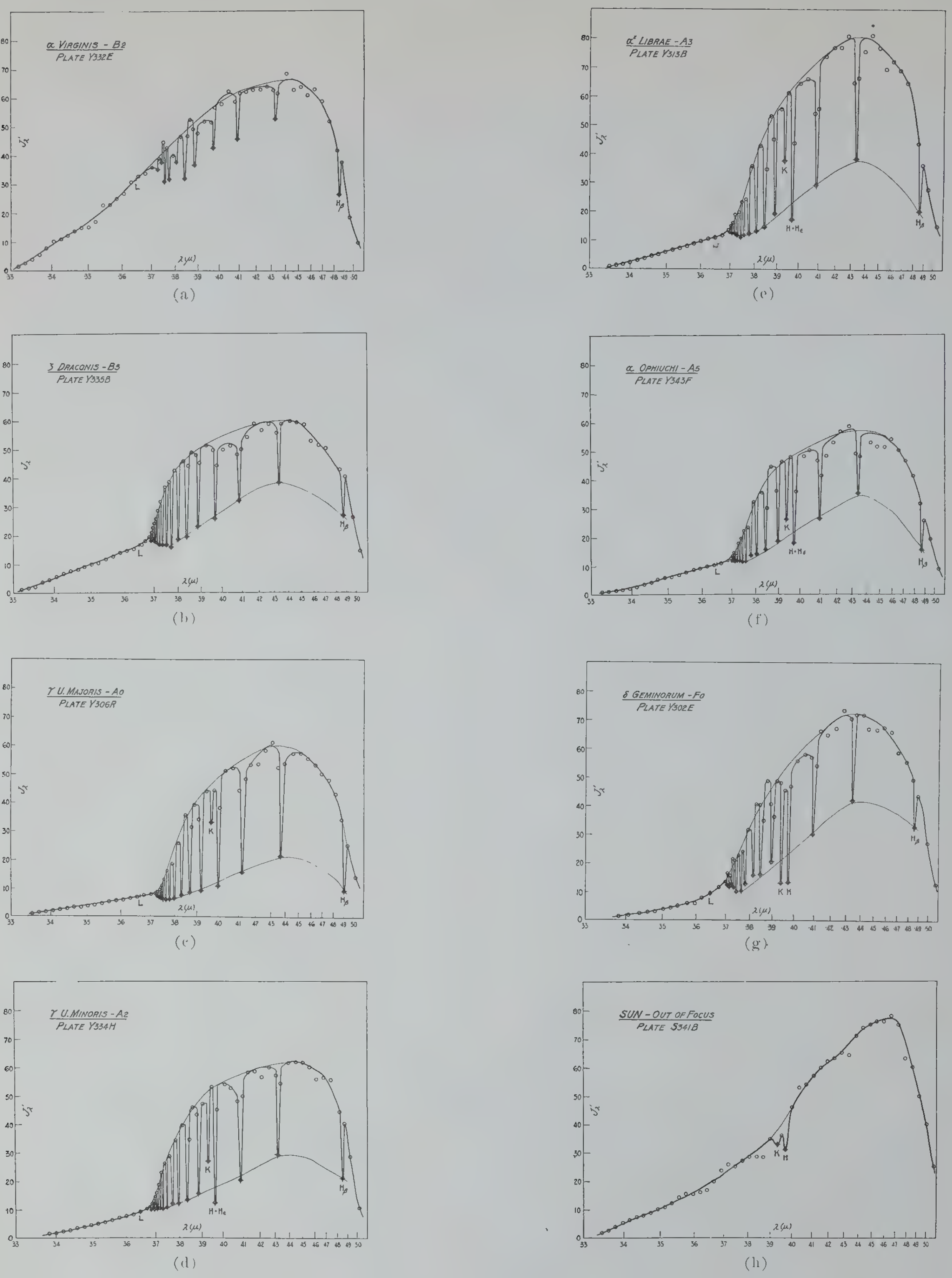


Fig. 4. *β U. Majoris* A0 (Slitless).



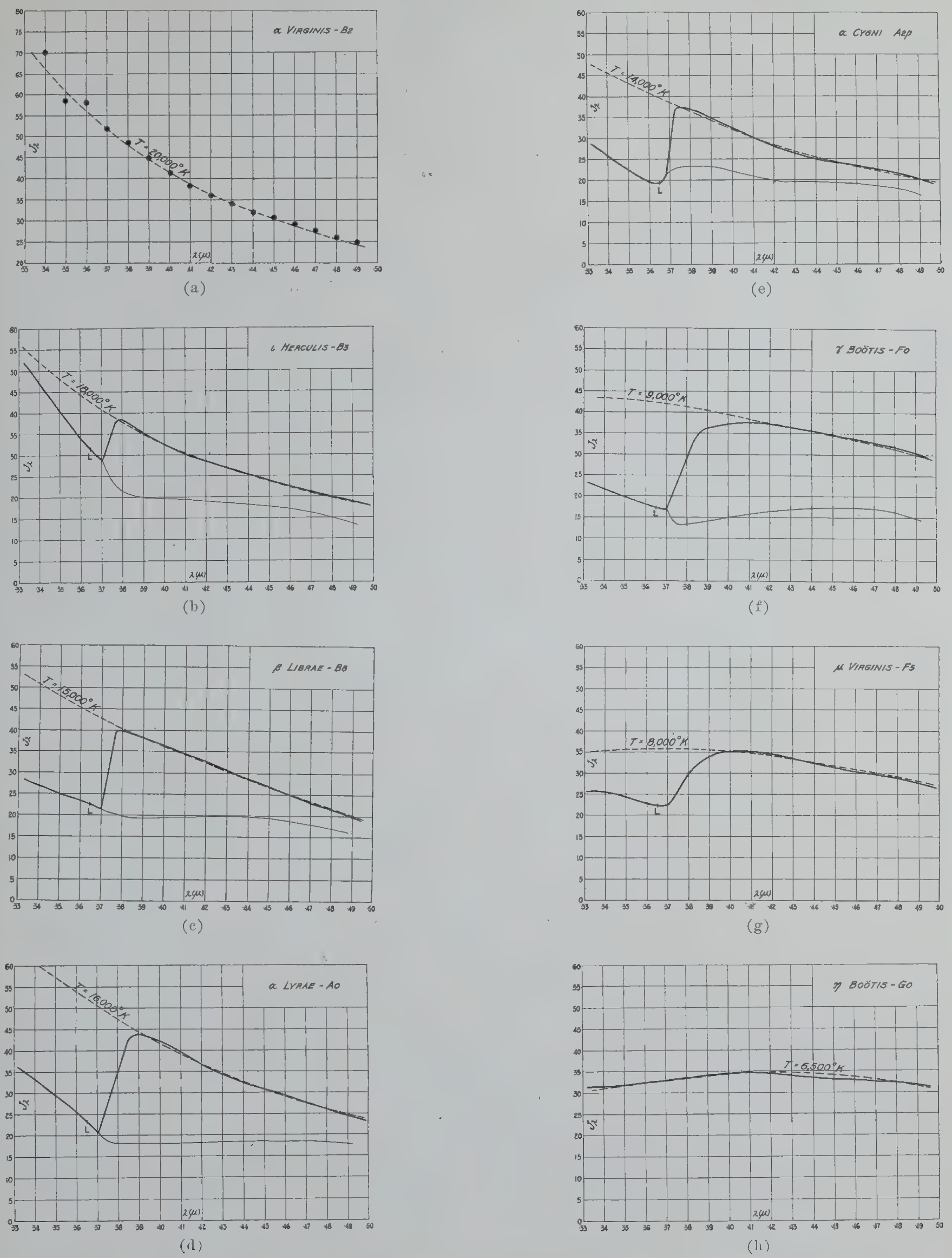
(a) (e)

(b) (f)

(c) (g)

(d) (h)

Fig. 6. Apparent Intensity Graphs.
(Slit Observations)



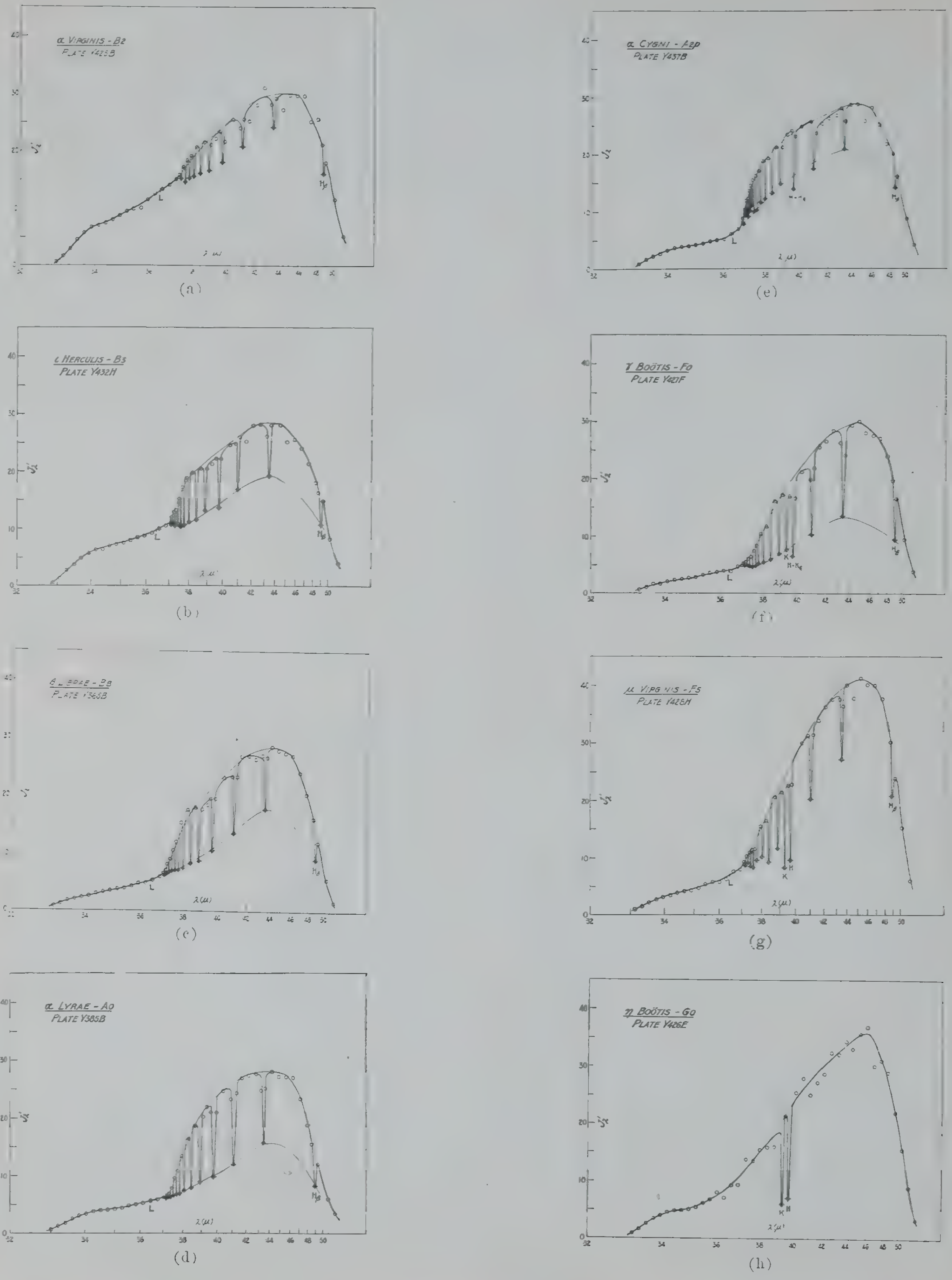
(a) (e)

(b) (f)

(c) (g)

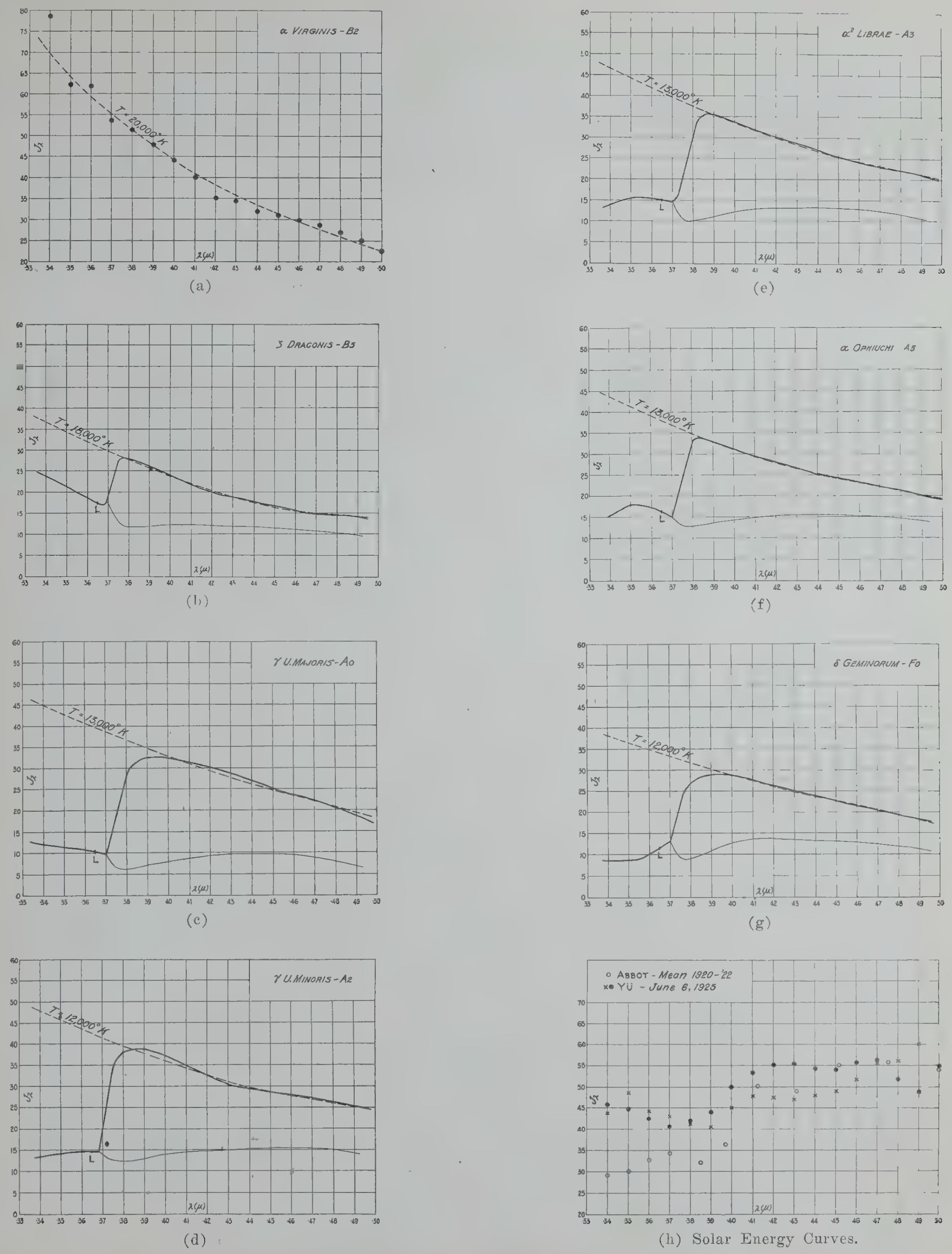
(d) (h)

Fig. 9. Stellar Energy Curves.
(Reduced from Fig. 8.)



(a) (b) (c) (d) (e) (f) (g) (h)

Fig. 8. Apparent Intensity Graphs.
(Slitless Observations)



(a) (b) (c) (d) (e) (f) (g)

(h) Solar Energy Curves.

Fig. 7. Stellar Energy Curves.
(Reduced from Fig. 6.)

Hence they can be treated together, that is, we may write

$$J'_\lambda = J_\lambda a_\lambda (rbds)_\lambda \tag{10}$$

For any star, J'_λ is known. a_λ can be computed. Hence if we know $(rbds)_\lambda$, J_λ follows at once from equation (10).

To determine $(rbds)_\lambda$, we employ a standard star as a comparison. The star ζ *Ophiuchi* (B0) was selected for this purpose (see Figure 4). For this star, we have

$$[J'_\lambda] = [J_\lambda][a_\lambda](rbds)_\lambda \tag{11}$$

or

$$(rbds)_\lambda = \frac{[J'_\lambda]}{[J_\lambda][a_\lambda]} \tag{12}$$

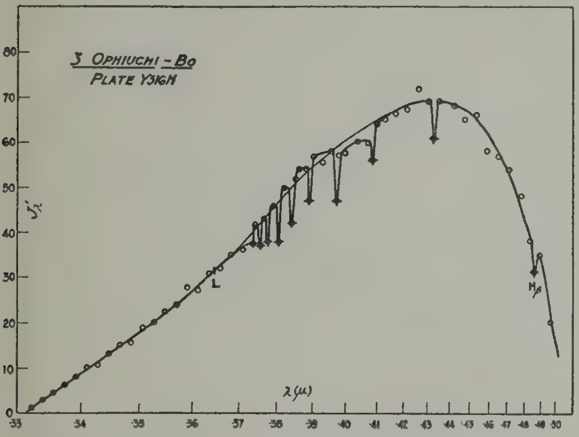


Fig. 4. The Comparison Spectrum for Slit Observations.

Everything on the right hand side of equation (12) is known except $[J_\lambda]$. Now in order to obtain $[J_\lambda]$, the two assumptions already mentioned are made (1) relating to the temperature of the comparison star, and (2) to its temperature-radiation law. The temperature assumed for ζ *Ophiuchi* is 22,000° K, and the law is assumed to conform to the black body radiation formula of Planck.

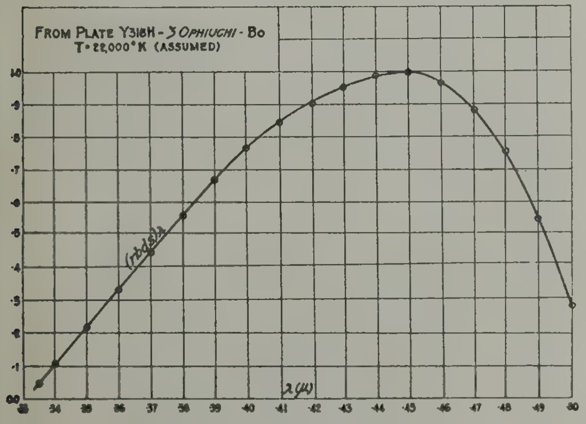


Fig. 5. Correcting Factor Curve.

The numerical values of the correcting factor as obtained from equation (12) are shown in the last column of Table II, and plotted in Figure 5. They were derived from the preceding column in the table by making the maximum value at λ.45(μ) unity. It should be remembered that this correcting factor holds only for the slit observations. For the others a similar, but independent solution like the one shown below has to be made.

Table II.—Computation of the Correcting Factor From ζ *Ophiuchi* (B0)—Plate Y316H

$\lambda(\mu)$	J'_λ	a_λ	J_λ(22,000°)	$(rbds)_\lambda$	
.335	3.6	.371	3.65	2.68	.042
.34	9.0	.388	3.50	6.64	.107
.35	18.0	.420	3.24	13.22	.212
.36	26.5	.450	2.99	19.7	.317
.37	37.0	.483	2.79	27.5	.442
.38	48.0	.514	2.59	34.6	.558
.39	55.0	.545	2.39	42.2	.678
.40	60.5	.580	2.20	47.5	.774
.41	65.0	.612	2.05	51.9	.835
.42	68.0	.640	1.91	55.7	.895
.43	69.0	.660	1.78	58.7	.945
.44	69.0	.679	1.65	61.7	.992
.45	66.5	.695	1.54	62.2	1.000
.46	61.0	.712	1.43	60.0	.965
.47	54.0	.730	1.34	55.2	.888
.48	43.5	.747	1.25	46.6	.750
.49	30.0	.765	1.17	33.5	.538
.50	15.0	.780	1.09	17.6	.283

Thus the true intensity distribution (J_λ) for any star can be derived from equation (10), or after transposing,

$$J_\lambda = \frac{J'_\lambda}{a_\lambda (rbds)_\lambda} \tag{13}$$

The right member of this equation is now completely known.

The true intensity graphs derived in this manner from the Figure 6 and Figure 8 series are shown in Figure 7 and Figure 9, respectively. The latter series was reduced independently, although for the sake of avoiding further assumptions the same star was used as a comparison. The intensity scale is an arbitrary one. It will be seen by comparing Figure 7 and Figure 9 that the two sets of results agree closely in many respects. The dotted lines were derived from Planck's law of radiation at the temperatures indicated. It is of importance to note that the stellar curves away from the region of continuous absorption follow the Planckian curves remarkably well. On account of this there appears to be no reason to doubt that if the absorption did not exist, the ultra-violet part of the energy curve would also obey the same law. In fact this is fully borne out by such a star as α *Virginis* (see Figure 7a and Figure 9a) where no absorption is found. It will be noticed also that the effective temperatures derived agree well with those obtained by recent investigators, notably Rosenberg and Sampson, already referred to earlier in the paper. One can, of course, argue that our results were based on the assumptions made respecting the comparison star. If the comparison star deviates from black body conditions, and if the same deviation affects all stars alike, it is clear that the discrepancy will mask itself in the process of reduction. Fortunately we have a simple way to test this, where no assumption is involved. Two spectra of the center of the Sun were taken on an exceptionally transparent day, one 4 mm and one 8 mm out of focus. Both were measured and reduced in the same way as the others. Their true intensities are denoted by dots (.) and crosses (x), respectively, in Figure 7h. Now Abbot[23] and his collaborators have obtained the solar energy measures in absolute units. They are represented by circles (o) in the same figure. These refer to the center of the Sun. The value for λ4753 was brought arbitrarily to agree with ours. Since no temperature or black body assumption is involved in Abbot's data, and since our results are in close agreement with his, from λ4000 to λ5000 at least, it seems reasonable to assume that our assumptions are substantially correct. The disagreement in the ultra-violet region is not perhaps a serious matter, for in a personal letter which Professor Abbot was kind enough to write in reply to our inquiry he has stated that too much weight should not be placed on the bolometric data in the ultra-violet, as they are not very reliable. Nevertheless it may be noted that in the region of disagreement our results are consistently higher than those of Abbot. [This disparity seems to be real. Recent observations by Pettit,[24] published since the writing of this paper, show that the ultra-violet radiation of the Sun fluctuates violently, the variation being as much as 80 percent. This range is of the order of the discordance between Abbot's curve and ours. The inconsistency of the solar radiation in the ultra-violet proves the unsuitability of the solar spectrum as a standard of reference in that region, and emphasizes the need of providing a suitable comparison spectrum for the study of energy distribution at short wave-lengths.]

Other features of the stellar energy curves will be discussed in the next section. For the moment mention will briefly be made of the way in which the atmospheric transmission was obtained.

The atmospheric transmission.—The atmospheric transmission coefficient a_λ was computed according to the method developed by Fowle.[25] For this purpose the barometer and dry and wet bulb thermometers were read during the periods of observations. An elevation of 1200 meters was assumed for the Crossley telescope. A typical reduction for ζ *Ophiuchi*, plate Y316H, is shown below.

TABLE III.—COMPUTATION OF THE ATMOSPHERIC TRANSMISSION COEFFICENT

For ζ *Ophiuchi*, Plate Y316H

λ	a_B	a_w	$a_o\lambda$	a_λ
3300	.563	.930	.515	.355
3400	.594	.940	.547	.388
3500	.619	.945	.573	.420
3600	.645	.950	.602	.450
3840	.700	.960	.662	.525
4130	.773	.965	.735	.622
4520	.832	.967	.796	.701
5030	.879	.977	.851	.778

DISCUSSION OF RESULTS

The chief purpose of this investigation, as stated in the first part of the paper, is to study the continuous absorption spectrum, to determine its limits and its law of distribution of intensity, with the view of establishing more definitely its origin, and to see how closely it satisfies the modern theory of spectroscopy. The final results obtained, as shown in Figures 7 and 9, permit such a study.

We have already noted that the stellar energy curves of Class A stars follow closely the law of black body radiation on the less refrangible side of λ3800, and it is highly probable that the energy curve in the ultra-violet would obey the same law, if the continuous absorption did not exist. Perhaps a more direct way of expressing it would be to say that the deviation from black body radiation in this region is due to the presence of the continuous absorption. Therefore the difference of ordinates at any wave-length between the black body curve and the stellar curve yields at once the amount of absorption.

The results disclose that the absorption begins in every case on the redward side of the highest observed series line. It increases rapidly but uniformly in intensity until the last line of the observed series is reached, then continues toward the ultra-violet in general with no sensible increase or decrease of intensity. In other words, the magnitude of absorption from the last observed series line toward the shorter wave-lengths is practically constant* within the region observed. In late Class A stars, there is a tendency for the absorption to increase slightly in intensity toward the ultra-violet. However, it seems probable that the numerous metallic absorption lines present in this region of the spectrum of late Class A stars[26] are largely responsible for the phenomenon, and while special care was taken to make measurements in places apparently free from lines, the latter are generally broad and are known to be numerous so that freedom from their influence cannot be assured. There is some indication (see Figure 9b) that in early Class B stars the absorption begins to decrease in intensity as the shorter wave region is reached. The end of the continuous absorption unfortunately appears to lie outside the spectral range of observation and therefore cannot be determined here.

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[23] *Smith. Misc. Coll.*, 74, No. 7, 1923.

[24] *Publ. Astron. Soc. Pac.*, **38**, 21, 1926.

[25] *Astrophys. Journ.*, **38**, 392, 1913; also *Smithsonian Physical Tables*, p. 419, 1921.

* Milne (See *Monthly Notices*, **85**, 770, 1925) has recently deduced from theoretical considerations a law of intensity distribution for this continuous hydrogen absorption. The law states that the intensity of absorption varies inversely as ν^3, but with this result our observations do not seem to agree.

It is interesting to note that the intrinsic intensities of the Balmer lines are about on the same level in all the stars examined, and there is, in many cases, a rise in intensity near the place where the absorption commences. This effect is very evident in the energy graphs. It is even more strikingly shown in the original apparent intensity curves, especially those reduced from plates taken with the slit spectrograph, a feature to which attention has already been called. The fact that the curve drawn through the bottom of the Balmer series lines joins, in all cases, the continuous spectrum at the same intensity and forms a smooth curve with it, is also remarkable. This was remarked by Wright in the spectrum of *α Cygni* and is found to be generally true in all the spectra here observed. It indicates clearly that the continuous absorption spectrum varies in intensity, from type to type, in fact, from star to star, according to the strength of absorption of the series lines. Referring again to Figures 7 and 9, we find that the continuous absorption is absent in early Class B stars, in which the Balmer lines of hydrogen are weak. The absorption spectrum begins to appear at about B3 and increases in intensity with advancing type until about A0 where the maximum absorption is attained, then decreases and finally fades out at about G0. The variation, it will be noticed, parallels exactly the variation of the intensity of the hydrogen series absorption lines in the spectral sequence. There appears, therefore, to be very little room for doubt that the continuous absorption spectrum is closely related to the Balmer series and like the latter is due to hydrogen, as it has generally been assumed to be.

The interpretation of the phenomenon of continuous absorption beyond the limit of a line series has been given by Bohr,[27] and elaborated, with special reference to emission and to its less refrangible limit by Wright.[28] Briefly stated it is as follows: According to the Bohr theory of atomic structure, a neutral hydrogen atom consists of a positively charged nucleus and an electron revolving around it in any one of an infinite number of orbits fixed by quantum relations. A line of the Balmer series is absorbed when a transfer of energy occurs during which the electron is lifted from the second orbit to some outer one. The frequency of the absorption is proportional to the amount of energy absorbed, defined by the following equation:[29]

$$h\nu = W_n - W_2 \tag{14}$$

where W_2 and W_n are the total energies of the electron in the 2nd and nth orbit respectively, and h is Planck's universal constant of action.

For a line absorbed at the limit of the Balmer series, we have this relation;

$$h\nu_L = W\infty - W_2 \tag{15}$$

where ν_L is the frequency at the limit. Since $W\infty = 0$, equation (15) becomes

$$h\nu_L = -W_2 \tag{16}$$

That is, the amount of energy W_2 is absorbed in order to bring to rest the electron from the 2nd orbit to the orbit at infinity. But nothing can stop the atom from absorbing energy greater than W_2 if the energy is available. In that case the extra energy E absorbed will be given over to the electron at infinity in the form of kinetic energy. Therefore

$$h\nu = E - W_2 \tag{17}$$

But from (16) $-W_2 = h\nu_L$, hence (17) becomes

$$\nu = \nu_L + \frac{E}{h} \tag{18}$$

It is obvious that E may have an infinity of positive values. Therefore $\nu \geqq \nu_L$ and is continuous. In other words, the absorption is continuous beyond the limit of the Balmer series, in perfect agreement with observation.

As the electron possesses kinetic energy at the infinity orbit it therefore has velocity and "escapes" or becomes "unbound" from the atom. In short, the atom is ionized through photo-electric action. E thus also represents the photo-electric energy.

The intensity of absorption at any frequency or wavelength depends upon the total number of electrons set free possessing a corresponding initial kinetic energy E, or velocity v. Now the uniformity of absorption beyond the series limit, as disclosed by our observations, would seem to indicate that, within the region investigated, the number of electrons set free in the various states of initial kinetic energy is practically a constant, or the probability of an atom absorbing any energy greater than the amount required to just ionize it is about even. Whether there are more or less electrons set free with higher velocities it is not possible to say, as our data do not extend beyond λ3300.

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[26] *Lick Obs. Bull.*, **10**, 100, 1921; **12**, 81, 1926.

[27] *Phil. Mag.*, **26**, 1, 1913; *et seq.*

[28] *Pro. Amer. Phil. Soc.*, **49**, 530, 1920; also *Nature*, **109**, 810, 1922.

[29] See also Sommerfeld, *Atomic Structure and Spectral Lines*, 3rd ed., p. 448.

From the above considerations, the continuous absorption should begin exactly at the theoretical limit of the Balmer series and not before or to the redward of it. However, observation shows conclusively in every case that it commences at a considerable distance before the limit is reached, and in fact before the Balmer series lines run out. In other words, the continuous absorption spectrum and the series line spectrum overlap. The same phenomenon is observed in the continuous emission of this spectrum from the solar chromosphere, and from the gaseous nebulae, to which reference has already been made. Its reality might well be questioned, for it is conceivable that the reduction of intensity on the red side of the theoretical limit of the Balmer series is due to the crowding together of high order series lines, assuming that they are present, on account of low dispersion and resolution. But this seems extremely improbable, for, as was noted in a preceding paragraph, the last line observed is sufficiently separated from its less refrangible neighbor for easy resolution, and the absorption occurs, in fact, between well separated lines near the head of the Balmer series. The phenomenon must, therefore, be assumed to be real. The overlapping observed here amounts to as much as 150A, a discrepancy which would appear to throw considerable doubt on the tenability of the view that the continuous absorption spectrum is a continuation of the line spectrum, predicted by the theory discussed above. However, a slight modification and restriction of the Bohr hypothesis, first proposed by Wright,[30] seems not only to remove the difficulty, but to turn the implications of the evidence in the direction of corroboration. Wright's argument is formally expressed only for the case of emission, and for the completeness of this paper, which deals with absorption, the theory will be sketched explicitly in relation to absorption.

If we consider an isolated hydrogen atom which can possess orbits extending to infinity the simple Bohr theory no doubt applies in its entirety to the atom. But in nature we deal with multitudes of atoms which are in effect not infinitely far apart, for all the minuteness of atomic dimensions, and which may be expected to exert disturbing effects on one another. It is conceivable that under such conditions quantized orbits larger than a certain size cannot exist. This would account for the absence of very high order lines in a series, a fact already pointed out by Bohr.[31]

Now since the number of orbits is restricted, an electron lifted just outside the outermost effective orbit will become free, as it no longer belongs to the parent atomic system. Therefore it can have a set of continuous values of kinetic energy and possess velocities varying from 0 upwards. Moreover it requires less energy to transport an electron from the 2nd orbit to just outside the last effective orbit with 0 velocity, than to lift it to the last effective orbit with its usual orbital velocity. Hence a line of less frequency will be absorbed and occur on the redward side of the last series line observed. Since this is the limiting case, it represents the redward limit of the absorption, and since the velocity of electrons necessarily varies from 0 upwards, the total effect will be a continuous band extending toward the ultra-violet. This explains effectively the overlapping of the series line spectrum and the continuous absorption spectrum as observed. Furthermore the extent of overlapping can be determined theoretically by the following considerations.

Let the nth orbit be the largest effective orbit of the atomic system; ν_s the highest line frequency absorbed. Then according to Bohr

$$\nu_s = N\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \tag{19}$$

where N is the series constant. The second term in the parenthesis represents the absorption of energy required in order to lift the electron from the nth orbit to infinity. It is also the kinetic energy of the electron due to motion in its orbit. Hence for an electron brought to rest at the nth orbit, the energy absorbed will be less by twice $\frac{1}{n^2}$. Therefore

$$\nu_b = N\left(\frac{1}{2^2} - \frac{2}{n^2}\right) \tag{20}$$

where ν_b is the frequency thus absorbed. It also marks the beginning of absorption. From (19) we have also

$$\nu_L = \frac{N}{2^2} \tag{21}$$

where ν_L is the Balmer limit.

Equations (19), (20), and (21) give at once the relation

$$\nu_s - \nu_b = \nu_L - \nu_s \tag{22}$$

This shows that the amount of overlapping, $(\nu_s - \nu_b)$, in frequencies and approximately in wavelengths also within the small range in question, is equal to the interval between the Balmer limit ν_L and the last observed series line ν_s. It is of extreme interest to see how far theory and observation agree in this matter. With the exception of a few instances where the beginning of the absorption is rather abrupt, it is difficult to locate with certainty the exact place where the absorption commences. The fall of intensity in this region is undoubtedly complicated by the influence of the absorption lines which are usually broad and hazy, characteristic of the Class A stars. As a result, the beginning of absorption seems to occur earlier than could be accounted for by theory. However, the agreement, as seen from Figures 6 and 8, is close considering the difficulties involved. As a matter of fact an exact agreement of this sort is not to be expected. For the observation does not represent the effect of a single layer of radiating gas. It is a sum total result produced by gases of widely different levels. From the higher level gases the prevalent lower density would favor the production of high order series lines, whereas in deep lying layers the high density condition and the atomic interference would shift the beginning of absorption further to the red and thus increase the overlap. Consequently the equality of interval as expressed by equation (22) would under such circumstances no longer hold with exactness.

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[30] *Loc. cit.*

[31] *Loc. cit.*

We have another and perhaps a better way of testing this relationship. When electrons are set free, they are sooner or later recaptured by the hydrogen nuclei. As a result energy is emitted. The intensity within the Balmer series lines is due in part at least to such subsequent re-emission, when electrons fall back from some outer orbit to the second one. We have already noted that the intrinsic intensity of the Balmer series is about the same. Now at the place where we expect the absorption to begin, the re-emission due to the capturing of free electrons should also occur, according to the principle discussed above. In other words, due to the re-enforcement of intensity, the intensity within the series lines beyond this point should be strengthened, or there should be a rise of the line drawn through the bottom of the lines at this juncture. This is suggested in explanation of what is actually observed. A glance at many of the original graphs (Figures 6 and 8) will verify these statements. Of course it must be remembered that this re-emission strengthens the background as well as the lines in this region, but this only contributes to the general form of the absorption curve.

MINOR RESULTS

As a sort of "by-product," there is some interesting and useful information to be derived from the present investigation. We may consider three items: (1) the effective temperatures of stars, (2) the reflectivity of the silver mirror, and (3) the sensitivity of the Seed 23 photographic plate. It should be emphasized at the outset that, due to the somewhat intricate nature of the problem, no great accuracy is claimed for the results to be given and too great reliance should not be placed upon them.

Effective temperatures of 69 stars.—Temperatures of stars, whose energy curves are given in this paper, are obtained directly from the fitted Planckian curves. A similar determination was carried out for 20 stars whose energy curves do not appear herewith. To reduce the remaining stars of the programme in like manner simply for the temperature determination would involve excessive and perhaps unnecessary labor. Therefore it was thought sufficient to adopt a less rigorous system for handling them. In the method adopted only two points were taken in each stellar curve, one at λ4000 and the other at λ4800. These points were then reduced to their true intensities, and their ratio compared with the computed ratio supplied by Planck's law of radiation. It is reasonable to assume that if the Planckian curve fits these two points it should not be far off at the intermediate section, a condition which has been generally verified by curves rigorously reduced. The temperatures are listed in Table IV. Where more than one observation was made on a star, the mean of results is given. In general, the temperature of a star obtained from the slit plates agrees closely with that secured from the slitless. Temperatures of such of these stars as have been determined by other investigators are entered in the table for comparison. It will be noticed that the agreement is as close as could be expected.

TABLE IV.—EFFECTIVE TEMPERATURE OF 69 STARS
(Based on an assumed temperature of 22,000°K for ζ *Ophiuchi*)

Star	Class	Yü	Sampson	Brill	Rosenberg	Wilsing
λ *Orionis*..............	Oe5	20,000°K				
ζ *Ophiuchi*..............	B0	(22,000)				
ε *Orionis*..............	B0	22,000	25,000°	20,800°	46,000°	
β *C. Majoris*..............	B1	19,000				
α *Virginis*..............	B2	20,000			23,000	
η *Tauri*..............	B5p	13,000	11,300	13,500		
ζ *Draconis*..............	B5	17,500		16,100		
β *Tauri*..............	B8	17,000		15,300	25,000	

TABLE IV.—EFFECTIVE TEMPERATURE OF 69 STARS—(Continued)
(Based on an assumed temperature of 22,000°*K* for ζ *Ophiuchi*)

Star	Class	Yü	Sampson	Brill	Rosenberg	Wilsing
β *C. Minoris*	B8	16,000°K		15,800°		
α *Leonis*	B8	15,000		13,400	20,000°	9,400°
γ *Corvi*	B8	17,000				
β *Librae*	B8	15,900				
α *Delphini*	B8	14,800				
λ *Aquilae*	B9	16,550				
γ *Geminorum*	A0	13,000		10,700	16,000	10,300
α *C. Majoris*	A0	13,200	12,800°		27,500	
κ *U. Majoris*	A0	15,000				
β *U. Majoris*	A0	16,800	20,000	11,900	17,500	
δ *Leonis*	A0	10,400		9,570		
γ *U. Majoris*	A0	13,000	16,200	12,300	27,500	
δ *Corvi*	A0	13,000				
107 *Virginis*	A0	12,600				
α *Cor. Bor.*	A0	17,000	16,900	12,500		9,600
μ *Serpentis*	A0	12,400				
λ *Ophiuchi*	A0	12,000				
ε *Herculis*	A0	15,000				
γ *Ophiuchi*	A0	13,700				
ο *Herculis*	A0	13,550				
α *Lyrae*	A0	16,000	11,600	11,900	22,000	
ζ *Aquilae*	A0	13,700	20,000	11,000		
δ *Cygni*	A0	13,000	10,900	11,900	23,000	
θ *Aquilae*	A0	12,600				
ε *Aquarii*	A0	13,500				
7 *Lacertae*	A0	13,750				
η *Leonis*	A0p	12,000				
ε *U. Majoris*	A0p	17,400	16,400	10,200	19,500	
α *Can. Venat.*	A0p	15,800	16,800	11,800	19,000	7,800
γ *Lyrae*	A0p	10,600		12,500		
θ *Geminorum*	A2	10,000				
λ *Geminorum*	A2	11,450				
λ *U. Majoris*	A2	10,200				
β *Leonis*	A2	12,500		10,200	14,500	8,100
δ *U. Majoris*	A2	11,400		10,900		
ζ *Virginis*	A2	12,200		9,700		
γ *U. Minoris*	A2	12,500		10,800		
β *Serpentis*	A2	9,450				
ε *Serpentis*	A2	13,600				
η *Ophiuchi*	A2	11,200				
δ *Herculis*	A2	10,000		10,000		
ι *Cygni*	A2	9,600				
α *Cygni*	A2p	14,000	10,900	11,000	20,500	9,400
δ *Leonis*	A3	9,600		9,570	10,500	6,900
α^2 *Librae*	A3	13,000				
72 *Ophiuchi*	A3	11,600				
β *Trianguli*	A5	9,800	10,700	9,630		
ι *U. Majoris*	A5	10,000	10,200	8,590		
α *Ophiuchi*	A5	12,000	10,900	9,320	14,000	7,100
ξ *Serpentis*	A5	9,450				
α *Aquilae*	A5	10,100		8,640	10,500	7,100
α *Cephei*	A5	9,700	8,900	9,380	10,500	
γ *Boötis*	F0	9,000		8,800		
δ *Geminorum*	F0	12,000		7,760		
α *C. Minoris*	F5	8,000	8,300	7,550	7,000	
μ *Virginis*	F5	8,000				
β *Virginis*	F8	7,000		6,640		
η *Boötis*	G0	6,500	6,000	6,380	5,500	5,000
η *Draconis*	G5	5,000	5,100	5,000	3,750	
α *Boötis*	K0	4,200	4,200	4,340	3,100	3,500
γ *Draconis*	K5	4,000	3,400	3,600		

The reflectivity of the silver mirror.—The reflectivity (r_λ) was very simply obtained by comparing the density curve of *a Cygni* taken with the mirror with one taken without the mirror. The latter was secured by Mr. Wright some years ago with the slitless spectrograph pointed directly to the star. The coefficients derived from this comparison are tabulated in Table V, third column, and plotted in Figure 10. It will be seen that the reflectivity of the silver mirror is fairly uniform from λ5000 to about λ4400 where it begins to fall off, the decrease in reflecting power being increasingly rapid beyond λ3600.

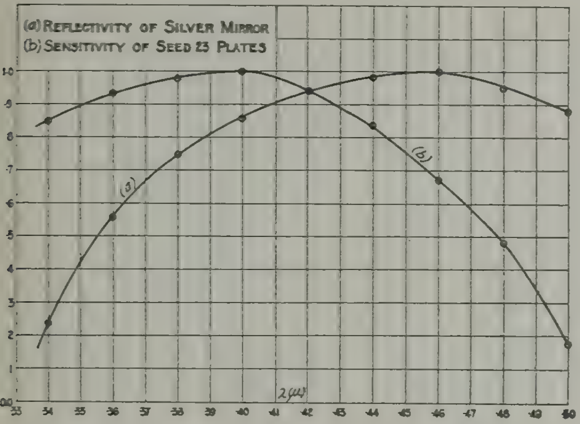

Fig. 10. Reflectivity and Sensitivity Curves.

The sensitivity of the Seed 23 plate.—The correcting factor curve, $(rbds)_\lambda$ (see Figure 5), contains four parts, the reflectivity coefficient r_λ, the instrument transmission coefficient b_λ, the coefficient of dispersion d_λ, and the sensitivity coefficient s_λ. In order to solve for s_λ, the first three parts must be known. The first was obtainable by the method described in the preceding paragraph. The second can be assumed, without appreciable error, to be constant. The third can be computed. Hence the solution is complete. The values of these various factors are shown in Table V below. The last column, which was obtained from the preceding one by making the maximum figure unity, gives the relative sensitivity of the Seed 23 plate for the different wave-lengths. This is also plotted in Figure 10. The maximum sensitiveness of the plate was found to be at about the region λ4000. It falls off gradually in both directions but much more rapidly toward the red. The sensitivity curve is derived for the purpose of a general check on the work and is not offered as a contribution to sensitometry.

TABLE V.—COMPUTATION OF THE SENSITIVITY COEFFICIENT

λ(μ)	$(rbds)_\lambda$	r_λ	d_λ	s_λ	
.34	.100	.234	.275	1.542	.850
.36	.325	.565	.341	1.690	.935
.38	.575	.745	.413	1.780	.980
.40	.770	.860	.494	1.815	1.000
.42	.905	.940	.580	1.660	.915
.44	1.000	.985	.676	1.501	.827
.46	.965	1.000	.789	1.222	.674
.48	.740	.945	.885	.885	.488
.50	.285	.880	1.000	.324	.178

SUMMARY

The continuous spectrum, in both absorption and emission, which occurs near the limit of the Balmer series of hydrogen lines, has been observed in celestial spectra by a number of investigators, notably Huggins and Wright. Because of the peculiar relationship which it bears to the Balmer series, it has generally been assumed to be due to hydrogen. This spectrum is of special interest in the light of the new Bohr and Quantum Theory. However, the observations so far obtained were limited in number, and determination of a quantitative nature was completely lacking. It is thus obvious that more accurate and comprehensive results are extremely desirable. Accordingly the writer undertook to investigate more thoroughly the character of this continuous absorption in spectra of all Class A stars brighter than the 4th magnitude, the main object being to determine the limits of absorption and the law of intensity distribution along the spectrum, and to see how far observation and theory agree.

One hundred and thirty-one spectrograms of 91 stars, including, for comparative study a few of types other than Class A, were taken with the two-prism quartz spectrograph attached to the Crossley reflector. Seed 23 plates were used. In order to calibrate the intensity distribution of the photographic spectrum, a set of six standard exposures of known intensities, increasing with the ratio 1 : 2, was impressed on each plate by means of a standardizing photometer, which is described.

Since the photographic blackening or density at different wave-lengths does not represent the intensity distribution of the stellar radiations, it is necessary, in order to obtain the latter, to adopt a standard source, the radiation of which is known and with which the unknown sources under study can be compared. After considering a few, including the Sun and several terrestial radiation standards, a star with assumed radiations was finally selected. For the purpose of a check, however, several spectrograms of the Sun were also secured.

The plates were measured on the Hartmann wedge microphotometer. These measurements yield the apparent intensity graphs, a set of which for typical stars is presented with the paper. From these graphs and with the aid of the comparison spectrum, the stellar energy curves were obtained, which reveal the following results:

(1) The continuous absorption begins, in every instance, on the redward side of the limit of the Balmer series.

(2) The continuous spectrum and the line spectrum overlap.

(3) The absorption increases rapidly in intensity until the last line of the observed series is reached.

(4) Beyond the last line the absorption is practically constant.

(5) The intensities at the centers of the Balmer series lines are about the same.

(6) The amount of the continuous absorption varies from star to star with the strength of the line absorption.

The above facts are satisfactorily explained on the basis of the Bohr Theory of the hydrogen atom, with the restriction that quantized orbits larger than a certain size do not exist. On this consideration the amount of overlapping of the continuous and the line spectrum was found to agree closely with observation. All evidence, both experimental and theoretical, seems to point unmistakably to the conclusion that the continuous absorption in question is due to hydrogen.

The fact that, away from the region of absorption, the stellar energy curves obtained in this investigation follow closely the law of black body radiation makes it highly probable that stars radiate as black bodies, a result in harmony with those of other investigators.

In addition to the principal results of the investigation, three minor ones are also presented: (1) the effective temperatures of 69 stars, (2) the reflectivity of the silver mirror, and (3) the sensitivity of the Seed 23 photographic plate. These estimates are, however, subject to some uncertainty and great accuracy is not claimed for them.

ACKNOWLEDGMENTS

It is a very pleasant duty to express my indebtedness to Director W. W. Campbell and Associate Director R. G. Aitken, who placed at my disposal the observatory equipment and supplies which made the undertaking of this investigation possible. My thanks are also due to Dr. H. H. Plaskett and Dr. C. G. Abbot, who have made helpful and valuable suggestions through our correspondence. I am indebted also to Dr. H. N. Russell for a helpful discussion during a brief visit by him to the observatory. But above all I wish to express my deep sense of obligation to Mr. W. H. Wright, who, besides suggesting and outlining this investigation, showed the keenest interest in my work from start to finish, and was always ready to help and offer suggestions at a considerable sacrifice of time, when difficulties, of which there were many, arose. In concluding, I must not forget to mention that this investigation was made possible by a grant of the James M. Goewey Fellowship in the University of California.

Lick Observatory,
Mount Hamilton, Calif.,
December, 1925.
Issued March 23, 1926.

UNIVERSITY OF CALIFORNIA PUBLICATIONS
ASTRONOMY

LICK OBSERVATORY BULLETIN
NUMBER 376

THIRD ELEMENTS AND EPHEMERIS OF COMET *j* 1925 (VAN BIESBROECK)

BY

A. D. MAXWELL and L. C. DAMSGARD

Peliminary and second elements of Comet *j* 1925 are given in *Lick Observatory Bulletins* **12,** 370, 371. The following observations were made the basis of a third orbit:

U. T.	α(1926.0)	δ(1926.0)	Observer
1925, Nov. 18.5163	$11^{h}57^{m}28^{s}.67$	+34°33′19″.2	Jeffers, Lick
1926, Jan. 1.4612	11 59 09.42	+21 52 40.0	Jeffers
1926, Mar. 11.1964	10 00 46.67	+ 6 28 16.9	Jeffers

The present orbit was derived by differential correction of the second orbit, concentrating the residuals into the first declination. The removal of all residuals would give an hyperbolic orbit.

ELEMENTS

T	1925, Oct. 2.93610 U. T.	
ω	106° 25′ 32″.9	1926.0
Ω	334 34 39.3	
i	49 20 42.1	
$\log q$	0.195092	

CONSTANTS FOR THE EQUATOR 1926.0

$x = r[9.975653] \sin (179° 13' 11''.0 + v)$
$y = r[9.662879] \sin (47 \; 33 \; 48.9 + v)$
$z = r[9.975753] \sin (96 \; 01 \; 08.4 + v)$

The orbit represents various observations, as follows:

O—C	cos δΔα	Δδ	Observer
Nov. 17	+2″.2	+27″.9	Van Biesbroeck, Yerkes
Nov. 18*	+0.4	+27.0	Jeffers, Lick
Nov. 25	0.0	+20.1	Jeffers
Dec. 24	+0.7	+ 3.9	Jeffers
Jan. 1*	0.0	0.0	Jeffers
Jan. 22	−1.6	− 7.5	Jeffers
Jan. 26	−8.7	− 6.7	Jeffers
Feb. 24	−4.0	− 4.1	Van Biesbroeck
Mar. 10	+3.8	− 2.6	Van Biesbroeck
Mar. 11*	+0.8	− 1.6	Jeffers
Mar. 13	−1.0	− 3.4	Jeffers

*Observations on which the orbit was based.

The large declination residuals in November are due to the arrangement of the solution mentioned above.

EPHEMERIS FOR GREENWICH MIDNIGHT

1926, U. T.	True α	True δ	log Δ	Br.
Apr. 1.0	$9^{h} 40^{m} 09^{s}$	+3° 20′.1		
2.0	39 34	12.3	0.305	0.25
3.0	39 01	+3 04.6		
4.0	38 30	+2 57.0		
5.0	38 01	49.5		
6.0	37 34	42.1		
7.0	37 09	34.8		
8.0	36 46	27.5		
9.0	36 24	20.3		
10.0	36 05	13.2	0.339	0.21
11.0	35 47	+2 06.2		
12.0	35 31	+1 59.2		
13.0	35 17	52.3		
14.0	35 04	45.5		
15.0	34 53	38.7		
16.0	34 44	31.9		
17.0	34 36	25.2		
18.0	34 30	18.6	0.372	0.17
19.0	34 25	12.0		
20.0	34 22	+1 05.4		
21.0	34 21	+0 58.9		
22.0	34 20	52.4		
23.0	34 22	46.0		
24.0	34 24	39.6		
25.0	34 28	33.2		
26.0	34 33	26.9	0.405	0.14
27.0	34 40	20.5		
28.0	34 48	14.2		
29.0	34 57	08.0		
Apr. 30.0	35 07	+0 01.7		
May 1.0	35 18	−0 04.5		
2.0	35 30	10.8		
3.0	35 44	17.0		
4.0	35 59	23.2	0.435	0.11
May 5.0	9 36 14	−0 29.3		

Ephemeris for Greenwich Midnight—(Cont.)

1926, U. T.	True α	True δ	log Δ	Br.
May 6.0	$9^{h}\ 36^{m}\ 31^{s}$	$-0^{\circ}\ 35'.5$		
7.0	36 49	41.7		
8.0	37 07	47.8		
9.0	37 27	−0 53.9		
10.0	37 48	−1 00.1		
11.0	38 09	06.2		
12.0	38 32	12.4	0.465	0.09
13.0	38 55	18.5		
14.0	39 20	24.6		
15.0	39 45	30.8		
16.0	40 11	36.9		
17.0	40 37	43.1		
18.0	41 05	49.3		
19.0	41 33	−1 55.4		
20.0	42 02	−2 01.6	0.492	0.08
21.0	42 32	07.8		
22.0	43 02	14.0		
23.0	43 33	20.2		
24.0	44 05	26.4		
25.0	44 38	32.7		
26.0	45 11	38.9		
27.0	45 44	45.2		
28.0	46 18	51.5	0.518	0.07
29.0	46 53	−2 57.8		
30.0	47 29	−3 04.1		
May 31.0	48 05	10.4		
June 1.0	48 41	16.7		
2.0	49 18	23.1		
3.0	49 56	29.5		
4.0	50 34	35.9		
5.0	51 12	42.3	0.542	0.06
6.0	51 51	48.7		
7.0	52 31	−3 55.2		
8.0	53 11	−4 01.7		
June 9.0	9 53 51	−4 08.2		
June 10.0	$9^{h}\ 54^{m}\ 32^{s}$	$-4^{\circ}\ 14'.7$		
11.0	55 13	21.3		
12.0	55 55	27.8		
13.0	56 37	34.4	0.565	0.05
14.0	57 20	41.1		
15.0	58 02	47.7		
16.0	58 45	−4 54.4		
17.0	9 59 29	−5 01.1		
18.0	10 00 13	07.8		
19.0	00 57	14.6		
20.0	01 42	21.3		
21.0	02 26	28.1	0.586	0.04
22.0	03 11	35.0		
23.0	03 57	41.8		
24.0	04 43	48.7		
25.0	05 29	−5 55.6		
26.0	06 15	−6 02.5		
27.0	07 01	09.5		
28.0	07 48	16.5		
29.0	08 35	23.5	0.606	0.04
June 30.0	09 22	30.5		
July 1.0	10 10	37.6		
2.0	10 57	44.7		
3.0	11 45	51.8		
4.0	12 33	−6 58.9		
5.0	13 21	−7 06.1		
6.0	14 10	13.3		
July 7.0	10 14 58	−7 20.5	0.624	0.03

The unit of brightness is that of January 1.

Berkeley Astronomical Department,
March 22, 1926.
Issued March 23, 1926.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 377

MEASURES OF COMETS

By

H. M. Jeffers

The positions contained in this paper are mean places for the beginning of the year, obtained, in the case of visual observations, by applying the observed $\Delta\alpha$ and $\Delta\delta$ to the mean place of the comparison star.

For most of the visual measures, the differences comet-star were measured directly. When this was done, the usual procedure was to measure the differences in the order $\Delta\delta$, $\Delta\alpha$, $\Delta\delta$, $\Delta\alpha$, $\Delta\delta$, turning the micrometer 90° for each set. This method affords a useful check against errors in reading the micrometer head. When the $\Delta\alpha$ was measured by transits, the observations thus measured being distinguished by a "t" following the number of comparisons in α, the $\Delta\delta$ was measured in two positions of the micrometer only. For a measure by transits, a chronograph was generally used, occasionally a stop watch. Corrections for refraction were made to about one-third of the measures, whenever an inspection indicated that this might amount to $0''.2$. Except for the parallax factors and small refraction corrections, the numerical work has been checked by duplicate calculation.

The photographic measures are from plates taken with the Crossley reflector. These are distinguishable from the visual measures in the same list by the fact that for them no "No. of Comp.", and "Comet-Star" are given. The times given are those of mid-exposure. The calculated motion of the comet was used in guiding; the stars therefore trail. It was frequently possible to find in the photographic catalogues three or four stars very close to the comet, and, by orienting the plate in the measuring machine by means of another star some distance away, a very simple method of reduction sufficed. In the cases when no close stars were to be had, complete least square solutions were made.

For the visual measures, nearly all the recording was done by Mrs. Jeffers; this was a very great help.

Previous measures of comets *a*, *b*, *c*, and *d*, 1925, have already been published in *Lick Observatory Bulletin* 367.

Since a number of the comets for which observations are given below are still observable, the publication of the present list does not indicate that the series are ended.

COMET *a* 1925 (SHAJN-COMAS SOLA)

			No. of	Comet—Star				Log pΔ	
1925	U. T.	Star	Comp.	$\Delta\alpha$	$\Delta\delta$	α 1925.0	δ 1925.0	α	δ
Dec. 13	$11^{\mathrm{h}}\ 33^{\mathrm{m}}\ 32^{\mathrm{s}}$	2	10, 10	$-\ 2^{\mathrm{s}}.69$	$+2'\ 42''.3$	$9^{\mathrm{h}}\ 52^{\mathrm{m}}\ 33^{\mathrm{s}}.48$	$-12°\ 03'\ 07''.1$	9.085n	0.820
1926	U. T.					α 1926.0	δ 1926.0		
Jan. 10	12 02 17	3	10, 9	−19.23	+1 37.3	9 11 35.47	−13 46 52.5	9.385	0.819

MEAN PLACES OF COMPARISON STARS FOR 1925.0

	α	δ	
1.	$9^{\mathrm{h}}\ 51^{\mathrm{m}}\ 23^{\mathrm{s}}.01$	$-12°\ 06'\ 48''.6$	A. G. Cambridge U. S. 3796.
2.	9 52 36.17	−12 05 49.4	Referred to (1); 10t, 7.

MEAN PLACE OF COMPARISON STAR FOR 1926.0

3.	9 11 54.70	−13 48 29.8	A. G. Cambridge U. S. 3551.

The above measures were made with the 36-inch telescope. Comet about 12^{m}, with condensation at the center.

COMET *b* 1925 (REID)

1925	U. T.	Star	No. of Comp.	Comet—Star Δα	Comet—Star Δδ	α 1925.0	δ 1925.0	Log pΔ α	Log pΔ δ
Dec. 4	2^{h} 15^{m} 44^{s}	1	12, 9	+18^{s}84	−2′ 23″1	21^{h} 13^{m} 55^{s}51	−29° 51′ 50″5	9.376	0.888
Dec. 5	2 43 07	3	10, 9	−20.18	−2 36.2	21 16 1.31	−29 33 11.8	9.472	0.875

MEAN PLACES OF COMPARISON STARS FOR 1925.0

	α	δ	
1.	21^{h} 13^{m} 36^{s}67	−29° 49′ 27″4	Cordoba B 13996.
2.	21 17 22.18	−29 29 5.5	Cordoba B 14032.
3.	21 16 21.49	−29 30 35.6	Referred to (2); 12t, 7.

The measure of December 4 was made with the 36-inch refractor, and that of December 5 with the 12-inch. Comet about 20″ in diameter, with possibly a 14th magnitude nucleus. Total magnitude 11.5.

COMET *c* 1925 (ORKISZ)

1925	U. T.	Star	No. of Comp.	Comet—Star Δα	Comet—Star Δδ	α 1925.0	δ 1925.0	Log pΔ α	Log pΔ δ
July 14	5^{h} 48^{m} 36^{s}	1	10t, 10	+144^{s}76	−3′ 03″4	10^{h} 24^{m} 29^{s}60	+56° 50′ 18″8	9.924	0.605
July 15	5 32 43	2	15, 14	+ 9.68	+1 06.3	10 26 0.00	+56 23 03.0	9.922	0.570
Aug. 17	4 41 42	3	8, 8	+ 12.08	+0 33.9	11 04 41.89	+44 54 50.7	9.798	0.721
1926	U. T.					α 1926.0	δ 1926.0		
Jan. 12	12 29 37	4				11 25 52.65	+40 40 27.8	8.774	9.714n
12	13 19 38	4				11 25 49.71	+40 40 42.9	9.281	9.498n
Apr. 15	4 50 02	5				9 17 51.28	+38 53 25.7	9.162	9.124n
15	6 05 42	5				9 17 49.86	+38 53 01.4	9.514	9.646

MEAN PLACES OF COMPARISON STARS FOR 1925.0

	α	δ	
1.	10^{h} 22^{m} 04^{s}84	+56° 53′ 22″2	A. G. Hels-Gotha 6349.
2.	10 25 50.32	+56 21 56.7	Am. Eph., 1925, +cor.
3.	11 04 29.81	+44 54 16.8	Hels. ph. 11^{h} 00^{m}, +44°, 75; 11^{h} 00^{m}, +46°, 64; 11^{h} 05^{m}, +45°, 25.
4.	Five stars from Helsingfors photographic catalogue.		
5.	Four stars from Helsingfors photographic and A. G. Lund catalogues.		

The observation of July 14 was made with the 36-inch, the other two with the 12-inch. Measures on August 17 were made with difficulty, due to the faintness of the comet.

The first plate of January 12 was exposed 60^{m}, the second 30^{m}. The shorter exposure was sufficient to show a strong black image, of magnitude 14. The longer exposure shows nebulosity about 15″ in diameter.

The two plates of April 15 were exposed 62^{m} and 72^{m} respectively. The comet was then considerably fainter, of magnitude estimated at 16.5. The image does not suggest appreciable condensation, and its diameter is of the order of 10″.

COMET *d* 1925 (TEMPEL II)

1925	U. T.	Star	No. of Comp.	Comet—Star Δα	Comet—Star Δδ	α 1925.0	δ 1925.0	Log pΔ α	Log pΔ δ
July 14	7^{h} 25^{m} 34^{s}	1	16t, 10	+52^{s}19	+1′ 20″2	18^{h} 39^{m} 14^{s}56	−11° 57′ 08″1	8.120	0.822
15	6 39 17	2	10t, 10	−65.61	+5 36.5	18 40 1.37	−12 34 12.0	8.885n	0.825
Aug. 8	8 04 45	3	12, 13	−20.94	+5 45.6	19 14 22.54	−29 03 46.5	9.389	0.884
9	7 33 23	4	10t, 10	−50.96	+0 9.1	19 16 27.84	−29 39 45.8	9.262	0.896
9	8 14 59	4	10t, 10	−47.16	−0 59.5	19 16 31.64	−29 40 54.4	9.433	0.881
15	6 40 35	5	16,12	+19.97	−2 9.0	19 30 20.98	−32 59 28.9	8.943	0.915
15	7 08 32	6	16, 12	−30.76	−0 47.9	19 30 23.39	−33 00 05.8	9.188	0.910
Nov. 5	4 30 29	8	8, 9	+17.02	+1 00.3	23 01 24.23	−28 07 34.2	8.620	0.902
18	4 03 26	10	14, 12	+ 9.37	−1 09.7	23 25 50.80	−24 01 03.9	8.597	0.886
Dec. 4	3 39 04	12	8, 10	− 6.63	+0 43.5	23 54 14.16	−18 59 42.7	8.780	0.862
9	3 04 10	13				0 02 47.33	−17 28 19.6	7.946	0.855
9	3 47 10	13				0 02 50.42	−17 27 47.1	9.000	0.853

MEAN PLACES OF COMPARISON STARS FOR 1925.0

	α	δ	
1.	$18^h\ 38^m\ 22^s.37$	$-11°\ 58'\ 28''.3$	A. G. Cam. U. S. 6411; referred to 6394, $+2^m\ 54^s.00$, $-1'\ 39''.7$.
2.	18 41 6.98	−12 39 48.5	A. G. Cam. U. S. 6426.
3.	19 14 43.48	−29 09 32.1	Cordoba B 12632.
4.	19 17 18.80	−29 39 54.9	Cordoba B 12662.
5.	19 30 01.01	−32 57 19.9	Cordoba Gen. 26796; Perth ph −33°, $19^h\ 25^m$, 259.
6.	19 30 54.15	−32 59 17.9	Cordoba Gen. 26816; Perth ph −33°, $19^h\ 25^m$, 261.
7.	23 02 42.29	−28 07 51.3	Cordoba B 14792.
8.	23 01 7.21	−28 08 34.5	Referred to (7); 10t, 8.
9.	23 23 16.14	−24 00 12.8	Cordoba A 15702.
10.	23 25 41.43	−23 59 54.2	Referred to (9); 14t, 7.
11.	23 55 59.26	−18 55 29.4	Cincinnati (1890), 1992.
12.	23 54 20.79	−19 00 26.2	Referred to (11); 12t, 5.
13.	Four stars from Hyderabad ph.		

The second observation of August 9, at $8^h\ 14^m\ 59^s$, was made by Mr. C. S. Mundt. It is included here, through the courtesy of the observer, in order that the observations may all be given in one list.

Measures with the 36-inch were on July 14, November 5, November 18, and December 4. The other visual measures were with the 12-inch. During July and August the comet had a nearly stellar nucleus, with a short fan-shaped tail extending in direction North preceding. On November 5, an unusually clear night, the comet was distinctly visible in the 4-inch finder. On November 18, it was more diffuse, about 40″ in diameter, and of about magnitude 12. On December 4 it was very diffuse, about 20″ in diameter, and of magnitude about 14. The two plates of December 9 (exposures 60^m, 20^m, respectively) show a rather sharp condensation with a faint fan-shaped tail, about 1′ long, extending northeast. Magnitude estimated about 15. Two plates taken 1926 January 9, 30^m exposure, showed no certain trace of the comet.

As there was no good ephemeris for following this comet throughout this apparition, I derived new elements, using observations on 1925 June 15, July 14, and August 15. This orbit is given in *H. C. O. Bulletin* 826, in *Pop. Astron.*, Jan., 1926, and in the *Pub. A. S. P.*, Dec., 1925. As the comet appears to have been somewhat neglected in recent years, it is planned, as opportunity offers, to derive elements which best satisfy the observations made at this apparition.

COMET 1925 *e* (WOLF)

1925	U. T.	Star	No. of Comp.	Comet—Star Δα	Comet—Star Δδ	α 1925.0	δ 1925.0	Log pΔ α	Log pΔ δ
July 25	$8^h\ 33^m\ 16^s$	1				$23^h\ 01^m\ 02^s.96$	$+25°\ 06'\ 22''.3$	9.482n	0.381
Aug. 17	9 06 05	2				22 58 43.01	+24 55 14.3	8.613n	0.273
17	10 09 05	2				22 58 42.12	+24 55 02.5	9.001	0.284
Sept. 24	7 40 51	5	12, 12	$-1^s.49$	$-1'\ 43''.5$	22 42 55.13	+17 41 32.7	9.116	0.478
Oct. 9	3 41 19	7	8, 8	−3.69	+1 02.2	22 40 21.89	+13 19 45.0	9.369n	0.575
Dec. 10	2 28 39	8				23 22 51.08	+ 1 16 55.3	8.402	0.712
10	3 11 39	8				23 22 53.38	+ 1 16 47.8	9.048	0.712

MEAN PLACES OF COMPARISON STARS FOR 1925.0

	α	δ	
1.	Three stars in Oxford and Paris ph.		
2.	Three stars in Oxford and Paris ph.		
3.	$22^h\ 41^m\ 14^s.10$	$+17°\ 38'\ 08''.6$	A. G. Berlin A 9303 (with p.m.)
4.	22 44 52.31	+17 42 35.2	A. G. Berlin A 9328.
5.	22 42 56.62	+17 43 16.2	Referred to (3): $+1^m\ 42^s.50$, $+5'\ 07''.5$; to (4): $-1^m\ 55^s.66$, $+41''.2$.
6.	22 40 44.87	+13 17 28.5	A. G. Leipzig I 9076.
7.	22 40 25.58	+13 18 42.8	Referred to (6).
8.	Four stars in Algiers ph.		

Both visual measures were made with the 36-inch telescope. Several other plates were secured, but only the best ones were measured. The plates of July and August show a faint tail nearly 1′ long extending southwest, magnitude about 15. On December 9 the brightness is about the same, but only the faintest trace of a tail is to be seen. The plates were exposed 1^h, except the first of December 10, for which the exposure was 20^m.

COMET 1925 *f* (BORRELLY)

1925	U. T.	Star	No. of Comp.	Comet—Star Δα	Comet—Star Δδ	α 1925.0	δ 1925.0	Log pΔ α	Log pΔ δ
Sept. 27	12^h 39^m 23^s	1	12, 12	+ $1^s.85$	+0′ 24″.7	7^h 15^m $11^s.79$	+13° 41′ 40″.4	9.438n	0.579
Dec. 24	9 34 13	2	12, 12	+ 3.04	−1 14.9	11 28 46.94	+44 41 14.2	9.745n	0.064
1926	U. T.					α 1926.0	δ 1926.0		
Mar. 13	8 10 47	3	10, 10	+22.81	+0 17.9	10 57 54.94	+59 56 00.6	9.041	0.526n
15	8 07 18	4	10, 10	−11.93	+1 16.6	10 55 29.03	+59 37 39.5	9.134	0.518n

MEAN PLACES OF COMPARISON STARS FOR 1925.0

	α	δ	
1.	7^h 15^m $09^s.94$	+13° 41′ 15″.7	A. G. Leipzig I 2837.
2.	11 28 43.90	+44 42 29.1	Hels. ph. 11^h 25^m, +45°, 75; 11^h 30^m, +44°, 21.

MEAN PLACES OF COMPARISON STARS FOR 1926.0

3.	10 57 32.13	+59 55 42.7	A. G. Hels.-Gotha, 6590.
4.	10 55 40.96	+59 36 22.9	Rome ph. +59°.37878, +60°.21937.

The above measures were all made with the 36-inch telescope. On December 24 the magnitude was about 12.5, with a sharp, though not quite stellar, nucleus. It was elongated in position angle 120°. In March it was fainter, 14^m, and the elongation was no longer apparent. Measures were also obtained on September 26, 1925, and January 13, 1926, for which good star positions are not available.

COMET 1925 *g* (BROOKS)

1925	U. T.	Star	No. of Comp.	Comet—Star Δα	Comet—Star Δδ	α 1925.0	δ 1925.0	Log pΔ α	Log pΔ δ
Oct. 10	6^h 56^m 26^s	2	12, 12	$-15^s.46$	−2′ 49″.6	23^h 15^m $43^s.04$	−5° 49′ 51″.5	8.986	0.777
15	7 48 36	4	12, 12	+ 0.49	+2 58.7	23 16 11.26	−6 14 16.6	9.368	0.774
Nov. 5	3 01 53	5	12, 12	+ 4.06	−2 53.1	23 26 10.16	−6 43 29.6	9.280n	0.783
5	3 36 48	5	6, 6	+ 5.13	−2 50.5	23 26 11.23	−6 43 27.0	9.089n	0.783
18	7 05 48	7	8, 8	+ 6.98	+2 26.2	23 38 46.76	−6 03 34.4	9.535	0.762
Dec. 10	4 17 40	8				0 08 03.99	−3 37 03.9	9.186	0.757
10	4 59 10	8				0 08 06.53	−3 36 47.7	9.363	0.754

MEAN PLACES OF COMPARISON STARS FOR 1925.0

	α	δ	
1.	23^h 14^m $30^s.23$	−5° 48′ 28″.4	A. G. Wien-Ott. 8279.
2.	23 15 58.50	−5 47 01.9	Referred to (1); 13t, 9.
3.	23 16 48.87	−6 19 04.4	Zod. Stars (Hedrick), 1560.
4.	23 16 10.77	−6 17 15.3	Referred to (3); 12t, 11.
5.	23 26 06.10	−6 40 36.5	A. G. Wien-Ott. 8332.
6.	23 35 40.01	−6 01 28.0	A. G. Wien-Ott. 8366.
7.	23 38 39.78	−6 06 00.6	Referred to (6); 8t, 4.
8.	Three stars from San Fernando ph.		

All the visual measures except that on October 10 were with the 36-inch refractor. The comet was faint with 12-inch on October 10. It gradually faded, becoming more diffuse. On November 18 the measure was difficult in the 36-inch. The later plates on December 10 (exposures 1^h, 15^m, respectively) show the comet as rather diffuse, and with a distinct tail nearly 1′ long, in position angle 70°. Two plates taken 1926 January 8 still show the comet, magnitude about 16, but, since a star trail interfered with the image of the comet, these were not measured.

COMET 1925 *h* (FAYE)

1925	U. T.	Star	No. of Comp.	Comet—Star Δα	Comet—Star Δδ	α 1925.0	δ 1925.0	Log pΔ α	Log pΔ δ
Oct. 23	12^h 54^m 20^s	2	12, 12	$+3^s.41$	−0′ 54″.0	8^h 10^m $42^s.38$	+8° 52′ 29″.5	9.194n	0.626

MEAN PLACES OF COMPARISON STARS FOR 1925.0

	α	δ	
1.	8^h 09^m $42^s.14$	+8° 52′ 15″.8	A. G. Leipzig II 4432.
2.	8 10 38.97	+8 53 23.5	Referred to (1); 12t, 9.

Observed with 36-inch, magnitude 14. The comet was round and condensed in the center; measure not difficult.

Comet 1925 *i* (Peltier-Wilk)

1925	U. T.	Star	No. of Comp.	Comet—Star Δα	Comet—Star Δδ	α 1925.0	δ 1925.0	Log pΔ α	Log pΔ δ
Nov. 23	3^{h} 06^{m} 10^{s}	1	10t, 8	−77^{s}.90	−0′ 03$''$.0	18^{h} 04^{m} 34^{s}.22	+24° 33′ 45$''$.1	9.696	0.612
25	3 33 17	2	12, 12	−15.10	+1 18.1	18 25 50.81	+19 44 49.1	9.688	0.657

Mean Places of Comparison Stars for 1925.0

	α	δ	
1.	18^{h} 05^{m} 52^{s}.12	+24° 33′ 48$''$.1	A. G. Berlin B 6324.
2.	18 26 05.91	+19 43 31.0	A. G. Berlin A 6831.

The first measure was with the 12-inch; for the second the 36-inch was used. Comet strongly condensed in the center; magnitude 7.

Comet *j* 1925 (Van Biesbroeck)

1925	U. T.	Star	No. of Comp.	Comet—Star Δα	Comet—Star Δδ	α 1925.0	δ 1925.0	Log pΔ α	Log pΔ δ
Nov. 18	12^{h} 23^{m} 30^{s}	1	10t, 8	+78^{s}.83	+6′ 34$''$.8	11^{h} 57^{m} 25^{s}.59	+34° 33′ 39$''$.4	9.683n	0.356
19	12 24 58	2	14t, 18	−183.80	+2 58.5	11 58 14.79	+34 13 08.1	9.675n	0.351
21	13 03 30	3	10, 11	+ 2.45	+2 43.2	11 59 47.98	+33 32 02.5	9.603n	0.236
25	12 57 15	5	8, 8	−11.41	−3 10.0	12 02 27.21	+32 13 00.9	9.584n	0.257
Dec. 8	13 06 58	6	10, 9	+ 3.20	−6 4.0	12 07 16.08	+28 14 02.8	9.423n	0.257
24	11 05 06	7	10, 9	+12.24	−3 24.4	12 04 29.99	+23 53 54.3	9.550n	0.449
1926	U. T.					α 1926.0	δ 1926.0		
Jan. 1	11 04 09	8	14t, 10	−79.98	+0 23.7	11 59 09.42	+21 52 40.0	9.461n	0.446
16	13 27 50	9	14t, 8	−134.58	−1 46.6	11 41 12.50	+18 14 19.9	9.231	0.475
22	10 55 52	10	15t, 10	−40.38	+3 08.3	11 31 34.94	+16 50 35.1	8.908n	0.488
26	10 41 21	11	11t, 10	−89.62	+4 44.2	11 24 21.06	+15 54 03.4	8.798n	0.506
Mar. 5	11 23 14	12	12t, 6	−174.93	+0 5.1	10 09 24.63	+ 7 30 24.8	9.607	0.681
11	4 42 50	13	16t, 10	+80.54	+3 25.5	10 00 46.67	+ 6 28 16.9	9.409n	0.664
13	6 04 46	15	12t, 10	+40.61	−2 06.9	9 57 58.66	+ 6 07 05.7	8.904n	0.657
23	9 09 55	16	9, 9	+ 4.82	−0 42.0	9 46 40.23	+ 4 31 36.8	9.550	0.693
Apr. 13	7 23 39	17	10, 9	+ 2.24	−4 02.1	9 35 13.09	+ 1 50 30.2	9.528	0.712
May 10	5 29 21	19	10, 10	+ 6.95	−1 56.3	9 37 53.75	− 1 01 05.4	9.507	0.732
13	5 54 55	22	8, 9	−14.85	+1 02.2	9 39 02.22	− 1 19 36.0	9.569	0.733

Mean Places of Comparison Stars for 1925.0

	α	δ	
1.	11^{h} 56^{m} 06^{s}.76	+34° 27′ 04$''$.6	A. G. Lund 5318, A. G. Leiden 4561.
2.	12 01 18.59	+34 10 09.6	A. G. Leiden 4584.
3.	11 59 45.53	+33 29 19.3	A. G. Leiden 4579.
4.	12 05 19.51	+32 20 21.0	A. G. Leiden 4603.
5.	12 02 38.62	+32 16 10.9	Referred to (4); 8t, 6.
6.	12 07 12.88	+28 20 6.8	Oxf. ph. +29°.32244, +28°.34362.
7.	12 04 17.75	+23 57 18.7	Oxf. ph. +25°.40207, Par. ph. +24°, 12^{h} 00^{m}, 85.

Mean Places of Comparison Stars for 1926.0

8.	12 00 29.40	+21 52 16.3	Boss P. G. C. 3150.
9.	11 43 27.08	+18 16 06.5	A. G. Berlin A 4518.
10.	11 32 15.32	+16 47 26.8	A. G. Berlin A 4466.
11.	11 25 50.68	+15 49 19.2	Boss P. G. C. 3025.
12.	10 12 19.56	+ 7 30 19.7	A. G. Leip. II 5413.
13.	9 59 26.13	+ 6 24 51.4	A. G. Leip. II 5345.
14.	9 57 25.71	+ 6 12 18.3	A. G. Leip. II 5332.
15.	9 57 18.05	+ 6 09 12.6	Referred to (14); 6, 6.
16.	9 46 35.41	+ 4 32 18.8	Algiers ph +4°, 9^{h} 44^{m}, 92.
17.	9 35 10.85	+ 1 54 32.3	Alg. ph. +3°, 9^{h} 32^{m}, 210.
18.	9 36 38.33	− 1 03 0.9	A. G. Nic. 2911.
19.	9 37 46.80	− 0 59 9.1	Referred to (18); 8t, 6.
20.	9 36 46.27	− 1 23 35.0	A. G. Nic. 2912.
21.	9 38 11.28	− 1 24 56.5	A. G. Nic. 2916.
22.	9 39 17.07	− 1 20 38 2	Referred to (20): +2^{m} 30^{s}.85, +2′ 58$''$.0; to (21): +1^{m} 05^{s}.74; +4′ 17$''$.6.

The above measures, excepting those on November 18, November 19, and December 8, were made with the 36-inch telescope. The comet always had a sharp, though not stellar, nucleus. On November 21, the head was about 0′.5 in diameter, magnitude 9.5, and with a tail several minutes long, in position angle 330°. On January 1 it was easily seen in the 4-inch finder, notwithstanding a full Moon. On March 13, the tail was still visible, and the diameter of the head was about 1′. March 23, and April 13: comet becoming fainter, about $11^m.5$ on April 13. In May the magnitude of the comet was about 13. The diameter was about 15″. On May 13 the measure was difficult, due to a hazy sky.

Comet 1926 *b* (Blathwayt)

1926	U. T.	Star	No. of Comp.	Comet—Star Δα	Comet—Star Δδ	α 1926.0	δ 1926.0	Log pΔ α	Log pΔ δ
Jan. 20	13^h 22^m 58^s	2	10, 10	$-19^s.57$	−2′ 15″.3	11^h 53^m $43^s.65$	−27° 01′ 13″.2	9.250	0.888
21	13 36 49	3	10, 10	−13.48	−4 21.6	11 47 46.07	−25 55 36.4	9.354	0.877
22	12 32 33	5	10, 10	−14.50	−2 44.8	11 41 46.09	−24 47 25.4	9.034	0.886
Feb. 9	6 04 28	6	10, 10	−28.25	+2 42.5	8 43 43.32	+17 58 28.8	9.281n	0.486
17	8 39 06	7	9, 10	−29.27	−7 43.8	7 17 59.56	+33 42 42.1	9.600	0.224
Mar. 6	7 00 20	9	10, 10	+ 3.18	+0 27.1	5 46 22.30	+43 25 14.8	9.746	0.163
11	7 05 01	11	10, 11	− 3.84	+0 38.5	5 34 15.82	+44 22 24.8	9.786	0.314

Mean Places of Comparison Stars for 1926.0

	α	δ	
1.	11^h 54^m $57^s.44$	−27° 01′ 35″.6	Cordoba A 9101.
2.	11 54 3.22	−26 58 57.9	Referred to (1); 12t, 7.
3.	11 47 59.55	−25 51 14.8	Cordoba A 9038.
4.	11 42 01.14	−24 40 58.6	Cordoba A 8983.
5.	11 42 0.59	−24 44 40.6	Referred to (4); 8, 8.
6.	8 44 11.57	+17 55 46.3	A. G. Berlin A 3533.
7.	7 18 28.83	+33 50 25.9	A. G. Leiden 3108.
8.	5 46 41.09	+43 19 47.4	Hels. ph. 5^h 45^m, +43°, 142; 5^h 50^m, +44°, 8.
9.	5 46 19.12	+43 24 47.7	Referred to (8); 6, 6.
10.	5 34 57.24	+44 18 36.3	A. G. Bonn 4630; Hels. ph.
11.	5 34 19.66	+44 21 46.3	Referred to (10); 10t, 6.

The measures of January 20, February 9, and February 17, were made with the 12-inch, the others with the 36-inch. The comet was located on the morning of January 20, after some hunting, for the daily motion had increased since discovery, and some nebulae in the neighborhood tended to confuse matters. It was nearly round, brighter in the middle, and of about 11.5 magnitude. This comet faded quite rapidly. The observations of February 9 and 17 are both uncertain because of this, the latter particularly so. The comet was nearly invisible when the faintly illuminated micrometer wire was near by. The measures of March 6 and 11, with the larger instrument, are thought to be better. The magnitude on March 11th was about 15, total diameter about 10″, and brighter in the center. The comet was again seen with difficulty, magnitude about 16, on March 15, but no measures were attempted.

May 18, 1926.

Issued May 27, 1926.

UNIVERSITY OF CALIFORNIA PUBLICATIONS
ASTRONOMY

LICK OBSERVATORY BULLETIN
NUMBER 378

A PHOTOELECTRIC STUDY OF *u* HERCULIS

By

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During the summer of 1922 the photoelectric photometer was generously assigned to the writer for use with the 12-inch telescope of the Lick Observatory. It seemed well to employ this opportunity to renew acquaintance with the eclipsing binary u *Herculis* under these favorable conditions. The present paper is a report of the investigation.

The photometer has been described in detail by Miss Cummings.[1] Mechanically it leaves little to be desired, but at the time these observations were made certain problems of insulation against moisture and electrical effects had not been fully solved. On this account the degree of precision which I attained does not do justice to this fine instrument. Doubtless an important contributing factor was my inexperience in the special requirements of this exacting type of observing. While the individual observations are not of the highest accuracy from the photoelectric standpoint, nevertheless they have considerable merit relative to the usual photometric standards; and in the mean they establish a light curve which meets very well the requirements of the theory.

The history of 68 u *Herculis* (R. A. $17^{h}\,13^{m}\!.6$, Decl. $+33°\,12'$) as an eclipsing binary star began with the determination of the spectroscopic orbit in 1908 at the Allegheny Observatory.[2] The period of revolution was found to be 2.05102 days. This star had long been known as a light variable, the discovery having been made by Schmidt[3] in 1869; but the true character of the variation escaped the early observers. Subsequent to the publication of the spectroscopic results a number of light curves appeared, both from the current observations and from earlier ones which were now assembled on the basis of this period. Notable among these is Hertzsprung's discussion of more than four thousand observations by Schmidt between the years 1869 and 1879. He found a slightly improved value of the period, 2.051027 days, which is likely to stand for some time to come. Among more recent investigations of u *Herculis*, a photographic light curve was published by the writer from the University of Missouri Observatory and two photoelectric studies of the star were made at the Berlin-Babelsberg Observatory, by Hügeler from Guthnick's observations, and by Pavel. The last named appeared after the present observations were made. In all three photoelectric series the same comparison star, 69 e *Herculis* was used. The discussion in this paper is based on the combined photoelectric data.

Available observed epochs of the primary minimum of u *Herculis* are compared in Table I with those computed with Hertzsprung's period. It appears that the period is very nearly constant over an interval of fifty years. But the residuals in the last column can not all be readily ascribed to uncertainties in determining the times of the minima from the observations. The three photoelectric light curves are systematically displaced in the time direction; they have accordingly received the corrections:

Guthnick Δ Phase = −.006 day
Pavel −.006
Baker +.0025

The methods of observation and reduction are adequately described in Miss Cummings' paper and her Figures 8, 9, 10 show the characteristic curves of

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[1] *Lick Obs. Bull.* **11**, 349, 1923.
[2] *Publ. Allegheny Obs.* **1**, 77, 1909.
[3] *Astron. Nach.* **74**, 229, 1869.

TABLE I

Minima of u *Herculis* Observed and Calculated with the Elements

$\text{Min.}=2419928.790+2\overset{\text{d}}{.}051027\cdot E$

Observer	Reference	E	O	C	O−C days
Schmidt	*A. N.* **189**, 245, 1911	−6874	2405830.032	.030	+.002
Plassmann	*A. N.* **207**, 241, 1918	−2128	2415564.202	.205	− 3
Enebo	*A. N.* **182**, 309, 1909	−1503	2416846.105	.096	+ 9
Ichinohe	*A. N.* **183**, 193, 1909	− 879	2418125.93	.937	− 7
Wendell[4]	*A. O. P.* **2**, 51, 1910	− 879	2418125.933	.937	− 4
Hertzsprung	*A. N.* **189**, 255, 1911	− 574	2418751.500	.501	− 1
Nijland	*A. N.* **216**, 97, 1922	− 380	2419149.411	.400	+ 11
Dziewulski	*A. N.* **207**, 241, 1918	− 130	2419662.154	.156	− 2
Plassmann	*A. N.* **207**, 241, 1918	− 63	2419799.596	.575	+ 19
Baker	*Laws O. B.* **2**, 209, 1917	000	2419928.790	.790	0
Guthnick	*A. N.* **213**, 145, 1921	+1100	2422184.926	.920	+ 6
Pavel	*A. N.* **217**, 177, 1922	+1467	2422937.653	.647	+ 6
Baker	This Bulletin	+1634	2423280.166	.168	− 2

the quartz potassium cell. The cell was one of the best made by Professor Jacob Kunz of the University of Illinois; it performed well on a faint star with an anode potential as high as 310 volts. It is a matter of much regret to the writer that the cell was subsequently flashed and its effectiveness destroyed while he was working with it.

The separate photoelectric observations of u *Herculis* are given in Table II. The observed times are taken from Greenwich mean noon; the phases are corrected

TABLE II

Photoelectric Observations of u *Herculis*

Julian Day G. M. T.	Corrected heliocentric phase	Magnitude difference u−e	Residual from computed curve O−C
	Berlin, 1919		
2422	days	mag.	mag.
136.424	0.726	+.029	+.034
141.455	1.655	−.001	+.005
142.416	0.565	−.022	+.003
147.440	1.487	−.021	+.004
150.421	0.365	+.012	+.011
153.487	1.380	−.029	−.012
156.401	0.192	+.076	+.016
158.428	0.168	+.109	+.007
159.400	1.140	+.087	−.014
161.372	1.061	+.200	−.009
.385	1.074	+.177	−.016
.400	1.089	+.171	−.002
.498	1.187	+.051	+.005
164.371	2.009	+.560	+.009
.383	2.021	+.611	+.012
.395	2.033	+.630	+.002
.408	2.046	+.650	+.005
.426	0.013	+.634	−.002
.457	0.044	+.528	−.015
.503	0.090	+.338	−.015
.530	0.117	+.254 :	+.005
169.422	0.907	+.120	+.022
.440	0.925	+.135 :	+.014
175.424	0.755	−.010	−.010

TABLE II—Continued

Julian Day G. M. T.	Corrected heliocentric phase	Magnitude difference u−e	Residual from computed curve O−C
	Berlin, 1919		
2422	days	mag.	mag.
183.364	0.490	−.032	−.010
.374	0.500	−.027	−.004
.479	0.605	−.010	+.015
186.393	1.468	−.033	−.008
.489	1.564	−.032	−.011
190.401	1.374	+.003	+.018
.463	1.436	−.019	+.003
191.394	0.316	+.005	−.009
.448	0.370	−.013	−.013
196.359	1.179	+.046 :	−.008
198.447	1.216	+.020	−.005
199.348	0.066	+.454 :	.000
203.419	0.034	+.578	−.008
.428	0.043	+.528	−.022
204.331	0.946	+.177	+.026
.351	0.966	+.190	+.010
.369	0.984	+.209	+.009
.385	1.000	+.240	+.024
.406	1.021	+.237	+.010
.421	1.036	+.228	+.004
205.349	1.964	+.370	+.006
.369	1.984	+.459	+.008
.390	2.005	+.540	+.002
207.339	1.903	+.182	+.032
.358	1.922	+.199	−.006
.378	1.942	+.275	−.002
.397	1.961	+.340	−.010
.416	1.980	+.411	−.020
208.358	0.871	+.054	+.001
.375	0.888	+.059	−.014
.394	0.907	+.092	−.006
.421	0.934	+.106	−.028
210.331	0.793	+.003	−.007
.347	0.809	+.022	+.008
212.316	0.727	+.023	+.028
.332	0.743	−.006	−.005
.348	0.759	+.011	+.011
.383	0.794	+.007	−.003
.402	0.813	+.010	−.005

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[4] *Harvard Annals* **69**, 127, 1914.

TABLE II—Continued

Julian Day G. M. T.	Corrected heliocentric phase	Magnitude difference u−e	Residual from computed curve O−C
	Berlin, 1919		
2422	days	mag.	mag.
213.317	1.728	−.008	−.021
.335	1.746	+.016	−.001
.366	1.777	+.008	−.013
.385	1.796	+.008	−.022
.404	1.815	+.016	−.020
214.318	0.678	−.017	−.003
.337	0.697	−.007	+.003
.356	0.716	+.001	+.008
.377	0.737	−.003	−.002
.398	0.758	+.003	+.002
215.325	1.685	+.007	+.007
.341	1.701	+.007	+.001
.361	1.721	+.023	+.013
.378	1.738	+.019	+.004
.396	1.756	+.042	+.022
217.327	1.635	+.008 :	+.019
218.304	0.561	−.014	+.011
.323	0.580	−.018	+.007
.342	0.599	−.031	−.006
.370	0.627	−.023	−.001
219.305	1.562	−.013	+.009
.320	1.577	−.012	+.008
.337	1.594	−.008	+.011
.379	1.636	+.006 :	+.016
222.312	0.467	−.015	+.004
.329	0.484	−.019	+.002
.348	0.503	−.021	+.003
.371	0.526	−.024	+.001
229.317	1.319	−.006	−.001
231.324	1.275	−.008	−.013
	Berlin, 1921		
2422	days	mag.	mag.
880.419	0.197	+.075	+.020
.496	0.274	+.051 :	+.025
881.427	1.205	+.034	.000
885.405	1.081	+.202	+.018
.508	1.184	+.078	+.029
886.391	0.016	+.639	+.007
.419	0.044	+.548	+.002
.459	0.084	+.361 :	−.016
.503	0.128	+.225	+.018
895.489	0.910	+.112	+.011
.501	0.922	+.134	+.017
903.445	0.662	−.019	−.002
.457	0.674	−.018	−.004
922.404	1.161	+.086	+.011
.414	1.171	+.059	−.002
924.358	1.064	+.198	−.007
.369	1.075	+.173	−.019
.381	1.087	+.162	−.014
.393	1.099	+.146	−.014
930.363	0.915	+.095	−.013
.378	0.930	+.144	+.016
932.369	0.869	+.054	+.003
939.361	1.708	+.017 :	+.011
940.324	0.620	+.014	+.009
942.346	0.591	−.006 :	+.019
949.344	1.435	−.017	+.005
951.328	1.368	−.020	−.005

TABLE II—Continued

Julian Day G. M. T.	Corrected heliocentric phase	Magnitude difference u−e	Residual from computed curve O−C
	Berlin, 1921		
2422	days	mag.	mag.
952.362	0.351	.000	−.005
953.313	1.302	−.006	−.003
.330	1.319	−.005	.000
954.312	0.250	+.026	−.007
963.283	1.017	+.230	+.004
.299	1.033	+.233	+.008
.306	1.040	+.228	+.006
.324	1.058	+.210	−.002
964.266	2.000	+.520	+.003
.283	2.017	+.610	+.024
.300	2.034	+.639	+.009
.315	2.049	+.651	+.004
965.270	0.953	+.157	−.003
.286	0.969	+.179	−.004
.300	0.983	+.195	−.005
.315	0.998	+.211	−.004
967.257	0.889	+.069	−.004
968.264	1.896	+.129	−.002
.277	1.909	+.169	+.003
.287	1.919	+.203	+.007
.297	1.929	+.237	+.006
.308	1.940	+.260	−.018
.318	1.950	+.311	+.005
.329	1.961	+.348	−.006
.343	1.975	+.412	+.002
969.257	0.837	+.033	+.006
.273	0.853	+.049	+.011
.286	0.866	+.063	+.014
.302	0.882	+.070	+.006
970.261	1.841	+.057	+.008
.278	1.858	+.073	+.013
972.263	1.792	+.029	.000
.281	1.810	+.044	+.009
973.268	0.746	−.012	−.011
.283	0.761	−.009	−.011
979.262	0.587	+.008 :	+.033
.280	0.605	−.014	+.010
981.262	0.536	−.038	−.013
982.264	1.538	−.010 :	+.013
	Lick, 1922		
2423	days	mag.	mag.
256.748	1.197	+.044	+.005
.785	1.234	+.032	+.016
.838	1.287	+.004	+.001
.868	1.317	+.015	+.020
257.699	0.097	+.368	+.044
.706	0.104	+.317	+.022
.739	0.137	+.223	+.042
.751	0.149	+.146	−.002
.766	0.164	+.115	+.006
.783	0.181	+.121	+.045
.802	0.200	+.110	+.057
.816	0.214	+.052	+.006
.822	0.220	+.060	+.015
259.693	0.040	+.546	−.018
.706	0.053	+.516	+.007
.764	0.111	+.206	−.062
.786	0.132	+.192	.000
.806	0.153	+.139	−.027

TABLE II—Continued

Julian Day G. M. T.	Corrected heliocentric phase	Magnitude difference u−e	Residual from computed curve O−C
	Lick, 1922		
2423	days	mag.	mag.
259.823	0.170	+.103	+.004
.831	0.178	+.070	−.012
262.694	0.990	+.222	+.016
.699	0.995	+.222	+.011
.730	1.026	+.258	+.031
.744	1.039	+.221	−.002
.757	1.052	+.226 :	+.011
.773	1.069	+.199	.000
.787	1.083	+.219	+.037
.826	1.122	+.123	−.005
.860	1.156	+.093	+.013
.871	1.167	+.050	−.017
263.686	1.982	+.404	−.036
.698	1.994	+.463	−.028
.730	2.025	+.612	+.002
.743	2.039	+.650	+.013
.768	0.013	+.660	+.024
.810	0.055	+.486	−.015
.834	0.078	+.426	+.025
.852	0.097	+.345	+.021
266.806	0.999	+.206	−.009
.820	1.014	+.231	+.006
.836	1.029	+.224	−.002
267.684	1.877	+.083	−.007
.689	1.882	+.112	+.011
.710	1.904	+.180	+.029
.721	1.914	+.178	−.035
.732	1.925	+.240	+.022
.747	1.940	+.282	+.011
.760	1.953	+.324	+.003
.774	1.967	+.386	+.006
.790	1.984	+.465	+.006
.804	1.998	+.546	+.038
.816	2.010	+.595 :	+.039
.829	2.022	+.595	−.006
.843	2.037	+.620	−.014
270.678	0.769	+.018	+.014
.683	0.774	+.007	+.002
.701	0.792	+.023	+.013
.710	0.801	+.004	−.009
.720	0.811	+.010	−.005
.732	0.823	+.026	+.007
.755	0.846	+.026 :	−.008
.765	0.856	+.066	+.026
.776	0.867	+.070	+.020
.789	0.880	+.062 :	.000
.804	0.895	+.104	+.023
.815	0.906	+.081	−.014
.825	0.917	+.097	−.013
.837	0.929	+.118	−.008
271.677	1.768	+.004	−.019
.699	1.791	+.047	+.017
.710	1.801	+.019 :	−.013
.724	1.815	+.060	+.024
.733	1.824	+.058	+.018
.742	1.833	+.062	+.018
.754	1.845	+.068	+.019
.765	1.856	+.056	−.001
.779	1.870	+.074	−.002
.789	1.880	+.092	+.001

TABLE II—Continued

Julian Day G. M. T.	Corrected heliocentric phase	Magnitude difference u−e	Residual from computed curve O−C
	Lick, 1922		
2423	days	mag.	mag.
271.801	1.892	+.099	−.024
.813	1.904	+.143	−.008
273.696	1.736	+.002	−.013
.707	1.747	+.020	+.002
.723	1.763	+.006	−.016
.735	1.776	+.043	+.018
.748	1.788	+.042	+.013
.760	1.800	+.039	+.007
.772	1.813	+.064 :	−.028
274.692	0.681	−.012	.000
.702	0.691	.000	+.010
.715	0.704	−.026	−.016
275.687	1.676	−.012	−.011
.696	1.685	+.007	+.007
.705	1.694	−.007	−.009
.718	1.707	+.017	+.011
.728	1.717	+.004	−.005
.739	1.728	+.018	+.006
276.688	0.626	−.043 :	−.020
.711	0.649	+.002	+.022
.722	0.659	−.043	−.025
.732	0.670	−.018	−.003
.796	0.734	−.005	−.002
.806	0.744	−.030	−.029
277.685	1.623	−.017	−.004
278.683	0.570	−.002 :	+.023
.696	0.582	−.046	−.021
280.686	0.522	−.032	−.007
281.672	0.507	−.019 :	+.005
.709	0.544	−.023 :	+.002
.720	0.556	−.026 :	−.001
.732	0.568	−.016	+.009
.743	0.578	−.006	+.019
.758	0.594	−.044	−.019
.769	0.604	−.034	−.009
282.669	0.454	−.023 :	−.005
.678	0.463	−.028 :	−.009
.692	0.476	−.041 :	−.021
.703	0.487	−.072 :	−.051
.712	0.497	−.025	−.003
.723	0.507	−.056	−.032
.738	0.523	−.033	−.008
.750	0.535	−.040	−.015
.760	0.545	−.038	−.013
.775	0.559	−.043	−.018
.786	0.571	−.036	−.011
.799	0.583	−.060	−.035
283.693	1.478	−.010	+.015
.703	1.488	−.028	−.003
.714	1.498	−.032	−.007
290.685	0.265	+.028 :	.000
.697	0.277	+.046 :	+.022
.711	0.291	+.020	.000
.721	0.301	−.010	−.028
.741	0.321	+.012	.000
.754	0.334	+.012	+.002
.775	0.355	+.003	−.001
298.658	0.033	+.529	−.059
.673	0.048	+.509	−.019
.684	0.059	+.466 :	−.017

TABLE II—Continued

Julian Day G. M. T.	Corrected heliocentric phase	Magnitude difference u−e	Residual from computed curve O−C
	Lick, 1922		
2423	days	mag.	mag.
298.695	0.070	+.399 :	−.036
.705	0.081	+.388 :	−.002
.730	0.105	+.287	−.003
.741	0.116	+.226	−.024
.753	0.128	+.199	−.007
301.664	0.988	+.184	−.021
.679	1.003	+.173	−.046
.690	1.014	+.210	−.010
.701	1.025	+.222	−.005
.716	1.040	+.224	+.002
.725	1.049	+.211	−.007
.736	1.061	+.208	−.001
.750	1.074	+.156 :	−.036
303.682	0.955	+.128	−.036
304.699	1.972	+.361 :	−.044
.734	2.007	+.500 :	−.040
.746	2.019	+.537	−.053
306.662	1.884	+.081	−.022
.681	1.903	+.122 :	−.010
.706	1.928	+.229	+.004
.722	1.944	+.280	−.003
.732	1.954	+.311	−.010
.740	1.962	+.349	−.008

as indicated above. The magnitude differences are referred to the single comparison star e *Herculis*. The Berlin series of 1919 combines with the Lick observations without change, but the series of 1921 gives a curve systematically higher and with a slightly greater scale. Consequently Pavel's magnitudes, as they appear in the table, have received the correction +0.060 magnitude and near primary minimum the additional correction $-0.015 \cdot n$ magnitude, where n is the percentage of obscuration in terms of that of mid-eclipse. Half weight is assigned the magnitudes marked as doubtful. In addition, two of Guthnick's observations which are marked "very doubtful" are rejected, and for like reason 14 of the Lick series. The present study is therefore based on 158 observations at Berlin and 161 at the Lick Observatory, which are combined for convenience in Table III into 57 normal places. The derivation of the third column of this table will now be explained. The analysis is made by the methods of Russell and Shapley.[5]

For the parts of the light curve which are certainly unaffected by the eclipses, the normal magnitude differences (numbers 11 to 20 and 38 to 45) are changed to intensities relative to the arbitrary value $m_0 = -0.031$. We have for each normal intensity the expression

$$l = a - b \cos \theta - c \cos^2 \theta$$

where a would be the constant intensity of the system between eclipses, if the stars were uniformly luminous spheroids; $2b$ is the difference in the brightening of the opposing hemispheres of the two stars due to reflection

TABLE III

NORMAL MAGNITUDES OF u *Herculis*

No.	Corrected heliocentric phase	Rectified intensity	Observed magnitude	Residual from computed curve O−C
	days			mag.
1	0.0069	0.6115	+.647	+.004
2	0.0389	0.6671	+.545	−.020
3	0.0500	0.6870	+.510	−.010
4	0.0697	0.7326	+.434	−.003
5	0.0907	0.7857	+.351	+.002
6	0.1108	0.8497	+.258	−.012
7	0.1313	0.8830	+.210	+.013
8	0.1585	0.9444	+.127	+.005
9	0.1801	0.9692	+.092	+.014
10	0.2077	0.9788	+.074	+.023
11	0.2774	1.0008	+.025	.000
12	0.3446	1.0016	+.004	−.003
13	0.4886	1.0073	−.030	−.008
14	0.5439	1.0055	−.031	−.006
15	0.5827	0.9999	−.024	+.001
16	0.6187	0.9967	−.018	+.005
17	0.6766	1.0028	−.017	−.002
18	0.7307	0.9992	−.003	+.001
19	0.7627	0.9999	+.003	.000
20	0.8045	0.9992	+.013	.000
21	0.8484	0.9791	+.046	+.011
22	0.8683	0.9701	+.060	+.009
23	0.8875	0.9610	+.074	+.004
24	0.9102	0.9417	+.100	−.001
25	0.9279	0.9210	+.127	+.002
26	0.9513	0.9007	+.154	−.003
27	0.9800	0.8687	+.196	.000
28	0.9990	0.8584	+.210	−.005
29	1.0196	0.8422	+.231	+.004
30	1.0363	0.8460	+.226	+.002
31	1.0556	0.8606	+.207	−.006
32	1.0761	0.8723	+.191	.000
33	1.0920	0.8964	+.160	−.008
34	1.1393	0.9411	+.101	−.001
35	1.1694	0.9725	+.060	−.004
36	1.1894	0.9710	+.058	+.013
37	1.2185	0.9913	+.029	+.004
38	1.3031	1.0006	−.001	.000
39	1.3990	0.9985	−.016	+.003
40	1.4838	1.0002	−.025	.000
41	1.5670	0.9939	−.015	+.006
42	1.6451	0.9944	−.003	+.005
43	1.6997	0.9966	+.008	+.003
44	1.7376	0.9995	+.016	+.001
45	1.7868	1.0027	+.027	−.001
46	1.8276	0.9916	+.054	+.013
47	1.8653	0.9851	+.072	+.002
48	1.8869	0.9635	+.103	−.007
49	1.9066	0.9171	+.164	+.005
50	1.9246	0.8745	+.222	+.008
51	1.9415	0.8374	+.275	−.002
52	1.9557	0.8019	+.327	−.003
53	1.9683	0.7688	+.377	−.005
54	1.9850	0.7291	+.440	−.012
55	2.0040	0.6685	+.542	+.009
56	2.0211	0.6375	+.598	.000
57	2.0356	0.6184	+.634	.000

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[5] *Ap. J.* **35**, 315, 1912; **36**, 54, 239, 285, 1912.

or whatever cause; and c is a constant function of the ratio between the major and minor equatorial axes of the stars and describes the mutual tidal effect. The two stars are supposed to rotate and revolve in the same period. A least-squares solution gives:

$$a=0.9935\pm.0009$$
$$b=0.0175\pm.0012$$
$$c=0.0758\pm.0031$$

The effects of reflection and ellipticity may now be removed from the normal intensities during the eclipses. I have used the formula

$$l_R=\frac{l_0+b(1+\cos\theta)+c\cdot l_R\cos^2\theta+\tfrac{1}{2}c^2\cdot l_R\cos^4\theta}{a+b}$$

By this convention the stars become uniformly luminous and they are spheroids with equatorial axes equal to the actual major axes. Thus we have the rectified intensities of the third column of Table III which vary, aside from accidental errors, only because of the eclipses.

The losses of light at the two minima are

$$1-\lambda_p=0.388 \text{ and } 1-\lambda_s=0.156$$

relative to the total light of the system. We have also the simultaneous relations

$$\alpha_0=1-\lambda_1+\frac{1-\lambda_2}{k^2}$$

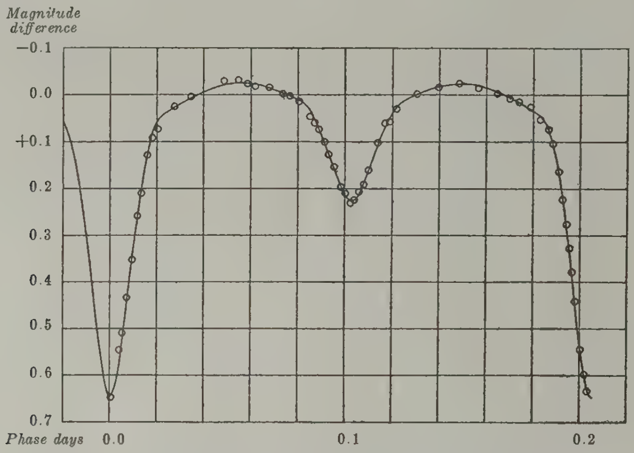

Fig. 1. Normal Magnitudes and Theoretical Light Curve of u *Herculis*

and $\chi\left(k,\ \alpha_0,\ \frac{1}{4}\right)$ whose values are tabulated by Russell, from which k and α_0 are to be found. But in this case the possible combinations of these quantities give values of the χ-function so nearly the same that the solution is practically indeterminate as it stands. However, the spectroscopic evidence suggests that the two stars have a difference of brightness of the order of one magnitude. This and the relation

$$L_2=\frac{1-\lambda_1}{\alpha_0}$$

make it probable that k is not less than 0.80. Within these limits we have no choice. Accordingly it is proper to put $k=1$. Then $\alpha_0=0.544$ and $\chi=1.854$. The definitive light curve is readily calculated from these data. That it represents the normal magnitudes faithfully is shown in the last column of Table III and in Figure 1. The probable error of a normal magnitude is ±0.0048 magnitude. The probable error of a single observation for the different observers is: Guthnick ±0.0084, Pavel ±0.0075, Baker ±0.0130. The last value should be interpreted in the light of the introductory remarks.

Aside from the reflection effect I have supposed in the analysis that the disks of the two stars are uniformly luminous. This assumption is, of course, not true for the Sun and it may well be untrue for the stars we are considering. Russell and Shapley have provided appropriate formulae and tables for the other extreme, namely, for disks entirely darkened at the limb.

The details of the system of u *Herculis* appear in Table IV and Figure 2; the dimensions are given in terms of the distance between the stars, a_b+a_f, and also in terms of the Sun's radius. This system shares with not more than twenty eclipsing binaries at present the special interest that the elements of the system may be determined absolutely. For in these cases the spectra of both components are measurable so that $a_b \sin i$ and $a_f \sin i$ become known, the value of the inclination being furnished, of course, by the photo-

TABLE IV

ELEMENTS OF THE SYSTEM u *Herculis*
UNIFORM SOLUTION

		Relative	Absolute
Period of revolution	P	2.051027 days	
Epoch of primary minimum	T	2419928.790 G.M.T.	
Semi-duration of eclipses		0.201 day	
Eccentricity of orbit	e	0 (assumed)	
Inclination of orbit	i	77° 39′	
Radius of relative orbit	a_b+a_f	1	14.60 $\odot$
Ratio of radii	k	1	
Ratio of major and minor axes	$\frac{\mathrm{b}}{\mathrm{a}}$	0.915	
Major semi-axis	$\mathrm{a}_b=\mathrm{a}_f$	0.318	4.64 $\odot$
Minor semi-axis	$\mathrm{b}_b=\mathrm{b}_f$	0.291	4.25 $\odot$
Mass of brighter star	m_b		7.3 $\odot$
Mass of fainter star	m_f		2.8 $\odot$
Density of brighter star	ρ_b		0.094 $\odot$
Density of fainter star	ρ_f		0.036 $\odot$
Maximum obscuration	α_0	0.544	
"Radiation effect" constant	b	0.018	
Light of brighter star	L_b	0.713	
Light of fainter star	L_f	0.287	
Ratio of surface intensities	$\frac{J_b}{J_f}$	2.48	

TABLE V

COMPARISON OF ELEMENTS FROM DIFFERENT OBSERVATIONS AND SOLUTIONS

Character	Visual		Photographic	Photoelectric	
Solution by	Schlesinger Baker	Shapley	Baker	Pavel	Baker
Observations at	Harvard	Harvard	Missouri	Berlin	Berlin Lick
Solution	uniform	darkened	darkened	uniform	uniform
Semi-duration (day)	0.30	0.20	0.31	0.21	0.20
i	75.4	74.4	73.5	76.0	77.6
a_b+a_f ($\odot=1$)	14.66	14.81	14.88	14.70	14.60
k	1	0.85	1	0.82	1
a_b ($\odot=1$)	5.89	4.56	6.43	4.30	4.64
b_b ($\odot=1$)	5.89	4.25	6.29	3.94	4.25
m_b ($\odot=1$)	7.5	7.6	7.7	7.4	7.3
m_f ($\odot=1$)	2.9	2.9	3.0	2.8	2.8
ρ_b ($\odot=1$)	0.037	0.084	0.049	0.094	0.094
ρ_f ($\odot=1$)	0.014	0.030	0.017	0.020	0.036
α_0	0.60	0.59	0.63	0.61	0.54
L_b	0.72	0.72	0.63	0.66	0.71
L_f	0.28	0.28	0.37	0.34	0.29

metric results. Since the spectroscopic elements are used in the calculation of the last column of Table IV, it will be proper for completeness to give them in this place. The spectrosopic elements[2] of u *Herculis* are:

$$P=2.05102 \text{ days}$$
$$T=2418125.80$$
$$e=0.053$$
$$\omega=66^{\circ}.15$$
$$a_b \sin i=2{,}800{,}000 \text{ km.}$$
$$a_f \sin i=7{,}120{,}000 \text{ km.}$$
$$m_b \sin^3 i=6.8\ \odot$$
$$m_f \sin^3 i=2.6\ \odot$$

Thus far we have not considered the effect of eccentricity of the orbit. The spectroscopic elements as they stand require a longer duration for the secondary minimum and a displacement of this minimum so that it occurs 0.029 day after the time midway between primary minima. This value of the displacement was used by Schlesinger and the writer in their discussion[6] of Wendell's photometric observations. Shapley[7] employed one-half this displacement to represent the same points. The Missouri photographic observations appeared to show a displacement in the same direction amounting to 0.020 day, corresponding to $\omega=73^{\circ}.8$. On the other hand, the three photoelectric series agree in placing the secondary minimum precisely one-half the period after primary minimum. Either there is now no considerable eccentricity or else the major axis of the ellipse has moved forward so that ω is in the neighborhood of 90°. This situation, however, would demand a duration of secondary minimum more than ten per cent greater than that of the primary; the photoelectric observations appear not to meet this requirement. Consequently, for present purposes, I assume the eccentricity of the orbit to be zero.

Schlesinger and Baker[6] found in the case of u *Herculis* that the primary minimum occurs 0.015 day

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[6] *Publ. Allegheny Obs.* **2**, 51, 1910.

[7] *Contrib. Princeton Obs.* **3**, 67, 1915.

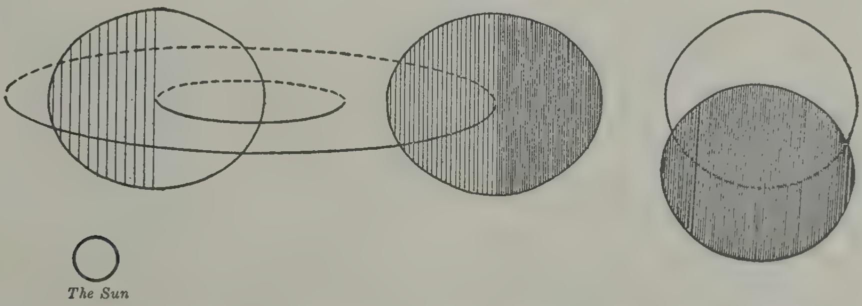

Fig. 2. The System of u *Herculis* at Greatest Elongation and Primary Minimum

after the time of principal spectroscopic conjunction. Differences of this sort are still on record against other eclipsing binaries, though some have disappeared with better values of the period.[8] So far as I know, the discrepancy remains for u *Herculis*. In this connection, however, it may be well to notice in Table I that the epoch of primary minimum appears to have shifted .008 day in the short time between Pavel's and the Lick observations.

UNIVERSITY OF ILLINOIS OBSERVATORY
April 12, 1926.
Issued May 27, 1926.

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[8] See, for example, *Publ. Missouri Obs.* **3**, 16, 29, 1921.

UNIVERSITY OF CALIFORNIA PUBLICATIONS
ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 379

A RE-DETERMINATION OF THE RADIAL VELOCITY CURVES OF CERTAIN CEPHEID VARIABLE STARS

By

THEODOR SIEGUMFELDT JACOBSEN

INTRODUCTION

In recent years the problem of Cepheid variation has received the attention of a number of investigators; a considerable amount of observational data has been accumulated concerning the variation in light, spectral type and radial velocity of these stars, and serious efforts have been made to interpret these phenomena on the basis of a theory which would account successfully for the known facts and at the same time be free from the obvious objections of the older binary hypothesis. To a certain extent the hypothesis which assumes as the cause of Cepheid variation the periodic pulsation of a single giant star of very low density, appears to meet these requirements. Its theoretical development on the basis of the known laws of gases, by Eddington[1] and the more recent modified analysis by Jeans[2] have added evidence of considerable weight that in these stars we are dealing with some form of pulsation. It must be admitted, however, that there are still certain difficulties in the way of the simple pulsation hypothesis.

In the present state of the subject further observational data of the highest attainable precision are of prime importance, and it was for the purpose of obtaining such data that this investigation was undertaken. The author has endeavored to determine the precise form of the curves representing the variation in radial velocity with time for four of the brighter Cepheids, *δ Cephei*, *η Aquilae*, *ζ Geminorum*, and W *Sagittarii*, from observations secured with high dispersion and according to methods which, it is believed, eliminate as far as possible the systematic errors arising from instrumental and other causes. A comparison of these velocity curves with those determined with high dispersion spectrographs by earlier observers should furnish conclusive evidence concerning the apparent changes in the forms of the velocity curves of Cepheids, found by several investigators on the basis of observations made with one-prism spectrographs. Moreover, the spectrograms obtained afford an excellent basis for studying the question of change in width of the spectral lines with the phase of light variation reported by some observers. As a related problem, a critical investigation has been made of the displacements given by different groups of lines which presumably refer to very different absorption levels in the stars' atmospheres, and which certain investigators have found to give velocity curves differing in amplitude, form, and phase.

PART I. INSTRUMENTS AND METHODS

The majority of the radial velocity observations secured for the present investigation were made with the new Mills spectrograph, mounted on the 36-inch refractor. This instrument has been described by Campbell,[3] and it is sufficient to note here only a few of its characteristic features. The three 60° flint prisms are set for minimum deviation at λ 4500, and with a camera of 21 inches focal length give a linear dispersion at λ 4500, of 11 A to the mm. Careful tests by the author and others have shown that this spectrograph is free from the effects of differential flexure in different positions of the telescope. The principal parts of the spectrograph are surrounded by a light wooden case, the temperature within which is kept constant during the night's work by means of a heating current, automatically controlled by a thermostat. With a slit width of 0.0015 inch and using Seed 30 plates it was found possible to secure well exposed spectrograms of *δ Cephei*, *η Aquilae*, and *ζ Geminorum* throughout their light variation. To obtain satisfactory spectrograms of W *Sagittarii* at minimum light and under ordinary conditions of seeing, it was necessary to use a slit width of 0.002 inch and the rapid Seed 60 plates.

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[1] *M. N.*, **79**, 2, 1918.

[2] *M. N.*, **85**, 797, 1925.

[3] *Stellar Motions*, page 47, et seq.

Since the lines in Cepheid spectra are sharp and on the present spectrograms are in excellent focus throughout the region measured, it was found advantageous to use the highest available magnifying powers in all the measuring instruments. Most of the radial velocities depend on measurements with the Hartmann spectrocomparator. In the few cases when the Gaertner machine was used they depend upon about 35 star lines, on the average.

Although the spectra of the Cepheids investigated do not differ much from the solar spectrum in the region measured, it has been found that measures can be made with greater accuracy by using a standard spectrogram of each star with which to compare all other spectra of that star. For this reason a standard spectrogram of each of the four Cepheids investigated was taken at a point intermediate between maximum and minimum of the radial velocity curve, where the spectral type would be near the mean.

Systematic Errors in Radial Velocity Determinations

An investigation which has as its object the determination of individual radial velocities with the highest degree of precision, in order that these may serve as a basis for studying the exact forms of velocity curves or apparent changes in the forms of these curves, must of necessity give serious consideration to the elimination of every possible source of systematic error by which such observations may be affected. In addition to those considered by other observers, arising from mal-adjustment of the spectrograph or from errors either in the process or method of measuring the plates, several other possible sources of such errors have been investigated, and their probable amounts, where present, have been evaluated. Although this work was merely incidental to the main problem, it is considered here in some detail since certain results of this special investigation will be of interest to all radial velocity observers.

(*a*) *Effect of photographic density.*—In order to investigate what effect the photographic density of the spectrograms has on the measured positions of the lines or blends, the residual radial velocities for individual lines from the mean of all the lines were listed and compared—(*a*) for three complete measures of the same sky plate of low density, two of them several months apart; (*b*) for two similar measures of a sky plate of high density; and (*c*) for two underexposed and one overexposed plate of the Moon's spectrum. This investigation led to the following conclusions:

(1) There is good agreement between the residuals for individual lines from different measurements of the same plate. The agreement is much better than between mean residuals for the same lines on plates of different photographic density. Thus a plate is always measured very nearly in the same way.

(2) For individual blends the effective exposure is of some importance. The range of displacements of corresponding blends on plates of high and low density is 2 or 3 km./sec. on the average. Many lines keep their position undisturbed by varying density.

(3) The effective exposure, however, has little effect on the mean position of all the lines or blends, about as many being displaced one way as the other on spectrograms having very different photographic densities.

(*b*) *Blend effect and spectral type.*—On reducing measurements of correctly exposed spectrograms of the two K type stars, *α Aquarii* and *α_2 Capricorni*, with the new Mills reduction table, based on solar wavelengths, the largest residuals for individual lines are greater (about 7 km./sec.), than from sky and planet plates. Evidently there is a change of wavelength of some of the blends with spectral type. For work on mean radial velocities the assumption was made that with the small range of spectral type exhibited by any one Cepheid the number of blends which have their wavelengths increased is about equal to that for which they are decreased, so that there is no systematic effect of variable spectral type on the mean radial velocity. This assumption is in harmony with the results found by Albrecht[4] in his measures of the radial velocity of the Cepheid l *Carinae*. A determination for each spectral type of the difference between the radial velocities of a great number of stars given by the original and the new Mills spectrographs is also in accordance with this assumption, although it is not a confirmation of it. From the values quoted below it will be seen that the variation for one type is small. Since the spectral regions of the two spectrographs are different the effect of blending due to spectral type can not well be large in either one.

Spectral type	Original *minus* new Mills
G	−0.19 km./sec.
K	−0.27
M	−0.38

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[4] *Ap. J.*, **54**, 163, 1921.

(*c*) *Effect of chromospheric absorption level.*—According to Mitchell's[5] values for the heights in the chromosphere to which individual lines extend the mean level in the Sun for 49 lines listed in the new Mills reduction table ordinarily used for the radial velocities is 583 km. The mean level for 47 lines listed in the original Mills table and most commonly measured on the earlier spectrograms of η *Aquilae* and ζ *Geminorum* is 580 km.

Thus for the purpose of the comparisons to be made later in this paper the original and new Mills radial velocities may be assumed to refer very nearly to the same effective absorption levels.

(*d*) *Curvature of the spectral lines.*—Since the slit of the new Mills spectrograph is ground so as to make its image in light of wavelength 4500 A accurately a straight line and since about as many lines are measured on one side of this wavelength as on the other, the mean curvature correction for the whole spectrum is zero. A simple computation shows that the omission of lines at one end of the spectrum, as, for instance, by the failure to measure 7 lines at the violet end on underexposed plates, introduces no appreciable (<0.019 km./sec.) curvature correction.

(*e*) *Guiding error.*—The importance of this error has been pointed out by a number of investigators. Without exception all flexure and check plates were obtained by allowing the star image to drift centrally along the slit as seen through a deep blue glass. In work on the variable stars the correcting lens was adjusted before starting each exposure by means of the blue glass so that the visual and blue images were coincident, and the guiding was then done by visual light. This adjustment was frequently checked during the exposure, especially when the star was observed at a large zenith distance.

(*f*) *Effect of flexure.*—It has been known since 1920 that there is a systematic difference between the new Mills radial velocities obtained from spectrograms exposed with the telescope east, and from those exposed with the telescope west of the pier. An examination of the differences Tel. E. *minus* Tel. W. for 18 plates of various objects, taken for checking the spectrographic adjustments, but not planned particularly for bringing out the effect, showed that this is one of the most important systematic errors existing in the spectrograph. When the differences E–W for 16 plates, taken close to the meridian, with the same, or very nearly the same, slit width, and exposed especially for its determination, were tabulated in order of zenith distance, it appeared that the effect is very small at the zenith, changes sign there, and increases numerically with the zenith distance. Since the E–W effect is also noted on the spectrograms of the planets and other extended surfaces, it is obviously not due to atmospheric refraction, but to a change in collimation of the spectrograph arising from flexure in the telescope tube.

If the optical parts of the spectrograph are not perfectly homogeneous, it is clear that the flexure effect should vary in a complicated manner with zenith distance, parallactic angle, slit width, and the optical non-homogeneity. To determine its value an approximate formula may be developed. Let it be assumed that the flexure of the telescope varies as the sine of the zenith distance, and that the flexure displacement of the star spectrum perpendicular to the slit is proportional to the component of flexure in this direction. Then, since the slit of the spectrograph may be considered parallel to the equator, the component of flexure perpendicular to the slit in any position of the telescope, f, will be given by

$$f=f_0 \sin z \cos q=f_0 (\cos \delta \sin \varphi-\sin \delta \cos \varphi \cos t) \ldots (1),$$

where

f_0 = the maximum amount of the flexure effect for any particular slit width; i.e., the effect when the telescope is in the meridian, east of the pier, and pointed to the south point of the horizon,

Z = zenith distance,
q = parallactic angle,
φ = latitude of observer,
δ = declination of the object,
and t = hour angle of the object.

The effect of flexure is zero at all points in the sky defined by the curve $\sin z \cos q=0$. This curve is the locus of the points where the declination circles intersect the vertical circles at right angles. It is approximately a circle, symmetrically placed with respect to the meridian, and passing through the zenith and the celestial pole.[6]

That this formula gives a fair approximation to the truth, is shown by the splendid reduction of the probable errors, caused by its application, not only to the observations for flexure, but also to the observations in general.

In practice the effect of flexure has usually been eliminated by observing the stars at equal hour angles east and west of the meridian, with the telescope on opposite sides of the pier for the two observations. Since it is not always possible, or convenient, to secure two such observations, it seems very desirable to be able to eliminate the effect of flexure from a single observation.

Since each observation furnishes an equation of the form

$$f_0=f \operatorname{cosec} z \sec q \ldots\ldots (2),$$

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[5] *Ap. J.*, **38**, 407, 1913.

[6] *Meddelanden, Lund*, **13**, 15, No. 19.

it is clear, that the best determination of the constant, f_0, by observation, can be obtained from stars at as large zenith distance, and as close to the meridian as possible. As the zenith distance decreases, the proportion of accidental error to flexure effect increases. From these considerations it was decided to assign weights to the observations proportional to $\sin z \cos q$. The reversal of the telescope is supposed to affect only the sign of f_0. For the short exposures under consideration the flexure displacement has been assumed to be that corresponding to the hour angle of mid-exposure.

The method of observing finally adopted and used throughout for the 1923–25 flexure plates was to observe bright stars with good lines close to the meridian with the telescope west of the pier. The instrument was then reversed and a similar observation taken with it on the east side. During such a complete pair of observations there was little or no temperature change, the slit widths were identical, and the conditions of seeing and exposure usually the same. The two plates were then developed at the same time in the same bath, and later measured and reduced by identical methods, 35 to 44 lines being generally used on each plate. In the 1924 series no pairs were formed by combining plates taken on different nights, to avoid the effect of change in observing conditions.

The observational data secured for the determination of the flexure effect, and its variation with slit width, rest upon six pairs of plates taken in 1920 by Moore and Thiele, and twelve pairs obtained by the writer in 1923 and 1924. These data are collected in Table I, in which the first four columns give, respect-

TABLE I

Data for the Determination of the Flexure Effect for the new Mills Spectrograph

Date of observation G.M.T. (observed)	Object	Position of telescope	Width of slit in .001 in.	δ	t (arc)	f (in km./sec.)	f_0 (in km./sec.)	Authority for E–W
1924, June 18.949	Mars	W.	0.5	−15° 39′	−24° 27′	−0.14	−0.18	
.986	Mars	E.	0.5	−15 39	−11 12			
26.939	Mars	W.	0.8	−15 8	−23 14	−0.30	−0.38	
.971	Mars	E.	0.8	−15 8	−11 45			
23.973	Mars	W.	1.0	−15 18	−12 30	−0.30	−0.38	
.992	Mars	E.	1.0	−15 18	− 5 30			
1920, Apr. 28.845	Mars	W.	1.4	− 9 36	+12 45	−0.44	−0.62	Mean of A.M.H., W.W.C., J.H.M.
.875	Mars	E.	1.4	− 9 36	+23 45			
21.818	β Crv	W.	1.4	−22 51	+16 0	−0.30	−0.36	A.M.H.
.869	β Crv	E.	1.4	−22 51	+33 45			
27.855	π Hya	W.	1.4	−26 12	+11 45	−0.38	−0.44	A.M.H.
22.897	π Hya	E.	1.4	−26 12	+22 20			
22.956	α Sco	W.	1.4	−26 13	+ 8 0	−0.68	−0.78	A.M.H.
21.954	α Sco	E.	1.4	−26 13	+ 5 45			
.012	δ Sgr	W.	1.4	−29 52	+ 0 45	−0.41	−0.45	A.M.H.
22.007	δ Sgr	E.	1.4	−29 52	− 0 15			
1923, July 16.970	ι Cep	W.	1.5	+65 40	+ 0 36	+0.40	−0.80	
17.005	ι Cep	E.	1.5	+65 40	+12 51			
20.909	α Aqr	W.	1.5	−12 49	− 6 36	−0.38	−0.44	
.953	α Aqr	E.	1.5	−12 49	+12 14			
24.957	α Aqr	E.	1.5	−12 49				
1924, June 13.990	Mars	W.	1.5	−16 4	−12 42	−0.52	−0.64	
14.006	Mars	E.	1.5	−16 4	− 6 57			
July 2.966	Mars	W.	2.0	−14 53	− 9 0	−0.55	−0.70	
.984	Mars	E.	2.0	−14 53	− 3 0			
25.916	Mars	W.	2.0	−15 12	− 7 39	−0.68	−0.85	
.930	Mars	E.	2.0	−15 12	− 2 39			
27.968	Mars	W.	2.0	−15 20	+13 8	−0.22	−0.28	
.984	Mars	E.	2.0	−15 20	+18 38			
June 20.984	Mars	W.	2.0	−15 30	−10 42	−0.84	−1.07	
.998	Mars	E.	2.0	−15 30	− 5 27			
Aug. 2.910	Mars	W.	2.0	−15 48	− 1 12	−0.62	−0.77	
.924	Mars	E.	2.0	−15 48	+ 3 48			
June 25.852	δ Sgr	W.	2.0	−29 52	+ 5 45	−0.54	−0.60	
.932	δ Sgr	E.	2.0	−29 52	+34 15			

ively, the Greenwich Mean Time of observation, the name of the object, the position of the telescope east or west of the pier, and the slit width of the spectrograph in thousandths of an inch. The fifth and sixth columns contain the declination of the object and its hour angle at mid-exposure. In the seventh and eighth columns are given half the observed differences E–W and the flexure constant f_0 computed from them by (2). Considering the accidental error in a single determination the agreement of the values of f_0 for any particular slit width is rather satisfactory. Except where indicated in the last column the plates were measured by the author. The initials here indicate the following persons: A.M.H., Miss A. M. Hobe; W.W.C., W. W. Campbell; J.H.M., J. H. Moore.

The weighted mean of f_0 for each slit width is plotted in Figure 1.

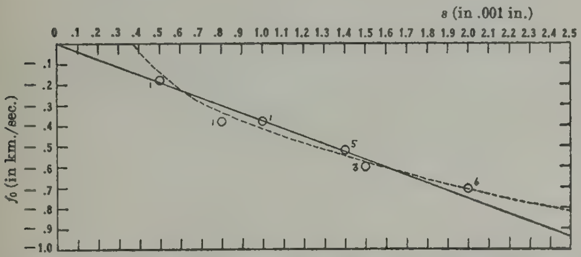

Fig. 1. Variation of flexure effect with slit width. Each circle represents a weighted mean of observed values. The number attached to each indicates the number of independent determinations entering into the mean. The full line represents a simple first approximation. The broken curve is suggested by theory.

As a first approximation the variation in the constant f_0 with slit width may be represented by a straight line through the origin whose equation is

$$f_0 = -0.375\,s \qquad (3),$$

in which, if s is the slit width expressed in thousandths of an inch, f_0 will be given in km./sec.

Regarding f_0 as a flexure correction the equation

$$f_0 = 0.375\,s \qquad (4)$$

will give its value for any observation taken with the telescope east of the pier. With the telescope west of the pier the correction is of the opposite sign.

Whether the relation between flexure effect and slit width is strictly a linear one is difficult to determine from the few observations available at present. Although of little practical importance, as a slit narrower than 1.0 or wider than 2.0 is rarely used, it is, however of considerable theoretical interest to determine the shape of the curve more precisely. The variation in the proportion of diffracted to non-diffracted light in the spectrograph as the slit width is changed can be surmised from a curve expressing the loss of light by diffraction for different slit widths, determined by Moore.[7] This curve shows, that with a very wide slit diffraction plays only a subordinate rôle in the image formation in the spectrograph, while, as the slit is gradually narrowed, its importance becomes relatively greater and increases very rapidly for slit widths smaller than 0.7. For this reason it is to be expected that the flexure effect will begin to decrease rapidly at slit width 0.7, that it will practically disappear before an infinitely narrow slit is reached, and that it will approach more and more to a constant value as the slit is widened. This is indicated by the broken curve in Figure 1.

As a "by-product" from the flexure determination, the systematic correction, which, by its algebraic subtraction, will reduce observed radial velocities to theoretical spectrographic velocities, was obtained. This correction refers strictly to the mean of a pair of observations, one being taken in the position Tel. E., the other, Tel. W.; but since the flexure effect seems to be present to about the same extent on both sides of the pier, it is probably also the one to be applied to any single observation. The application of this systematic correction will of course remove effects due to erroneous wavelengths in the reduction table, personal equation of measurement, etc., but its application to all late type spectra involves the assumption considered in section (*b*). The values of O−C (observed *minus* computed radial velocity) for the *Mars* observations, before and after correction for the effect of flexure are given in Table II.

The ten plates with slit width 2.0 afford the most homogeneous set of data for the determination of O−C. This determination refers strictly to plates measured on the Gaertner machine. But the measurement of two planet and four Moon plates both on this and on the Hartmann machine gave a mean difference of $G_n - H_n = +0.06$ km./sec. The smallness of this result in combination with the general agreement of the values given by the two machines for the Hartmann standard plates, shown in a later section, permitted the use of the same systematic correction to measures by the two. Consequently, the final adopted value, which was applied to all measured radial velocities, is, for the new Mills spectrograph, −1.09 km./sec.

(*g*) *Effect of differential flexure.*—The effect of differential flexure is merely a broadening of the spectral lines, which, in the extreme possible case; namely, of an exposure at the celestial pole starting on the meridian and lasting for six hours, can never amount to more than

$$f_0 \cos \varphi.$$

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[7] *L. O. B.*, **3**, 44, 1904, curve E.

TABLE II

Determination of O−C from Mars Series

a. With flexure

Position of instrument \ Slit width (in .001 in.)	0.5	0.8	1.0	1.5	2.0
W.	+1.66	+1.49	+1.36	+1.82	+2.00
E.	+1.37	+0.89	+0.77	+0.79	+0.76
W.					+1.27
E.					+0.83
W.					+2.29
E.					+0.92
W.					+1.28
E.					+0.18
W.					+1.55
E.					−0.14
				Mean	+1.09±1.14

b. Flexure eliminated

Position of instrument \ Slit width (in .001 in.)	0.5	0.8	1.0	1.5	2.0
W.	+1.51	+1.25	+1.06	+1.37	+1.40
E.	+1.52	+1.13	+1.07	+1.24	+1.36
W.					+0.67
E.					+1.43
W.					+1.69
E.					+1.52
W.					+0.69
E.					+0.77
W.					+0.96
E.					+0.45
				Mean	+1.09±0.38

That the actual broadening found in practice is not greater than that predicted by theory has been proven by measuring the widths of lines on plates of *δ Cephei* taken at very different hour angles, exposed to nearly the same photographic density, and referring to very closely the same phase in the star's variation.

Determination of the Radial Velocities for Hartmann Standards

When spectrograms of the sky, Moon or planets are used as Standards, with reference to which stellar spectrograms are measured on the Hartmann spectrocomparator, the radial velocity corresponding to the displacement of the absorption lines on the standard plate is usually assumed to be the theoretical relative velocity of the object and observer. Experience has shown that various systematic errors in the determination of stellar radial velocities have been introduced by such a procedure. An extreme case is afforded by the spectrogram of the sky, T 1923, June 27, used as a Hartmann standard in the present investigation. The plate was taken with the spectrograph out of collimation, through failure properly to tighten the bolts by which the spectrograph is attached to the telescope. Although the theoretical relative velocity of the Sun and observer at the time of observation was only +0.19 km./sec., measures of this spectrogram in the usual way by Mr. Kholodovsky, Mr. Pearce, and the writer gave the following results:

Sky Plate T 1923, June 27

Radial velocity	Measured by
+4.07 km./sec.	Kholodovsky
+3.70	Pearce
+3.71	Jacobsen
+3.73	Jacobsen
+3.59	Jacobsen
Mean = +3.76	

As a further check upon the radial velocity given by this spectrogram, the plate was measured on the Hartmann spectrocomparator against a standard sky plate, whose constants were well determined, with the following results:

+3.78 km./sec.
+3.76
Mean = +3.77

The value of +3.76 km./sec. was adopted for this spectrogram.

The following are the individual measures of the plates of *δ Cephei*, *η Aquilae*, and W *Sagittarii*, which were used as standard reference plates for the measurement of the spectrograms of these stars. In the case of the *δ Cephei* and *η Aquilae* standards two of the measures were made with reference to a spectrogram of the Moon, whose velocity had been determined by comparison with the standard sky plate. To these measures half weight has been assigned.

Plate 12827 O, *δ Cephei*

Radial velocity	Machine	Weight
−29.94 km./sec.	Gaertner	
−30.67	Gaertner	
−30.84	Hartmann	
−30.35	Hartmann	½
−30.00	Hartmann	½
Weighted mean = −30.41		

Plate 12811 Q, *η Aquilae*

Radial velocity	Machine	Weight
−29.92 km./sec.	Gaertner	
−30.34	Gaertner	
−29.39	Hartmann	
−30.49	Hartmann	½
−29.83	Hartmann	½
Weighted mean = −29.95		

Plate 12786 W, W *Sagittarii*

−26.72 km./sec.	Gaertner
−27.09	Gaertner
−26.33	Hartmann
Mean = −26.71	

Since no planets were in favorable position in 1923 for checking the spectrographic adjustments, a number of spectrograms were secured of stars whose radial velocities had been previously determined on the regular Mills program. The comparison of the 1923 radial velocities with the earlier values gave satisfactory evidence that the systematic correction to be applied to the 1923 measurements is the same as that for 1924.

Part II. The Observations

For the sake of uniformity all times of observation of the variable stars have been reduced to heliocentric times by the well known formula

$$\text{Helioc. time } minus \text{ Obs. time} = -0.0058 \cos\beta \cos(\lambda - \odot) \qquad (5)$$

in which the unit is the mean solar day.

It has been found by experience that practically all of the observations which fall markedly off a mean observed curve determined by the total number of observations depend upon spectrograms taken under the condition of large temperature changes in the spectrograph during the exposure. In only a few cases is the opposite true; namely, that an observation affected by a large temperature change fits the curve well. Accordingly, the following system of assigning weights to the observations has been adopted:

Temperature change	Weight
0 to 0°05 C	1
0°06 C to 0.10	½
>0.10	0

In this system of weighting an observation of weight zero is to be regarded, not as being of no interest at all, but as one, which, although not dependable, may be expected to give an approximate value of the radial velocity.

δ Cephei

α (1900) $22^{h}\ 25^{m}4$; δ (1900.0) +57° 54′.

In Table III are collected the data for the radial velocity observations of *δ Cephei*, obtained in 1923 and 1924 with the new Mills spectrograph. Column 1 contains the Greenwich Mean Time (heliocentric) of the observation. The second column gives the phase of the observation reckoned from the epoch J.D. 2417888.428 (epoch of periastron), using Meyermann's[8] period 5.366404 days, which is based upon a discussion of the light observations extending over a period of many years.

TABLE III

Radial Velocities of δ Cephei

Date of observation G.M.T. (helioc.)	Helioc. phase from periastron passage, by Meyermann's period	Radial velocity	Weight
		km./sec.	
1923, Aug. 3.954	$0^{d}108$	−24.35	
14.842	0.323	−32.65	0
July 14.005	0.624	−34.80	
Aug. 20.876	0.930	−32.52	
July 8.957	0.943	−32.31	
Aug. 31.889	1.211	−30.18	
July 19.970	1.223	−30.15	
Sept. 27.811	1.301	−27.95	0
1924, July 2.909	1.346	−28.16	
1923, Oct. 24.735	1.393	−28.06	
Dec. 1.835	1.928	−20.55	0
July 9.955	1.941	−23.31	
20.873	2.126	−20.53	
21.001	2.254	−19.93	
31.963	2.483	−17.96	
Oct. 9.902	2.659	−14.58	
Sept. 23.874	2.730	−14.37	
July 10.936	2.922	−12.49	
21.992	3.245	−10.74	
Sept. 13.826	3.415	− 9.14	
Nov. 6.620	3.545	− 6.62	
July 11.993	3.979	− 3.66	
Oct. 16.800	4.190	− 1.19	
July 22.990	4.243	− 0.11	½
Aug. 2.977	4.497	+ 1.11	½
13.990	4.777	+ 4.71	
Oct. 22.822	4.846	+ 6.14	0
July 12.990	4.976	+ 2.99	
Nov. 2.767	5.058	+ 3.16	
Oct. 1.683	5.173	− 4.32	
July 7.932	5.284	−11.95	0

Column 3 contains the radial velocities as obtained by the process described earlier in this paper. The weights in column 4 are assigned in accordance with the system explained above.

The observations are plotted in Figure 2.

The heavy curve drawn among the observations is based on the assumption of elliptic motion. Its coördinates are derived from the first, second, and last columns of Table IV. The 1923–24 observations were plotted with the same epoch and period as the 1907–08 series. The broken line shows the exact position with respect to my observations of an elliptic curve deduced by Moore[9] from his observations in 1907–08, after a systematic correction has been applied to it. It is at once obvious, that by a small adjustment of the systemic velocity and the period, this curve, derived

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[8] *A. N.*, **167**, 3, 1904.

[9] *L. O. B.*, **7**, 153, 1913.

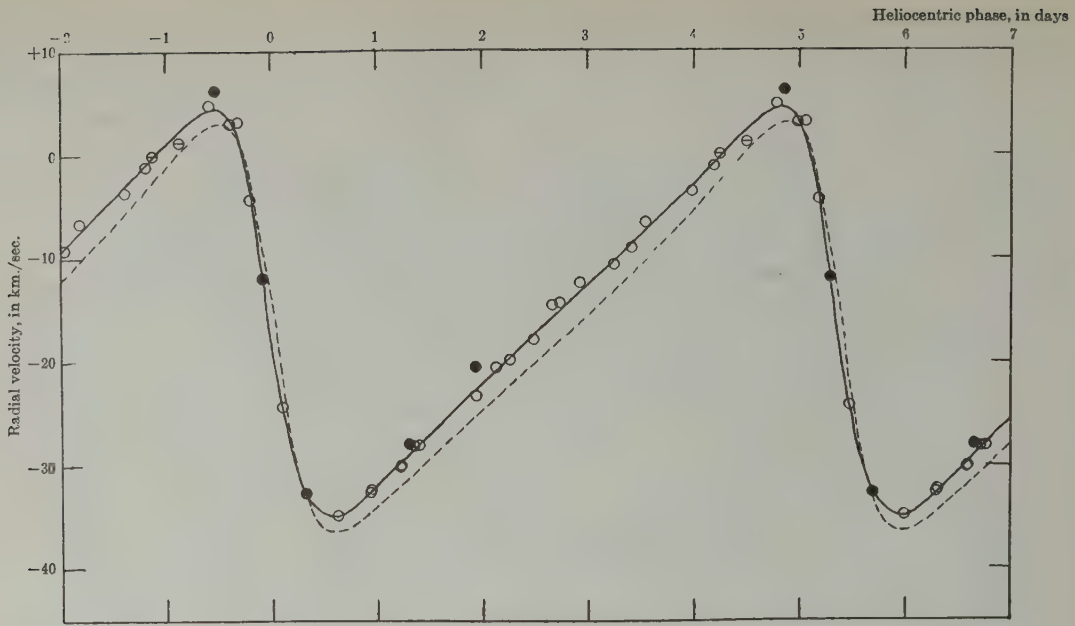

Fig. 2. Radial velocity curve of *δ Cephei.* The open, barred, and full circles represent single observations with the new Mills spectrograph, of weight 1, ½, and 0, respectively.

from elliptic motion, will give a striking representation of my radial velocity observations. The apparent change in the velocity of the system is so small, that it may possibly be due to an unexplained systematic difference between the two sets of observations.

There is, however, some evidence of a real change in the velocity of the system. The best available material for an investigation of its reality was a spectrogram of *Venus* and one of the Moon, both of good quality, taken just before and just after the earlier *δ Cephei* series. On measuring and reducing these by methods identical with those used for the 1923 plates of *δ Cephei,* the values (O−C) measured *minus* computed velocity were −0.55 and −0.48, mean = −0.52 km./sec. By treating in the same way five plates belonging to the earlier series of *δ Cephei* and well distributed over the radial velocity curve, the following values of Jacobsen *minus* Moore measured velocities were found.

Phase d	Jacobsen *minus* Moore km./sec.
0.009	−0.47
0.774	−0.65
2.026	−0.20
3.219	−0.45
4.892	−1.21
Mean =	−0.60

The close agreement of this with the mean for the two check plates indicates that both observers measure in the same way, and that the systematic difference is a purely instrumental one. It is known that in 1907–08 the collimator was not set, as it is now, to make the beam of light falling on the collimator lens concentric with the lens when the telescope was pointed to the zenith, but when it was on the east side of the pier, and pointed to the Sun. Since two-thirds of the plates of the earlier series were exposed with the telescope east of the pier and one-third west of the pier, with a rather narrow slit, no considerable smoothing out of the radial velocity curve can be effected by a reduction of the individual observations for flexure. The reduction would be more like a constant correction whose value, it is believed, would not differ greatly from the one found above. It is of interest to note in this connection that Moore has found that a systematic correction of −0.50 km./sec. should be applied to the value of the Hartmann standard plate used by him and therefore to his observed velocities of *δ Cephei* in order to reduce them to the present Lick Observatory system of velocities. Hence, before plotting the broken curve above, its ordinates were diminished algebraically by 0.60 km./sec. The apparent periods of earlier observers tacitly assume a constant velocity of the system. If P denotes the apparent period, P_0 the true period, and V_0 the velocity of the system, we have

$$P=P_0\left(1+\frac{V_0}{300000}\right).$$

It is of interest to see how great a change in the apparent period will be produced by a small change in the velocity of the system. Differentiating the above expression, we have,

$$d\,P = \frac{P_0\ dV_0}{300000}.$$

Assuming the velocity of the system to change uniformly, which is a sufficient approximation for the comparisons made in this paper, the lengthening of the apparent period in 16 years can be found approximately by multiplying dP by the number of periods elapsed. Since, on this assumption, changes occurring in the apparent periods are exactly duplicated in the true periods, the custom of using apparent periods has been retained.

A superposition of the two curves shows a change in velocity of system of +1.8 km./sec. in the sixteen years covered by the observations. The spectroscopic apparent period which will superpose the two curves is $5^{d}366331$, and refers to the epoch 1916.0 with the velocity of the system assumed increasingly uniformly. It is interesting to note in this connection that according to Chandler[10] the period of this star is certainly shortening. Using Hertzsprung's[11] formula for the period, $5^{d}3663770 - 0^{d}000000916\ (t-1883)$, the value for 1916 is found to be $5^{d}3663468$, which is very close to the spectroscopic period. The computed average rate of change in apparent period due to the observed increase in systemic velocity is $+0^{d}00000198$ per year. Considering Hertzsprung's value for the period in 1908 correct, the average spectroscopically observed rate of shortening in later years is $0^{d}00000088$ per year. Thus the actual average rate of shortening in later years is $0^{d}00000286$ per year.

In order to determine whether there was an appreciable change in form of the velocity curve in the interval of 16 years, the recent observations were used as a basis for a re-determination of the orbital elements. Observations affected by large temperature changes were excluded and the others combined into 19 normal places, the data for which are given in Table IV. The fourth column of this table contains the residuals from the curve computed from Moore's elements, after applying +1.7 km./sec. to his value for the velocity of the system, while the last column gives the values of O−C computed from the elements resulting from a least squares solution.

TABLE IV

NORMAL PLACES AND RESIDUALS FOR δ CEPHEI

Helioc. phase from periastron passage	Radial velocity	Weight (assigned for least squares solution)	O−C (original)	O−C (final)
	km./sec.		km./sec.	km./sec.
$0^{d}128$	−24.35	½	+1.60	+0.10
0.644	−34.80	½	−0.40	+0.24
0.957	−32.42		−0.27	+0.18
1.238	−30.16		−0.66	−0.44
1.394	−28.11		−0.12	−0.01
1.961	−23.31	½	−0.68	−0.83
2.210	−20.23		+0.11	−0.09
2.504	−17.96	½	−0.30	−0.53
2.716	−14.48		+1.25	+1.01
2.942	−12.49	½	+1.15	+0.87
3.265	−10.74	½	−0.16	−0.33
3.436	− 9.14	½	−0.25	−0.37
3.567	− 6.62	½	+0.95	+0.87
3.999	− 3.66	½	−0.74	−0.59
4.238	− 0.65		−0.45	−0.03
4.518	+ 1.11	½	−1.78	−1.22
4.798	+ 4.71	½	0.00	+0.49
5.038	+ 3.08		+1.20	+0.78
5.194	− 4.32	½	+0.89	−0.85

By the least squares adjustment the weighted sum of the squares of the residuals is reduced from 8.787 to 4.658.

The final elements for the 1923 solution are given in the last column of Table V, while those for 1907 are quoted in column two for comparison.

TABLE V

SPECTROSCOPIC ELEMENTS OF δ CEPHEI

	Moore, 1907		*Jacobsen*, 1923	
U (assumed)	$5^{d}366404$		$5^{d}366331$	
μ (assumed)	$67^{\circ}08403$		$67^{\circ}08495$	
T	J.D. 2417888.428	$\pm 0^{d}009$	J.D. 2417888.484	$\pm 0^{d}012$
ω	$85^{\circ}385$	$\pm 1^{\circ}048$	$89^{\circ}153$	$\pm 1^{\circ}214$
e	0.484	±0.006	0.487	±0.006
γ	−16.83 km./sec.	±0.26 km./sec.	−15.64 km./sec.	
K	19.675 km./sec.	±0.158 km./sec.	19.750 km./sec.	±0.173 km./sec.
r		±0.59 km./sec.		±0.49 km./sec.

The last entry in the table is the probable error of a single observation of weight unity, or, in the 1923 solution, of a normal place of half weight.

Before comparing the two sets of elements, Moore's value of the velocity of the system, γ, should be corrected by −0.60 km./sec. Besides the change in period the only changes in the elements that are large enough to be considered real are an increase of $3^{\circ}8$ in the longitude of periastron, ω, with a corresponding change in the epoch of periastron passage, T, and an increase of 1.8 km./sec. in the systemic velocity.

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[10] *Ast. Jour.*, **16**, 161, 1896.

[11] *A. N.*, **210**, 21, 1919.

η Aquilae

α (1900.0) $19^h\ 47^m.4$; δ (1900.0) $+0^\circ\ 45'$.

The following radial velocity observations have been made recently with the new Mills spectrograph:

TABLE VI

Radial Velocities of η Aquilae

Date of observation G.M.T. (helioc.)	Helioc. phase from maximum, by Schur's[12] elements	Radial velocity	Weight
		km./sec.	
1923, July 2.963	0^d033	−29.57	
Oct. 18.626	0.050	−29.96	
July 24.913	0.454	−31.47	
Nov. 2.661	0.733	−30.61	
July 10.937	0.831	−30.82	
1925, Nov. 26.629	1.181	−29.85	
1923, Sept. 6.810	1.293	−28.80	
Aug. 30.853	1.522	−26.32	
Sept. 28.748	1.702	−25.10	
July 11.933	1.827	−24.19	
1923, July 4.943	2.013	−22.83	
Aug. 2.917	2.282	−23.39	0
July 12.947	2.841	−18.01	
Oct. 28.609	2.857	−16.58	½
1925, Dec. 5.616	2.991	−18.33	
1923, Aug. 3.897	3.262	−15.87	
Sept. 30.743	3.697	−14.66	0
23.770	3.900	−13.60	0
July 6.948	4.018	−14.25	
Sept. 16.747	4.053	−13.90	
Oct. 15.641	4.242	−12.94	½
July 21.925	4.642	− 6.51	½
14.958	4.852	− 3.67	
Aug. 19.918	4.930	− 2.59	
June 30.889	5.136	− 0.17	½
Oct. 16.628	5.229	+ 1.64	
Aug. 20.824	5.836	+ 9.10	
1925, Dec. 8.621	5.996	+ 8.76	½
1923, July 1.975	6.222	+ 5.63	
Oct. 10.706	6.483	− 1.86	
1925, Dec. 15.618	6.817	−15.80	
1923, July 9.962	7.026	−24.85	

The observations are plotted in Figure 3.

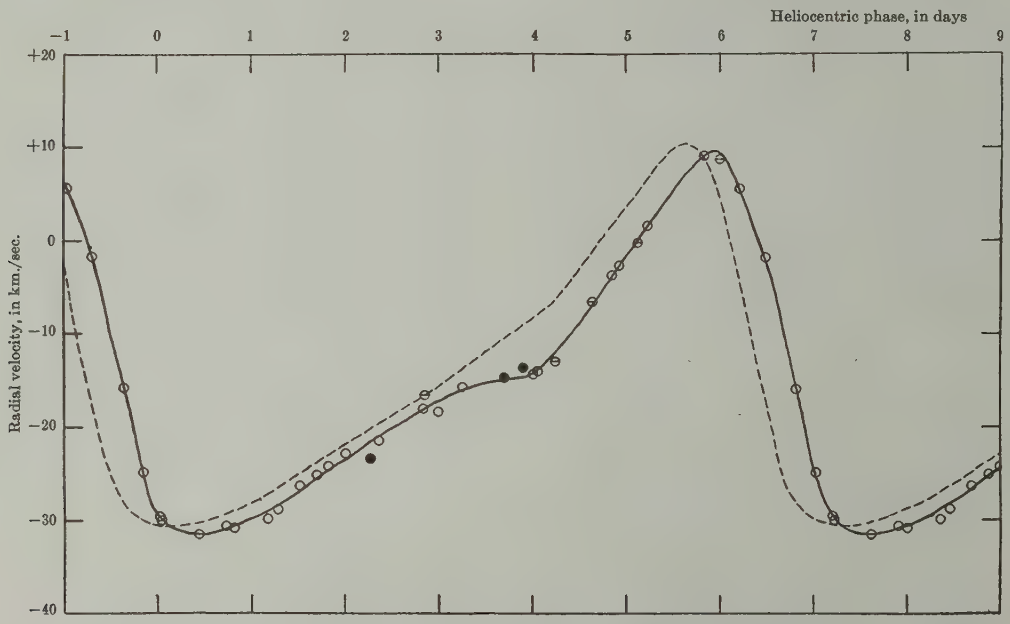

Fig. 3. Radial velocity curve of η *Aquilae*. The open, barred, and full circles represent single observations with the new Mills spectrograph, of weight 1, ½, and 0, respectively.

The full line drawn among the plotted points is an observational curve. The broken line shows the exact position of an elliptic curve used by Wright[13] to represent his series of observations taken in 1898. This curve, derived from elliptic motion, is also as good an observational curve as one could desire, except at phase 3^d773, where one of his points falls markedly below the curve. Within the errors of observations the two curves are of identical shape at the top and bottom and along the descending branch. The portions of the ascending branches between phases 0^{d}5 and 4^{d}5 differ considerably in shape. Due to the sparsity of observations in this part of the earlier curve and to the fact that the one outstanding observation at phase 3^{d}773 falls almost exactly on the 1923 curve when the two series are superposed, it is probable that the indentation shown by the 1923 curve existed also in 1898. Attention was first called to its existence by Rufus.[14] If we assume the shapes of the observational curves identical, the spectroscopic apparent period which will superpose the two was found to be 7^{d}176659, for epoch 1911.0, with the systemic velocity assumed uniformly decreasing. The observed change of −0.95 km./sec. in the systemic velocity, if real, will cause the period to be shortening at the average rate of 0^{d}000,000,92 per year. In connection with this determination of period it should be noted that Wylie,[15] from a photometric study of this star, has found a lengthening of the period in later years, and that his value between 1906 and 1920 is 0^{d}00004 longer than my value between 1898 and 1923. According to Miss Clerke[16] the period has probably slightly increased since 1784. Hellerich[17] also finds it increasing.

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[12] *A. N.*, **137**, 323, 1894.
[13] *Ap. J.*, **9**, 59, 1899.

ζ GEMINORUM

α (1900.0) 6^h 58^{m}4; δ (1900) +20° 43′.

A preliminary discussion of the radial velocity curve recently constructed for this star has already been published by the writer.[18] The individual radial velocities are contained in the following table:

TABLE VII

RADIAL VELOCITIES OF ζ GEMINORUM

Date of observation G.M.T. (helioc.)			Helioc. phase from minimum, by Chandler's[19] elements	Radial velocity	Weight
				km./sec.	
1923,	Mar.	21.759	0^{d}281	+ 6.17	0
		11.637	0.313	+ 5.87	
	May	1.680	0.587	+ 3.54	
	Mar.	12.644	1.324	+ 1.51	
		23.635	2.157	+ 0.64	
	Oct.	23.031	2.323	− 0.77	½
	Apr.	13.695	2.909	− 1.10	
	Mar.	14.631	3.307	− 2.87	
	Nov.	23.936	3.766	− 4.90	
	Mar.	25.653	4.175	− 5.68	
	Nov.	4.059	4.197	− 5.47	
	May	15.694	4.447	− 4.38	
	Mar.	5.668	4.498	− 5.41	0
		16.677	5.353	+ 0.87	
		27.665	6.187	+ 8.91	0
	Apr.	27.669	6.730	+12.16	
	Mar.	28.621	7.145	+14.37	
		18.660	7.336	+14.77	
		29.656	8.176	+20.67	½
	Apr.	19.677	8.891	+21.73	
	Mar.	20.736	9.412	+18.58	0
	Apr.	30.677	9.738	+13.14	0

The observations are plotted in Figure 4.

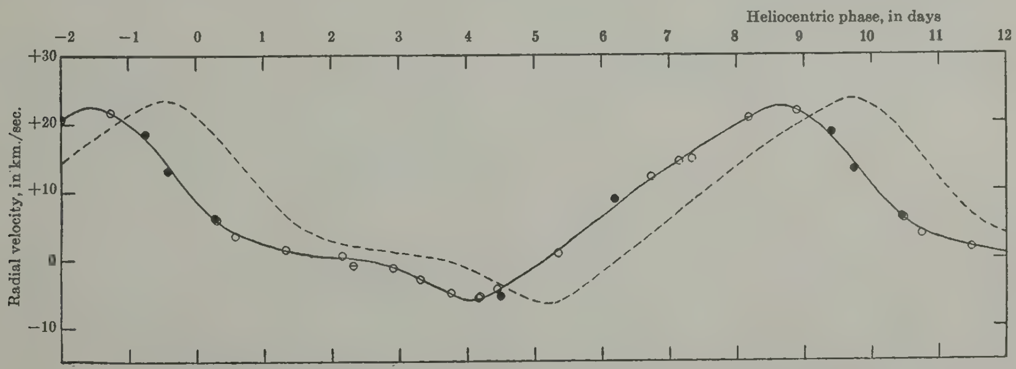

Fig. 4. Radial velocity curve of ζ *Geminorum*. The open, barred, and full circles represent single observations with the new Mills spectrograph, of weight 1, ½, and 0, respectively.

In figure 4 the heavy line is an observational curve for 1923 while the broken line shows the exact position of Campbell's[20] curve for 1899, both being plotted with Chandler's[19] epoch and period. By displacing one series of observations parallel to the time axis until the best possible fit is produced, the residuals of four adjacent 1923 observations near the point of greatest velocity of recession from Campbell's curve are positive; while those from five adjacent observations near the point of greatest velocity of approach are negative. If this is not fortuitous or due to some unexplained systematic error, it indicates a small decrease in the amplitude of the curve of not over one-half km./sec. in twenty-three years. Hence, the following conclusions are legitimate: (1) There is no change in the velocity of the system. (2) The period for the interval covered by the observations is shorter than 10$^{\mathrm{d}}$15382, given by Chandler. The spectroscopic apparent period which will superpose the two radial velocity curves is 10$^{\mathrm{d}}$15253, for epoch 1911.0, referred to a constant systemic velocity of +6.8 km./sec.

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[14] *Proc. N. A. S.*, **10**, 265, 1924.

[15] *Ap. J.*, **56**, 228, 1922.

[16] *The System of the Stars*, page 133.

[17] *A. N.*, **222**, 25, 1924.

[20] *Ap. J.*, **13**, 90, 1901.

[18] *A. S. P.*, **35**, 255, 1923.

[19] *Ast. Jour.*, **16**, 148, 1896.

The value of the period deduced differs a little from the one already published[18] on account of the preliminary character of the earlier result. In connection with any suspected change in period it should be noted that Henroteau's[21] recent spectroscopic observations of this star confirm the shorter period found. Several light observers, however, find a lengthening of the period. Miss Clerke[16] states that the period lengthened by ten minutes between 1847 and 1890. More recently Guthnick gives for the light period 10$^{\mathrm{d}}$15457.

THE WIDTH OF SPECTRAL LINES ON NEW MILLS SPECTROGRAMS OF ζ GEMINORUM

The widths of fourteen lines in the best exposed part of the spectrum were measured on all plates of *ζ Geminorum.* In order to make these comparable with each other they were all reduced to a standard photographic density by a correlation between width and photographic density derived from the plates themselves. The measured relation can be well represented by

$$y=40-1.8(x-5),$$

y being the average width, expressed in km./sec., and x the photographic density on an arbitrary scale running from 1 (underexposed) to 10 (overexposed).

The results are shown in Figure 5, in which the left and right vertical broken lines are, respectively, the points of maximum and minimum ionization as determined by Henroteau.[21] The left and right bars across the broken curves are respectively the points of maximum and minimum light as determined from Guthnick's[22] recent light curve.

The evidence for a variation of line width with phase is very slight, but definite. Part of it is undoubtedly due to the difficulty of assigning photographic densities. In the few cases where a reassignment was attempted the effect has been to smooth out the curve. The remainder of the phenomenon is undoubtedly due to a variation in intensity of the spectral lines because the curve for the spark lines shows a tendency to reach a minimum near the point of minimum ionization, and that representing the arc lines at maximum ionization.

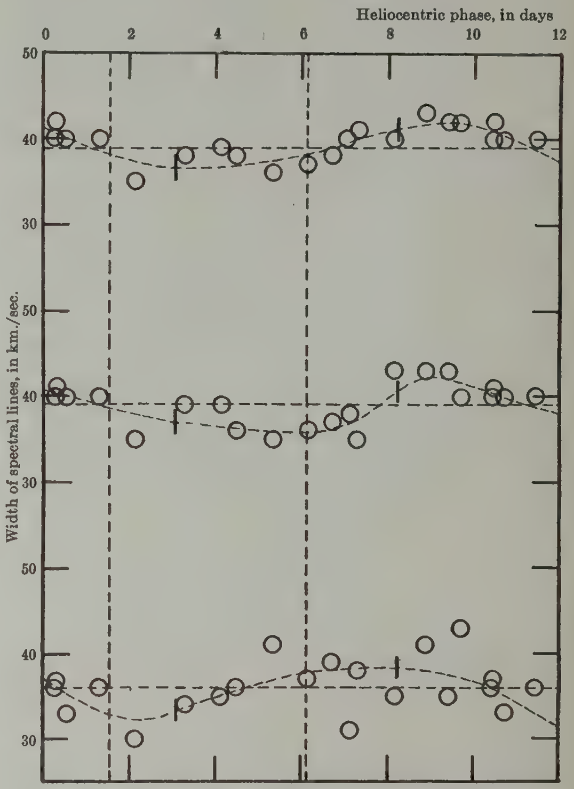

Fig. 5. Variation in width of spectral lines in *ζ Geminorum* with phase. Each circle in the upper part of the figure represents the mean width of 14 lines, 10 of which are spark lines, and 4 arc lines. The variation in mean width of the spark lines is shown in the middle curve, that of the arc lines in the lower curve.

A shading of the spectral lines, particularly on spectrograms taken at maximum and minimum light was also made the object of a careful search. Although the plates of *ζ Geminorum* have been measured on both the Gaertner and Hartmann machines for radial velocity, and on the Gaertner for width, and the effect has been looked for independently on the Toepfer; no general shading could be definitely detected. Pronounced shadings of individual lines were indeed noted, but these could be satisfactorily explained by known blends. According to Shapley and Nicholson[23] these results can be reconciled with the pulsation theory.

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[21] *Publ. Dom. Obs. Ottawa*, **9**, 111, 1925.

[22] *A. N., Jubilaeumsnummer*, 11, 1921.

[23] *Proc. N. A. S.*, **5**, 423, 1919.

W SAGITTARII

α (1900) $17^{h}\,58^{m}.6$; δ (1900) $-29^\circ\,35'$.

The following radial velocity observations have been obtained with the new Mills spectrograph:

TABLE IX

RADIAL VELOCITIES OF W SAGITTARII

Date of observation G.M.T. (helioc.)	Helioc. phase from maximum, by Chandler's[24] elements	Radial velocity	Weight
		km./sec.	
1923, July 4.836	$0^{d}344$	−41.04	
12.838	0.752	−37.22	
June 27.854	0.957	−37.84	½
12.927	1.219	−33.91	
Aug. 12.754	1.289	−35.34	½
14.743	3.278	−29.24	
Sept. 6.702	3.448	−29.14	0
Aug. 30.719	4.060	−20.02	0
July 8.851	4.359	−16.17	
1.860	4.963	− 7.74	½
June 16.907	5.199	− 5.47	
24.901	5.598	− 1.74	
Aug. 9.767	5.897	− 3.94	
July 2.860	5.963	− 6.12	½
June 25.868	6.565	−30.43	
10.868	6.755	−37.09	
Sept. 2.712	7.053	−41.98	
July 26.740	7.059	−42.57	½
1924, June 3.894	1.239	−33.73	0
4.880	2.225	−28.35	½
12.915	2.666	−27.04	½
20.871	3.027	−28.33	0
May 29.914	3.854	−20.10	0
30.895	4.835	− 7.94	
June 8.925	6.270	−17.64	0
1925, Aug. 2.738	0.786	−38.88	0
3.730	1.780	−33.23	0
July 19.794	2.031	−31.20	0
12.865	2.697	−26.40	0
20.760	2.997	−27.58	0
6.828	4.255	−16.70	
7.830	5.257	− 6.25	½
8.804	6.231	−14.04	½
9.836	7.263	−39.62	

The weights in column 4 are assigned as before, except for the observation of 1924 May 29.914 at phase $3^{d}854$. On the temperature basis alone this plate would have weight one, but the star spectrum is so underexposed that the quality of the lines is seriously impaired. The plate was consequently assigned low weight. The observations are plotted in Figure 6.

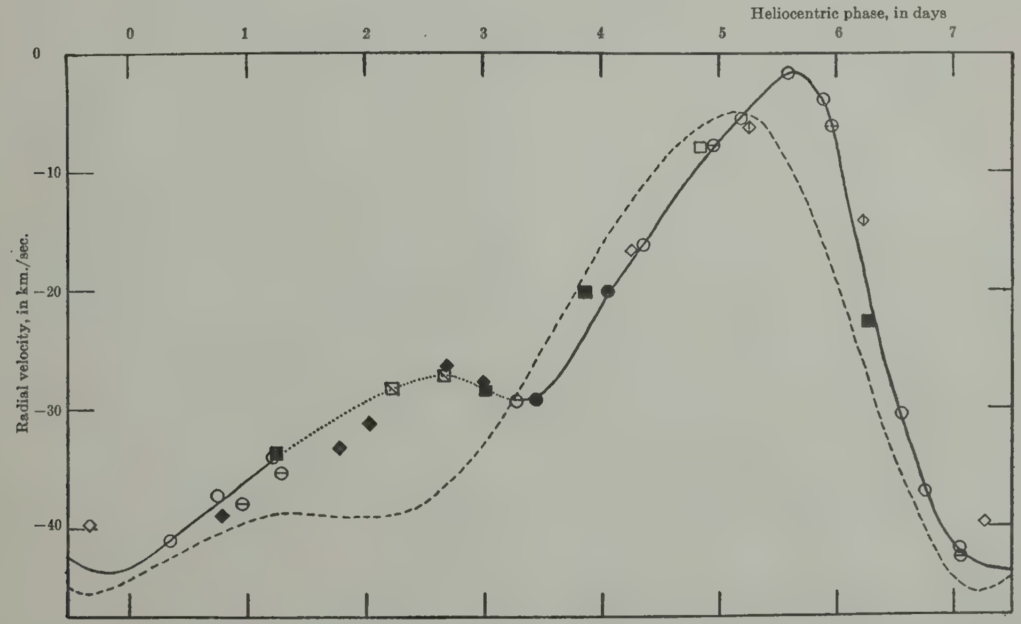

Fig. 6. Radial velocity curve of W *Sagitarii*. Single observations with new Mills spectrograph. Open, barred, and full figures represent observations of weight 1, ½, and 0, respectively. Circles, squares, and diamonds represent observations taken in 1923–24–25, respectively.

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[24] *Ast. Jour.*, **16**, 157, 1896.

The full line in the figure is an observational curve drawn through the 1923 observations except the dotted part between phases $1^{d}3$ and $3^{d}2$, which was obtained in 1924. The broken line shows the exact position of a curve used by Curtiss[25] to represent his observations made in 1903. It is the result of superposing a sine curve of half the period upon a curve representing elliptic motion, and it is a faithful expression of what we know about the observational curve of 1903.

Since the shapes of the two curves differ very materially, it is not possible to determine the change in period or in systemic velocity by superposition, as in the cases treated earlier in this paper; but it will be seen from the relative position of the two curves in Figure 6 that it can not have changed greatly,—if at all, it has probably lengthened. The extreme amplitude of the variation has remained constant within the errors of observation. An increase of 3.6 km./sec. in the velocity of the system is indicated. Without a special investigation of the systematic errors of the one-prism instrument used in 1903, it is difficult to decide whether this is real or not.

While it is possible that variations in blend effect with changing spectral type cause part of the differences in shape revealed by the comparison of curves constructed with one and three prism instruments, this difference is in the present case so large as to leave little doubt of its reality. In all cases where comparisons between three prism observations have been made, however, the observed changes in shape are so small, that their reality has not been proved conclusively by the observations.

THE DISPLACEMENTS OF INDIVIDUAL LINES

A new method of attacking the Cepheid problem by sorting out distinct absorption levels in the stars' atmospheres has already led to interesting results in the hands of several investigators.[26] Not only do the curves for different levels show pronounced differences of amplitude, shape, and phase, but the phenomenon has been given a plausible physical interpretation. Because of the importance of this new method of analyzing radial velocity curves, it seemed desirable to check the observational results with other instruments of the highest practicable dispersion, and a choice of lines which will as far as possible avoid blend effects with changing spectral type.

An inspection of Mitchell's[5] list of heights of lines in the solar chromosphere indicates that no great difference of level exists between heights of lines recorded on the new Mills spectrograms. But in the spectral region further toward the violet a number of good lines extending to great heights in the Sun's atmosphere and many low and intermediate level lines could be photographed on the same plate. Consequently, a three-prism spectrograph for the violet region, having a dispersion of 21 A pr. mm. at λ 4200 A, was constructed during the summer of 1925. The prism box was built of wood, and, although the instrument had no temperature control, plates of excellent definition were obtained even with exposures as long as four hours and temperature changes of the air immediately surrounding the prism box of several degrees Fahrenheit.

From the known optical constants the correction for the curvature of the spectral lines was computed. It was then independently determined by measurement of a sky plate taken with long slit and reduced to the standard slit width. The two methods agreed in showing that to a very close approximation the values of the correction, x, expressed in km./sec., could be represented by the equation

$$x = -3.66 - 0.0030\,(\lambda - 3940) \ldots\ldots\ldots\ldots (5).$$

Here x is the correction to be added algebraically to the measured position of any spectral line, whose wavelength, λ, is expressed in Angstrom units.

Some of the sharpest lines in the spectrum were selected for measurement and grouped as follows:

Group 1, high level, >2000 km., λλ 4077, 4101, 4215, 4226, and 4340 A; mean level in Sun = 6600 km.

Group 2, intermediate level, between 500 and 2000 km., λλ 4012, 4013, 4014, 4034, 4045, 4063, 4071, 4118, 4163, 4177, 4191, 4202, 4312, 4318, and 4383 A; mean level in Sun = 777 km.

Group 3, low level, $\leqq$500 km., λλ 4096, 4157, 4171, 4210, 4213, 4218, 4219, and 4369 A; mean level in Sun = 475 km.

The mean level in the Sun to which all these 28 lines refer is 1730 km.

As a beginning it was planned to take only a few plates of each star, one at maximum and one at minimum, and also a plate in the middle of the ascending or descending branch of the radial velocity curve. Two excellent plates of each of the non-variables *α Persei* and *β Ceti*, stars with c-characteristics, were obtained for comparison. In the following table the third and fourth columns contain the differences of the radial velocities given by the groups listed at the heads of the columns.

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[25] *L. O. B.*, **3**, 35, 1904.

[26] *Pop. Ast.*, **32**, 218, 1924; *Proc. N. A. S.*, **10**, 264, 1924, or abstract *Pop. Ast.*, **32**, 228, 1924; Abstract *Pop. Ast.*, **32**, 547, 1924.

TABLE X

Velocity Differences of Groups of Lines Referring to Different Chromospheric Levels

Star	Phase from max.	High *minus* low	Intermediate *minus* low	Remarks
		km./sec.	km./sec.	
α Persei........		−1.8	+4.7	Spectral type F_5
α Persei........		+1.0	+6.1	Spectral type F_5
Mean......		−0.4	+5.4	Spectral type F_5
β Ceti...........		+5.2	+7.2	Spectral type K
β Ceti...........		+3.0	+5.4	Spectral type K
Mean......		+4.1	+6.3	Spectral type K
δ Cephei........	$0^{d}21$	+3.0	+6.1	
δ Cephei........	0.68	+0.4	+5.5	
δ Cephei........	2.53	+5.5	+8.8	
δ Cephei........	3.52	+1.1	+1.8	
δ Cephei........	3.56	+3.6	+8.2	
δ Cephei........	4.78	+0.8	+5.1	
η Aquilae......	0.24	−2.0	+5.9	
η Aquilae......	2.09	−0.7	+4.6	
η Aquilae......	2.35	−1.6	+3.6	
η Aquilae......	6.09	+3.6	+5.4	
η Aquilae......	6.23	+6.2	+5.1	Underexposed
η Aquilae......	7.15	−4.0	+2.7	

The residuals for *α Persei* and *β Ceti* are perhaps due to the fact that the blends measured in these stars have wavelengths which differ from the solar wavelengths used in the reduction. Inasmuch as the values for the Cepheids are of the same order, all or a large part of any variation with phase shown may possibly be attributed to the known variation in spectral type of these stars. The velocity differences for *δ Cephei* show no relation to phase. In *η Aquilae* there seems to be a high *minus* low effect of the nature found by Rufus.[27] According to Table X the result is in the direction to be expected from the spectral variation of this star. The present results are, however, so preliminary that the conclusions drawn may easily have to be changed or modified when more results with high dispersion spectrographs are at hand.

General Discussion

The general phenomena of Cepheid variation are so well known and the different hypotheses which have been advanced to interpret the observed facts have been so frequently discussed that it is not necessary to recall them here. It may be of interest however to call attention to one phase of the subject with regard to the pulsation hypothesis which is concerned with the results of the present investigation.

The closeness with which three-prism observations of approximately solar type Cepheids, taken at random during many successive cycles of variation, fit an observational curve indicates that these stars show very regular variation in radial velocity. That the probable error of a single observation is slightly larger than the smallest obtained from spectroscopic binaries of solar type, is to be expected on account of the variation in spectral type of the Cepheids. Practically, all outstanding discrepancies in their radial velocity curves for any one season can be explained as arising from instrumental causes. The best light curves produced by the most accurate instruments at present available are no less smooth than the radial velocity curves.

Moreover, the velocity curves for three Cepheids determined from observations made with high dispersion at epochs differing by from sixteen to twenty-five years, show little change in form. In *δ Cephei* a very small change appears to be indicated corresponding to a rotation of the line of apsides in the ellipse representing orbital motion. It is interesting that Jeans[28] has predicted a slow rotation of the line of apsides.

The periods of variation for these stars are apparently changing at a slow rate. The velocity of the system of one of them, *δ Cephei,* is probably variable. In the case of W *Sagittarii* the apparent change in form of the velocity curve rests upon a comparison of the one-prism observations made in 1903 with those obtained with three-prism dispersion twenty years later. Although these changes are large a careful analysis of the blending of lines with variations of spectral type in the one-prism instrument used is necessary to establish their reality beyond doubt.

The regularity of the Cepheid phenomenon would not present any grave obstacle to a pulsation theory if the radial velocity curves had a simple form. The pulsation theory indicates roughly the shapes of these curves, but it is as yet far from giving a quantitative representation of them. Eddington[29] has shown that the radial velocity curve of a pulsating star may be approximately represented by an equation of the form

$$a_1 \cos (nt) - 2\, a_2 \sin (2\, nt).$$

This may be regarded as a good second approximation. Harmonic analysis of some of the best determined radial velocity curves will show how far this approximation goes. For *ζ Geminorum,* Plummer[30] has published a fine twenty-four ordinate analysis of Campbell's observational curve. Twelve ordinate analysis, however, is sufficient to bring out the main features of such a curve. The following table contains the results of

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[27] *Proc. N. A. S.,* **10**, 264, 1924, or abstract *Pop. Ast.,* **32**, 228, 1924.

[28] *M. N.,* **65**, 811, 1925.

[29] *M. N.,* **79**, 183, 1919.

[30] *M. N.,* **73**, 661, 1912.

Plummer's analysis as well as those for twelve ordinate analyses of Curtiss' 1903 curve for W *Sagittarii* and three of my observational curves. Following Plummer's notation the curves are expressed by equations of the form

$$V = V_0 + \sum r_j \sin (j\, \mu t + \gamma_j).$$

The amplitudes, r_j, and phase angles, γ_j, for harmonics of order j are given in Table XI.

TABLE XI

Constants for Harmonic Analysis of Radial Velocity Curves

	ζ Gem		δ Cep		η Aql		W Sgr 1903		W Sgr 1923–24	
j	r_j	γ_j	r_j	γ_j	r_j	γ_j	r_j	γ_j	r_j	γ_j
1	12.6	111°	16.0	181°	15.6	190°	17.2	223°	14.3	225°
2	2.9	167	6.9	179	8.9	221	8.5	−28	9.5	−68
3	1.3	107	3.7	182	3.7	265	0.9	14	2.8	11
4	0.2	135	1.9	187	1.9	−80	1.3	58	1.7	50
5	1.3	93	0.9	203	1.2	−39	0.8	229	1.2	166
6	0.1	236								
7	0.8	272								
8	0.2	248								
9	0.5	91								
10	0.1	225								
11	0.3	280								

All amplitudes are expressed in km./sec. The values of V_0 for the five curves are +7.0, −15.6, −14.5, −29.8, and −26.2 km./sec., respectively. These four stars give a fair idea of the most extreme curves that have to be represented in connection with Cepheid variables. It will be noted, that in each case the terms given by Eddington are the most important ones, but also, that the curves are highly complex, so that a great number of terms, probably five or more, must be included, to get a representation comparable with the accuracy with which the curves are determined by observation. The radial velocity curve of δ *Cephei* presents an outstanding difficulty for the simple pulsation theory. On the assumption that this curve is the result of a number of harmonics operating simultaneously, not less than five, corresponding to ten constants, are necessary to define it quantitatively. On the assumption of elliptic motion, however, an excellent quantitative representation of the observations involves only five constants. Other radial velocity curves are not so simple, but Curtis[31] has shown, that in most cases the part outside the "hump" can be fitted well to a curve representing elliptic motion.

It is seen from Table XI that the observed change of shape in the radial velocity curve of W *Sagittarii* corresponds to a weakening of the first and a strengthening of all the higher harmonics up to the fifth. The phase of the first harmonic with respect to the whole curve is practically unchanged, while all the higher harmonics have developed a phase lag. If this is a physical reality, it indicates not only that the radial velocity curve is complex, but also, that it changes in a more complicated manner than the simple pulsation theory now in vogue has as yet been able to account for.

Summary of Results

Systematic errors which affect determinations of radial velocities have been investigated and in particular the one arising from change in the collimation of the spectrograph due to flexure of the telescope tube.

The radial velocity curves of four Cepheid variables have been re-determined with three-prism dispersion and compared with the best curves for these stars determined at earlier epochs. From this comparison the following conclusions have been drawn:

(1) In the case of the three Cepheids for which the radial velocity curves of the earlier and later epoch were determined from three-prism observations little or no change has been found in the form of the curves.

(2) The apparent difference in shape found in the velocity curve of W *Sagittarii* based upon comparing one-prism with three-prism observations may perhaps be partly accounted for by variation in blend effect

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[31] *Proc. N. A. S.*, **9**, 187, 1923.

with phase. It is, however, so large as to be probably real.

(3) In the interval 1908 to 1923 the average spectroscopic apparent period of *δ Cephei* was $5^{\mathrm{d}}.366331$, the longitude of periastron had increased by $3^{\circ}.8$, and the velocity of the system by 1.8 km./sec.

(4) Between 1898 and 1923 the average spectroscopic apparent period of *η Aquilae* was $7^{\mathrm{d}}.176659$.

(5) From 1899 to 1923 the average spectroscopic apparent period of *ζ Geminorum* was $10^{\mathrm{d}}.15253$.

These results indicate a shortening of the period of *δ Cephei* and a lengthening for *η Aquilae*, the numerical amounts of which check very well with the values derived from photometric observations. The shortening of the period found for *ζ Geminorum* is confirmed by other spectroscopic but not by photometric observations.

Measurements of the widths of spectral lines and search for a shading of lines in *ζ Geminorum* indicate that variation in width with phase is small, and that general shading does not exist.

An investigation of the so-called atmospheric pulsation effect, based upon the radial velocities given by lines of different levels, shows no effect of this character for *δ Cephei* and only slight evidence of its existence in *η Aquilae*. These results, however, should be considered as only preliminary.

In conclusion I wish to acknowledge my great indebtedness, first, to Director W. W. Campbell, who made my work here possible by putting at my disposal all the instrumental equipment needed; secondly, to Dr. J. H. Moore, whose advice and constant interest in the progress of this investigation has been invaluable to me; and finally, to several persons who have taken some of the spectrograms or who have offered helpful criticism.

April 9, 1926.
Issued September 3, 1926.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 380

A PROPOSED SPECTROSCOPIC METHOD FOR DETERMINING THE ABSOLUTE LUMINOSITIES OF CLASS A STARS

By

CH'ING-SUNG YÜ

One of the most notable advances in astrophysics made in recent years is unquestionably the application of spectral analysis to determine the absolute brightness and distances of stars. This ingenious method, the spectroscopic parallax method as it is now known, was originated by Adams and Kohlschütter[1] and applied by them and other Mount Wilson investigators with remarkable success in deriving the parallaxes of numerous "late" type stars (F to M). Briefly outlined, Adams' method consists of an empirical correlation of the relative strength of certain spectral lines with absolute magnitude. It was found that within a spectral class, enhanced lines like λ4216 (Sr$^+$) increased in intensity, while arc lines like λ4607 (Sr) decreased, with increasing absolute luminosity. By taking stars of known absolute magnitude, computed from measured trigonometric parallax, and by estimating the intensities of certain lines in the spectra of these stars, empirical curves were constructed for each spectral class showing a definite relation between luminosity and intensity differences for several pairs of lines. The reduction curves were then used to derive the luminosities and therefore the parallaxes of other stars by estimation of the line intensities from their spectra.

Besides Mount Wilson there are several observatories which have adopted this line of research. Among them may be mentioned the Victoria, and the Norman Lockyer Observatory. At the former Young and Harper[2] elaborated the method for the late type stars by including additional pairs of lines and by improving the line intensity estimates. At the latter Edwards[3] extended the method to early B-type stars.

It is evident from the brief review above that the basis of the spectroscopic method is empirical. Its physical significance, however, was early recognized and has recently found abundant verification in the theory of ionization, first propounded by Saha[4] and later modified and improved from independent considerations by Fowler and Milne.[5] The spectrum of a star is governed by the physical conditions existing in its atmosphere. The fact that the great bulk of stellar spectra can be arranged in a simple sequence leaves little doubt that the factors governing the change are few. There is also no question that among these few, temperature is the predominating agency along the spectral sequence. Thus, in general, ionized lines of an element appear or are strengthened and arc or neutral lines are weakened or absent in stars of higher temperature which promotes ionization. Conversely, arc lines are stronger in stars of lower temperature which discourages ionization, while enhanced lines either weaken or disappear. But Saha has shown that strong ionization, and therefore the development of enhanced lines, can be obtained by low pressure as well as by high temperature. In other words, within any spectral class the temperature of which is sensibly uniform,* differences in spectral characteristics are expected to occur especially in the relative strength of enhanced and arc lines, due to the difference in absolute luminosity, which is, within the class, a simple function of pressure. Indeed this is precisely what was found by Adams and made the basis upon which his method was built. We shall have occasion to mention these theories again particularly in connection with the method to be presently proposed.

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[1] *Astrophys. Journ.*, **40**, 385, 1914; *M. W. Contr.* No. 89.

[2] *Publ. Dom. Astrophys. Obs.*, **3**, 1, 1924.

[3] *Monthly Notices*, **83**, 47, 1922; *ibid.*, **84**, 366, 1924; *ibid.*, **85**, 439, 1925.

[4] *Phil. Mag.*, **40**, 472, 1920; *Pro. Roy. Soc. Lon.*, **99***A*, 135, 1921.

[5] *Monthly Notices*, **83**, 403, 1923; *ibid.*, **84**, 499, 1924.

* This is only approximately true in late type stars, and not at all so among any sub-class of A-type stars as we shall see later.

The spectroscopic method described in the preceding paragraphs can not in general be applied to Class A stars. The principal reason, as given by Adams and Joy,[6] is that stars of this type do not show the large variations in the intensities of the lines which appear in the later type. While this is no doubt true, inasmuch as the small variation of line intensities tends to reduce the accuracy, we seem to have evidence that the failure of the usual method for Class A stars can be attributed to a disturbing factor which is potently at work in this group of stars, namely, the large temperature range within any sub-class. It is clear that this variation of temperature will tend to mask the intensity-magnitude relation. It is of interest to note that the effect of temperature of the different classes of the late type stars was taken care of by treating each class separately. Any possible dispersion of temperature within each class was, however, left out of consideration, although it is well known now that a giant star is as much as ten per cent cooler than a dwarf star of the same spectral type. The reason that this variation of temperature did not play havoc with the usual method for late type stars is probably that the temperature itself is low and the variation too small to be serious in its effect. Such is certainly not the case with the Class A stars. At any rate might not the accuracy of the method be increased when the temperature of each individual star is taken into account?

Whether the consideration of temperature will make the original method applicable to Class A stars can not be said at present. But in view of the scarcity of suitable lines and their small range of intensity variation, it seems clear that a solution of the problem can be more hopefully attempted by seeking new criteria. One thing is certain, however, that is that any spectroscopic method to be attractive, or at least theoretically sound, must be compatible with the theory of ionization and the theory of stellar evolution.

Confronted with the difficulties just mentioned, Adams and Joy[7] devised another method for determining the absolute magnitudes of A-type stars, and later extended it to include those of B-type.[8] The method, in brief, consists of a correlation between absolute magnitude and spectrum, the magnitude being brighter the earlier the spectral type. For this reason, it is essential to classify spectra accurately into subdivisions. It was also found that stars whose spectral lines are sharp are consistently brighter than those of the same type whose lines are diffuse. But the spectral type factor is by far the more important one. The reduction curves were obtained from the study of the *Taurus* and the *Ursa Major* clusters. The method evidently leaves much to be desired. It is not surprising to find that the variation of absolute magnitude with type exists, for since the types were classified according to the strength of certain lines, they may well also be criteria for absolute luminosity, but the assumptions involved are incompatible with both the theory of stellar evolution and the theory of thermal ionization. The simple linear relation between absolute magnitude and type implies that there is no dispersion in absolute magnitude within any spectral subclass,—a point at variance with the theory of stellar evolution. This has already been pointed out by Shajn.[9] The relation also assumes that the temperature variation which is known to exist within subclasses has no influence on the spectral characteristics,—a fact in serious discord with the theory of ionization. It must be admitted that the correlation is probably true for stars belonging to the moving clusters on which the reduction curves were based, and in general correct for other stars taken as a group, for in the latter case the dispersion of temperature will cancel out in the mean. But it seems doubtful whether the simple correlation can be applied with safety to each individual star. The moving clusters which have an immediate common origin are evidently on the main sequence of stellar evolution. They therefore are probably not representative of other stars in general.

Lindblad,[10] from a spectrophotometric study of the intensity of the continuous spectrum, discovered another criterion of absolute luminosity for stars of spectral type A. The relative intensity of the two adjacent regions λ3895–λ3907 and λ3907–λ3925 was found to vary with the absolute magnitude. Since the variation as ascribed by Lindblad[11] is certainly dependent to a considerable extent on the intensities of some arc lines of Silicon and Iron in the region λ3895–λ3907, and since the Mount Wilson classification was based on the increased strength of the metallic arc lines, it is possible that this method and that of Adams and Joy have much in common. It is significant, however, to note that in Lindblad's method, his reduction curve, instead of being a smooth curve like that of Adams and Joy, consists of a series of lines covering an area, one for each subclass, and arranged in a progressive manner. This phenomenon clearly indicates that the temperatures of the stars must be taken into account, if a simple correlation is to be found between the intensity difference and absolute magnitude, a fact well supported by our method as will be seen presently.

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[6] *Astrophys. Journ.*, **56**, 242, 1922; *M. W. Contr.* No. 244.

[7] *Loc. cit.*

[8] *Astrophys. Journ.*, **57**, 294, 1923; *M. W. Contr.* No. 262.

[9] *Astrophys. Journ.*, **62**, 104, 1925.

[10] *Astrophys. Journ.*, **55**, 85, 1922.

[11] *Astrophys. Journ.*, **59**, 305, 1924.

While these methods for determining absolute luminosities of Class A stars may prove reliable in a general way, they can not be said to be entirely satisfactory. New criteria for these stars, especially those which have firm theoretical backgrounds, are therefore very inviting. In view of the fact that the Class A stars compose by far the greatest group of celestial objects, the need and importance of a satisfactory spectroscopic method for their absolute magnitude determination can hardly be over-emphasized.

It is the purpose of this paper to present an entirely new criterion for determining the absolute luminosities of Class A stars, although the fundamental principle involved, as it should be in all methods, is the same as in other cases, namely, ionization. It should be emphasized here that the method is to be regarded as preliminary and tentative in character. The materials on which our criterion is based are scanty. The success of the method will no doubt depend on the accumulation of more observations and also on the improvement of the estimates of the quantities affected. However, the data at hand though meagre show such an unmistakable relation between our criterion and absolute magnitude for Class A stars that it seems desirable to present the matter at this stage.

THE NEW METHOD

In a recent paper[12] entitled "On the Continuous Hydrogen Absorption in Spectra of Class A Stars," the writer, by the application of a spectrophotometric method, obtained a number of energy curves of A-type stars. These curves provided a measure of the continuous absorption which begins quite abruptly near the limit of the Balmer series lines and extends with sensibly uniform intensity far into the ultra-violet. This absorption, which can be plainly seen from the apparent intensity graph and energy curve of α *Lyrae* reproduced in Figure 1 and Figure 2 respectively, has been generally interpreted on the basis of the Bohr theory of atomic structure, as being caused by the photoelectric ionization of hydrogen atoms. It is therefore evident that the intensity of absorption, R,* at the observed series limit, for convenience expressed in terms of the background, is intimately related to the rate of photoelectric ionization of hydrogen in the stellar atmosphere, and according to the theory of ionization should be a function of both temperature and pressure. In fact it was stated in my paper cited above that R was found to vary with spectral class and therefore in a general way with temperature. But if in any spectral subclass, the temperature can be assumed to be constant, any variation in R must be attributed to the effect of pressure, and therefore absolute magnitude, assuming of course, that the stars considered have equal mass.

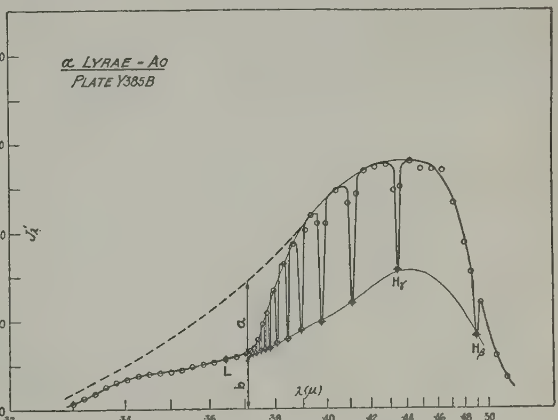

Fig. 1.—Apparent Intensity Curve

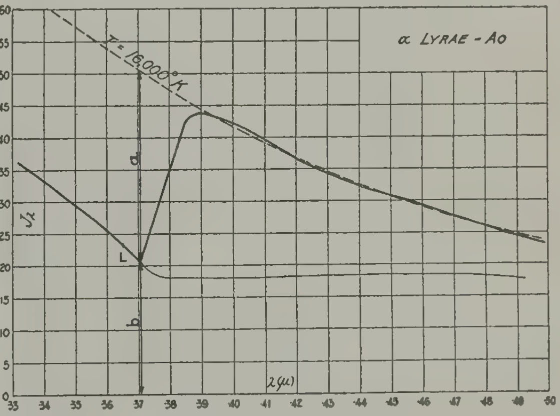

Fig. 2.—Energy Curve

In view of what has just been said, it is probable that an intensity-magnitude relation exists among Class A stars. Accordingly a careful search was made for this effect. The fundamental data for testing the relationship consist of several groups of Class A stars (B5 to F0 inclusive) for which we have spectrophotometric observations and of which the trigonometric parallaxes are more or less accurately known. These stars, numbering 39 in all, are listed among others in Column 2, Table I, and distinguished from the rest by asterisks (*). The trigonometric parallaxes were taken from Schlesinger's "Catalogue of Stellar Parallaxes" (1924 advanced copy). Where both the trigonometric and group parallaxes were available the latter was given preference.

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[12] Yü, *Lick Obs. Bull.*, **12**, 104, 1926.

* $R = \frac{a}{a+b}$ (see Figs. 1 and 2). It can be shown that $\frac{a}{a+b}$ in both figures is identical, provided of course the dotted line in Fig. 1 is drawn to correspond to the dotted line in Fig. 2. These dotted lines indicate where the stellar intensity curves would be, had the continuous absorption not existed.

By plotting the amount of absorption, R, as abscissa and absolute magnitude, M, derived from the trigonometric or group parallaxes, as ordinates, we obtained Figure 3. The symbols for the different groups of stars are indicated in the figure. It is at once evident that no simple relation exists between R and M for the entire group of A-type stars, or for any sub-type. There is some indication of a simple straight line relation in the two extreme sub-classes, B5–B9 and A5–F0. However this is rendered highly uncertain by the scarcity of material used. At any rate the A0 group, which greatly outnumbers the others in the figure, does not exhibit any systematic correlation.

Figure 3 indicates one of two things, either there is actually no simple relation existing between R and M, or else there is some other factor which has to be considered. The latter was found to be the case both from theory and observation.

It is significant to note that although the dots scatter seemingly at random all over Figure 3, those of the same subclass have a tendency to congregate on a certain area of the diagram, the earlier types falling on one section, the later ones on the opposite section, though there is considerable overlapping. Now in so far as the spectral type in general is a direct function of the temperature, this immediately gave the clue that temperature may be the new variable which has to be considered if any simple relationship is to be found between the amount of absorption and absolute magnitude.

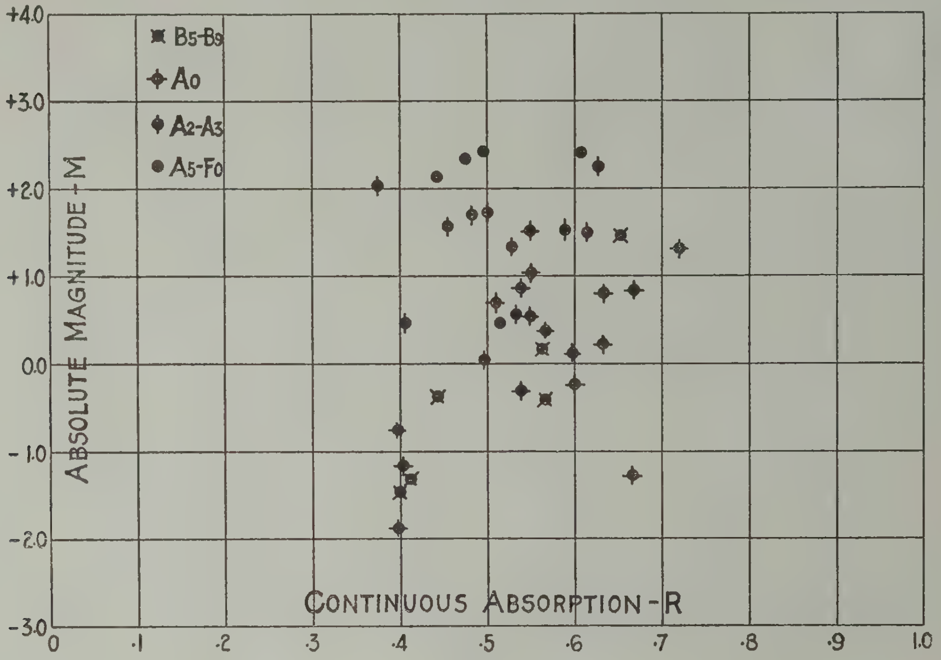

Fig. 3

The matter of taking temperature into consideration raises several points of difficulty. In the first place temperature determinations of individual stars are extremely scarce, and when they do exist, there are considerable uncertainties attached to them, especially those of early type stars. Moreover different investigators usually have different scales depending on their adopted standards. It happens that the temperature of each star in Figure 3 had been determined by the writer. These are taken directly from Table IV of my earlier paper, with a few additions and several improvements from more complete observations. Thus by attaching to each point its temperature, it became immediately apparent, as can be seen in Figure 4, that, with the exception of only a few outstanding discrepancies, there exists a gradation of temperature from one part of the diagram to another. In other words it is possible to draw in the figure a set of temperature "contour" lines as defined by the observational contour points. These lines are, as a first approximation, sensibly straight, parallel and equidistant. Their pronounced inclination to both co-ordinate axes shows that temperature is dependent both on R and M. It is clear that by including the temperature lines in Figure 4, we are introducing a new variable, T, the temperature.

The degree to which the platted data satisfy the temperature gradient, may perhaps be better seen in Figure 5, which was constructed by projecting the points on a plane perpendicular to the temperature lines, say the one whose trace is *ab* in Figure 4, and by plotting the temperature attached to each point as ordinate. The figure indicates that within errors of observation the points lie pretty well on a straight line, and this is taken as justifying the procedure in drawing a series of equidistant parallel lines in Figure 4.

It should be emphasized that the observations for the fundamental data upon which this diagram was built are none too numerous. Besides, each point in Figure 4 represents the mean of only two plates at most, one taken with the slit and the other with the slitless spectrograph. With the inclusion of more material, the inclination of the temperature lines, distances between them and their parallelism may need modification; but it is doubtful whether the change will prove serious, and there is little question but that the general features of the diagram are correct.

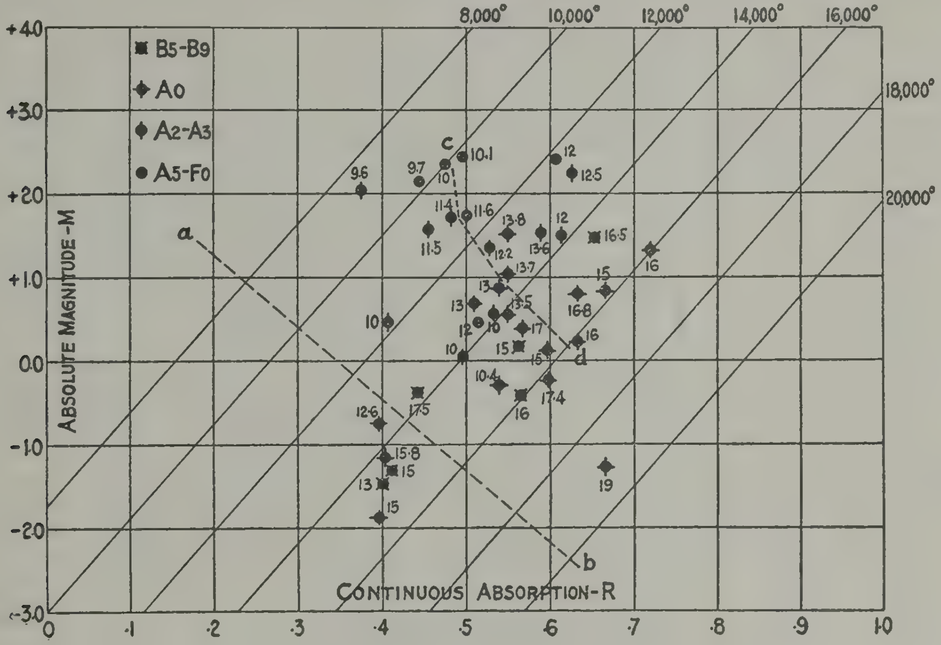

Fig. 4

Figure 4 thus constitutes the basis of the proposed method for determining the absolute luminosities of Class A stars. It involves three variables—a new feature in spectroscopic methods—two of which are independent. Knowing any two, the third is determinable. Here the continuous absorption R and the temperature T for each star were obtained from a spectrogram. These two quantities define a point in the graph, which corresponds uniquely to one value of absolute magnitude M. The diagram can also be conveniently represented by the following expression:

$$M = 11.15\,R - \frac{T}{1600} + 3.25 \qquad (1)$$

where R is expressed in percentage and T in degrees absolute. M is therefore determined by substituting R and T in the formula. In practice, however, the diagram was used as the absolute magnitude can be read off directly and more rapidly.

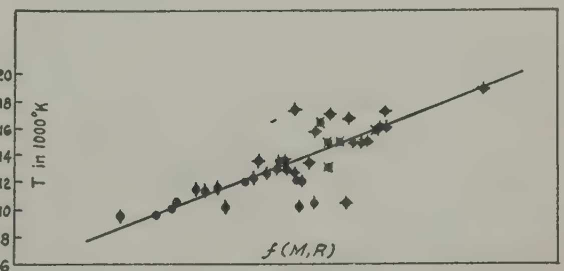

Fig. 5

Before applying the method to determine stellar parallaxes, it is important to examine more critically Figure 4 and be sure that there is no serious flaw in it. It must be admitted that observationally R and T are not quite entirely independent. Both these quantities for each star were obtained from the same plate. An error in T will bring about a corresponding error in R. As is evident from Figure 2, if the temperature T comes out too high the intensity of absorption, $R\left(=\frac{a}{a+b}\right)$, will as a consequence be too great, since the erroneous Planckian curve will be steeper and a bigger. In fact this is what is indicated in Figure 4, and offhand it would seem that the relationship of R and T exhibited there is spurious. Simple calculations, however, show that errors in T can only account for a small percentage of the variation in R as observed. The effect therefore, for the most part at least, is real.

But the strongest support for the reality of this correlation is evidenced from the identical effect presented by Figure 6, which was constructed in similar manner as Figure 4 and from the same observations except that here the intensity of the Hγ line is used instead of the intensity of the continuous absorption. For obvious reasons the slit observations alone were utilized in obtaining Figure 6. The intensity of Hγ, r, is expressed, like R, in terms of the background. Now since the background for Hγ is *observed*, in contradistinction to the continuous absorption background which is *computed* and thus subject to errors in temperature determinations, it is clear that the evaluation of r is independent of temperature. In view of the similarity in character of Figure 4 and Figure 6 it seems safe to conclude that the increase in R with the increase in T is not spurious.

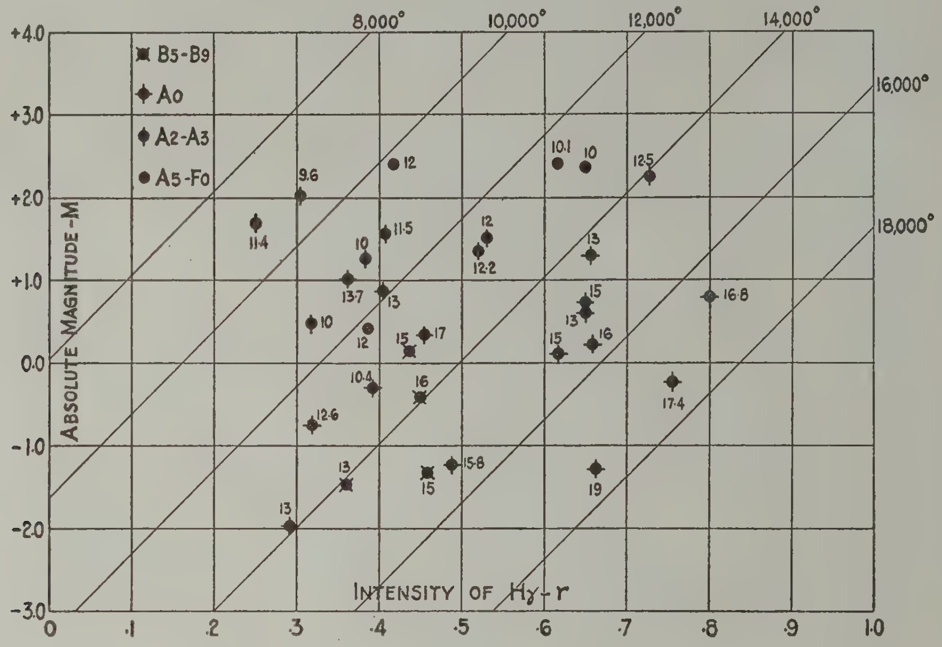

Fig. 6

It may be asked here whether the Balmer lines may not serve just as well as the continuous absorption as a criterion for the proposed method. In fact the use of lines may perhaps be better inasmuch as the line absorption shows a larger range in its intensity variation as evidenced by the further spreading out of points in Figure 6. This question can not be answered at present owing to the scarcity of slit observations which alone can be combined for line intensity treatment without introducing and involving uncertain assumptions. It is doubtful, however, whether it will be worthwhile to extend the slit observations, as too much time is required to expose fainter stars and with the special method of observation employed[13] the strenuous effort of guiding would prohibit taking them. The majority of my observations were taken with the slitless spectrograph, which therefore places much fainter and more numerous stars within reach. Unfortunately the intensity of lines here can have but little weight, for it is seriously affected by "seeing," guiding and focus, which may not be perfect.

The intensity of the continuous absorption, on the other hand, is free from these sources of error, and in the case of slit observations is independent of the slit width. Further, it is independent of instrument and dispersion. Thus the slit and slitless observations can be combined and treated together. The manifold advantage of the use of the continuous absorption over that of the lines is thus evident.

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[13] Yü, *Loc. cit.*, p. 109.

It is significant to note that the error (ΔT) in T with its accompanying error (ΔR) in R which is inherent in Figure 4 discussed above will hardly be felt in the determination of the absolute magnitude. For equation (1) shows that ΔT and ΔR, which have the same sign, are operating in opposite directions, since the terms involving R and T respectively have opposite signs.

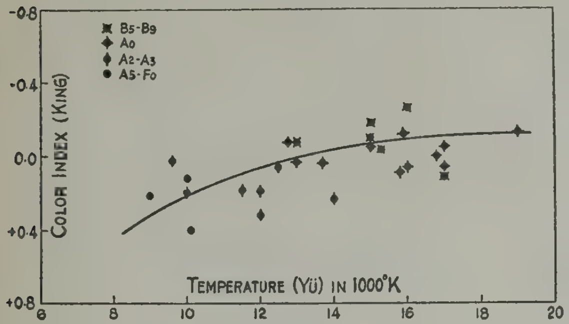

Fig. 7

Regarding the validity of Figure 4, another question is how much weight can be attached to the temperature of the individual stars. It will be recalled[14] that the determination of these temperatures was based on an assumed temperature of 22,000° K for *ζ Ophiuchi* (B0). For that reason the absolute character of the temperatures as appeared in Figure 4 and Figure 6 should be looked upon with caution. It is believed, however, that the relative values are qualitatively correct, as evidenced by the temperature gradient in the figures. The argument is further supported by a correlation between King's[15] color indices and my temperatures. Figure 7 shows the correlation not only for all Class A stars but also for subclasses, especially A0. This again strengthens the view that temperatures within subclasses do vary widely.

SPECTROSCOPIC PARALLAXES OF 63 CLASS A STARS

Absolute magnitudes of 63 Class A stars, for which we have temperature determinations and estimates of the amount of absorption, were derived according to the method described in the preceding section and listed in Column 7, Table I. These were then converted into parallaxes by the relation,

$$M = m + 5 + 5 \log \pi \tag{2}$$

where m is the apparent visual magnitude and π the parallax, and entered in the succeeding column of the table. For purpose of comparison, the spectroscopic parallaxes as derived by Adams and Joy whenever available, and the trigonometric and group parallaxes taken from Schlesinger's Catalogue are also listed in the table.

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[14] Yü, *Loc. cit.*, p. 114.

[15] *Ann. Harv. Coll. Obs.*, **85**, 58, 1923.

TABLE I

H.R.	Star	Class	Vis. Mag. m	Temperature T	Con. Absorp. R	Abs. Mag. M (Yü)	Spec. Para. π_s		Trig. Para. π_t	Group Para. π_g
							Yü	Adams		
1165	η Tau.*	B5p	2.96	13,000° K	.400	−0.4	0″.021	0″.029	0″.007	0″.013
1666	β Eri.*	A3	2.92	12,000	.615	+2.6	.087	.052	.052	
1791	β Tau.*	B8	1.78	15,000	.412	−1.6	.021	.040	.024	
2095	θ Aur.*	A0p	2.71	19,000	.666	−1.2	.016	.037	.016	
2421	γ Gem.*	A0	1.93	15,000	.600	+0.3	.048	.063	.043	
2491	α CMa.*	A0	−1.58	16,000	.720	+1.3	.380	.316	.371	.380
2540	θ Gem.*	A2	3.64	10,000	.500	+2.6	.063		.019	
2763	λ Gem.*	A2	3.65	11,500	.455	+1.1	.032	.038	.038	
2777	δ Gem.*	F0	3.51	12,000	.608	+2.5	.064	.055	.060	
2845	β CMi.*	B8	3.09	16,000	.567	−0.4	.020	.030	.020	
3569	υ UMa.*	A5	3.12	10,000	.475	+2.3	.070	.066	.070	
3594	κ UMa.	A0	3.68	15,000	.710	+1.8	.043	.026		
3975	η Leo.	A0p	3.58	12,000	.556	+2.0	.048			
3982	α Leo.*	B8	1.34	15,000	.564	+0.1	.058	.064	.058	
4033	λ UMa.	A2	3.52	10,200	.500	+2.4	.060			
4295	β UMa.*	A0	2.44	16,800	.634	−0.2	.029	.054		.047
4357	δ Leo.*	A3	2.58	9,600	.375	+2.0	.078	.066	.078	
4359	θ Leo.*	A0	3.42	10,400	.540	+2.7	.073	.032	.018	
4534	β Leo.*	A2	2.23	12,500	.626	+2.4	.108	.079	.101	
4554	γ UMa.*	A0	2.54	13,000	.510	+0.8	.045	.048	.004	.041
4660	δ UMa.*	A2	3.44	11,400	.482	+1.6	.043	.032		.045
4662	γ Cor.	B8	2.78	17,000	.434	−2.6	.002			

* Stars used in deriving the reduction curve.

TABLE I—(*Continued*)

H.R.	Star	Class	Vis. Mag. m	Temperature T	Con. Absorp. R	Abs. Mag. M (Yü)	Spec. Para. π_s		Trig. Para. π_t	Group Para. π_g
							Yü	Adams		
4757	δ Cor.*	A0	3.11	15,000	.400	−1.6	.011	.034	.010	
4787	κ Dra.	B5p	3.88	14,200	.410	−1.1	.011			
4905	ε UMa.*	A0p	1.68	17,400	.600	−1.0	.029	.069		.042
4915	α CVn.*	A0p	2.90	15,800	.402	−2.1	.010	.034	.015	.036
5107	ζ Vir.*	A2	3.44	12,200	.529	+1.5	.041		.038	
5291	α Dra.	A0p	3.64	16,000	.571	−0.4	.016			
5435	γ Boo.	F0	3.00	9,000	.588	+4.3	.182	.062		
5511	109 Vir.	A0	3.76	12,600	.518	+1.1	.030			
5531	α² Lib.	A3	2.90	13,000	.582	+1.1	.044	.100		
5685	β Lib.	B8	2.74	15,000	.492	−0.7	.020			
5735	γ UMi.	A2	3.14	12,500	.628	+2.4	.073			
5793	α CrB.*	A0	2.31	17,000	.567	−1.1	.021	.052	.053	.041
5867	β Ser.*	A2	3.74	10,000	.534	+3.0	.071	.030	.032	.023
5881	μ Ser.	A0	3.63	12,400	.500	+1.0	.031	.022		
5892	ε Ser.*	A2	3.75	13,600	.590	+1.3	.033	.042	.036	
6092	τ Her.	B5p	3.91	11,900	.340	−0.3	.014			
6149	λ Oph.	A0	3.85	12,000	.522	+1.6	.035	.029		
6324	ε Her.*	A0	3.92	15,000	.666	+1.2	.029		.023	
6378	η Oph.	A2	2.63	11,500	.500	+1.9	.073	.033		
6396	ζ Dra.*	B5	3.22	17,500	.442	−2.8	.006	.014	.019	
6410	δ Her.*	A2	3.16	10,000	.406	+1.5	.048	.035	.029	
6556	α Oph.*	A5	2.14	12,000	.515	+1.5	.074	.052	.049	.046
6561	ξ Ser.	A5	3.64	9,500	.538	+3.3	.085			
6629	γ Oph.	A0	3.74	13,700	.520	+0.5	.022			
6714	67 Oph.	B5p	3.92	10,000	.230	−0.4	.014			
6771	72 Oph.*	A3	3.73	11,600	.500	+1.7	.039	.042	.040	
6779	ο Her.	A0	3.83	13,600	.533	+0.7	.024			
7001	α Lyr.*	A0	0.14	16,000	.633	+0.3	.108	.123	.124	.104
7178	γ Lyr.	A0p	3.30	10,600	.474	+2.0	.055	.020	.011	
7235	ζ Aql.*	A0	3.02	13,700	.550	+0.8	.037	.036	.040	
7236	λ Aql.*	B9	3.55	16,500	.635	−0.1	.019		.038	
7420	ν Cyg.	A2	3.94	9,600	.409	+1.1	.028	.028	.005	
7528	δ Cyg.*	A0	2.97	13,000	.540	+0.9	.039	.042	.038	
7557	α Aql.*	A5	0.89	10,100	.496	+2.4	.204	.145	.204	
7710	θ Aql.*	A0	3.37	12,600	.400	+0.2	.019	.019	.015	
7852	ε Del.	B5	3.98	15,000	.348	−2.3	.005		.001	
7906	α Del.	B8	3.86	14,800	.464	−0.8	.012			
7924	α Cyg.	A2p	1.33	14,000	.451	−0.4	.044		.005	
7950	ε Aqr.*	A0	3.83	13,500	.550	+1.0	.027	.037	.022	
8162	α Cep.*	A5	2.60	9,700	.444	+2.2	.083	.056	.083	
8585	7 Lac.*	A0	3.85	13,800	.550	+0.8	.025		.034	

* Stars used in deriving the reduction curve.

The most direct test of the method is to compare parallaxes derived through its use with the trigonometric and group parallaxes. It will be seen from Table I that the agreement is generally satisfactory. The test as is evident is not quite independent, for most of the stars compared are those which were used in deriving the diagram (Fig. 4) or the "reduction curve"; still there can be little question that a correlation exists. Independent tests will of course be desirable. Perhaps a better idea of the test can be gained by inspection of Figure 5. The agreement will of course be perfect for points which fall on the line. The deviations can be accounted for by errors of the trigonometric parallax and errors of observation.

Four stars with extremely small trigonometric parallaxes (*α Cygni*, 0″.005; *γ Lyrae*, 0″.011; *ν Cygni*, 0″.005; *ε Delphini*, 0″.001) have not been used to derive the reduction curve. They do not seem to fit in the scheme. For this reason their absolute magnitudes found from this method are probably too faint. Since *α Cygni* (A2p) is a star with "c" characteristics and *γ Lyrae* (A0p) although not classified by Harvard observers in the "c" division has exceptionally sharp lines and other features similar to those possessed by *α Cygni*, it may indicate that stars of this peculiar class require separate treatment. However it is difficult to see why the method is not applicable to them on the general principle considered unless some factors other than temperature and pressure are exerting an appreciable influence on stars of this class. Perhaps improvement of the reduction curve and better determination of the temperature may ultimately remove the discrepancy.

It is of interest to note that a comparison of Adams and Joy's parallaxes with those obtained trigonometrically or from group motion, and a comparison of our figures with the same data for 33 stars in Table I common to the three lists yield the following results. The average difference of Adams and Joy's values regardless of sign is ±0″.0141, while that of ours is ±0″.0125. This, however, should not be taken too seriously, for the comparison is not quite a fair one since most of the stars concerned are the ones from which our reduction curve was derived.

THEORETICAL SIGNIFICANCE

We have pointed out earlier in the paper that on the basis of the theory of thermal ionization, the amount of continuous hydrogen absorption should show a definite relation to both temperature and pressure. The existence of such a relation in Class A stars seems highly probable as depicted by Figure 4. It will therefore be interesting to make a more detailed inquiry into the extent to which observations satisfy the theory at least from the qualitative point of view.

According to the simple theory of ionization, the strength of subordinate series lines (*e.g.*, the Balmer series of hydrogen including the continuous absorption) near its maximum is rather insensitive to changes in temperature. On the other hand it is very sensitive to changes in pressure. The maximum for the Balmer series occurs within the group of stars here considered. The Harvard classification placed it at A0, Menzel[16] estimated it to be at A3, while Miss Payne[17] is of the opinion that no significant maximum can be derived for the Balmer series. Our results (Figs. 4 and 6) seem to confirm the Harvard determination. Near maximum, about three-fourths of the hydrogen atoms are ionized. A fraction of the remaining neutral atoms are in the 2p-state giving rise to the Balmer series. If now the pressure is lowered, the rate of ionization will as a result increase. Therefore the number of neutral hydrogen atoms in the 2p-state will correspondingly diminish, or in other words the strength of the Balmer series should be weakened. This is in fact what was observed. For constant temperature, represented by any inclined line in Figure 4 and Figure 6, the intensity of either the continuous absorption or Hγ decreases as the absolute luminosity becomes greater or as the pressure decreases, since for constant temperature pressure is a simple function of absolute magnitude. In view of the familiar anomalous behavior of the hydrogen lines in the late type giant and dwarf stars, this result and interpretation must, however, be regarded with caution. It is possible that among Class A stars, where the temperature is high, hydrogen may behave differently from its behavior under low temperature conditions. But until more observations are obtained and until the theory is improved the question can not be satisfactorily settled. Moreover, Figure 4 and Figure 6 show, contrary to the prediction of theory, that the strength of the Balmer series near maximum is sensitive to changes in temperature as well as to changes in pressure. Further work is now in progress which may throw more light on the subject.

There is another interesting feature in Figure 4. If we group the observations in the figure into temperature classes and average each class, the points as a result will fall on a line approximately perpendicular to the temperature lines closely resembling and near the dotted line *cd* in the diagram. This line by virtue of its construction represents the mean of the observations considered. It is interesting to note that, if Adams' spectral classes are translated into mean temperatures, the line *cd* is precisely one of the two reduction curves derived by Adams[18] and used by him in determining the absolute magnitudes of some Class A stars. This lends further support to the view mentioned earlier in the paper that the method of Adams and Joy is applicable probably only to the average of sufficiently large numbers of stars and not to each individual one. The line *cd* which represents Adams' reduction curve for nebulous spectral lines,* was derived from Table II constructed as follows: The first and second columns were taken from Table V in Adams and Joy's paper.[19] The third column was obtained by averaging for each spectral class the temperatures determined by the writer and published in Table IV of my article.[20] For purpose of comparison the temperature scale as derived by Sampson[21] was

TABLE II

Class	*M* (Adams)	Mean Temperature	
		Yü	Sampson
B8	+0.2	16,000° K	14,300° K
A0	+0.9	13,700	13,100
A2	+1.7	11,200	12,100
A5	+2.3	10,200	11,200

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[16] *Harv. Cir.*, 258, 12, 1924.

[17] *Harv. Cir.*, 263, 9, 1924.

[18] *Astrophys. Journ.*, **56**, 248, 1922; *M. W. Contr.* No. 244, p. 7.

[19] *Loc. cit.*

[20] Yü, *Loc. cit.*, p. 118.

[21] *Monthly Notices*, **85**, 237, 1925.

* The other reduction curve for sharp spectral lines if plotted in the figure would lie a little below *cd* and be essentially parallel to it.

also added. Hence having T and M, the curve cd is uniquely located in the figure.

Investigators of stellar effective temperatures have been confronted with the fact that the temperature of stars although belonging to the same spectral subclass varies widely. The dispersion, which is especially pronounced among early type stars, decidedly exceeds what could be accounted for by errors of observation. It is therefore in part at least real. This discrepancy seems susceptible to explanation on the basis of the temperature and magnitude considerations here discussed. The spectral characteristic of a star, from which its type is determined, depends as we have seen on both temperature and pressure. Since both these factors vary widely, it is evident that spectral classes are not in this respect suitable for theoretical investigation. This emphasizes the need of a classification of stars according to their temperatures, or colors, as has been advocated by Milne[22] and Sampson.[23] The classification of A-type stars in this manner will probably solve some difficulties of this group of stars discussed and emphasized by Miss Payne.[24] At least it gives considerable promise in the direction toward which a solution of the problem may be accomplished.

ADVANTAGES AND LIMITATIONS OF THE METHOD

Aside from the theoretical advantages discussed above the proposed method also possesses a few practical conveniences worth noting. In the first place, spectral classification is not involved in our data. It can be discarded altogether as far as the determination of absolute magnitude is concerned. This therefore eliminates the personal factor in the estimation of classes. Again, the intensity of continuous absorption which constitutes our main criterion is not obtained from an arbitrary estimated scale, but has a direct photometric basis, and furthermore, as has already been pointed out, being continuous and unlike the line intensity, it is independent of slit width and dispersion, and in case of objective prism observations independent of "seeing," imperfect guiding and focus.

The method is not, however, without its limitations. The inclusion of a new variable naturally creates additional uncertainty. The time required in reducing the observations is rather considerable. However it can be much reduced with the aid of a self-registering microphotometer. But the most serious limitation of the method, one which is apt to keep it from general adoption, is that the region of continuous hydrogen absorption lies in the ultra-violet and therefore is not readily accessible; for not many observers are fortunate enough to be equipped with an ultra-violet spectrograph and reflector, such as must be used. Nevertheless, the Balmer lines of hydrogen can be utilized as well as the continuous absorption if the latter is not available, as we have seen from Figure 6. It is therefore possible that the proposed method may be developed in this direction, provided of course that the intensity of lines is based on a secure photometric scale, that constancy of the slit width can be assured, and that the temperature of individual stars is taken into account.

In conclusion, I take great pleasure in expressing my thanks to Mr. W. H. Wright, Dr. H. N. Russell and Mr. H. H. Plaskett for their helpful suggestions and criticisms.

June 10, 1926.

Issued September 4, 1926.

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[22] *Proc. Phys. Soc. Lon.*, **36**, 105, 1924.
[23] *Monthly Notices*, **85**, 212, 1925.
[24] *Stellar Atmospheres*, p. 168, 1925.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 381

THE ORBIT OF THE SPECTROSCOPIC BINARY 112 HERCULIS

By

W. F. MEYER

The variable velocity of 112 *Herculis* ($\alpha = 18^h\ 48^m.0$, $\delta = +21° 18'$, visual magnitude 5.3, spectral type B9) was announced by the writer[1] from a number of plates taken in March of this year. From March 11 to May 11, twenty-eight spectrograms were secured with the Mills three-prism spectrograph. The plates were measured on a Gaertner engine. The characteristic lines for 112 *Herculis*, which in general were measurable and which were used to determine the radial velocities are:

λ 4481.400	λ 4522.802
4491.570	4534.139
4508.455	4549.642
4515.501	4556.070
4520.397	4584.018

These lines are good for three-prism dispersion and this type of spectrum.

Table I contains all the observations secured at this observatory. The last column contains the final residuals from the elements which resulted from a least squares solution. The epoche from which the phases were calculated is J.D. 2424589.675.

Tables II and III contain the observations received by letter from the Mount Wilson and Yerkes observatories. The residuals were determined from the graph of the orbit.

The Mount Wilson observations were made with a 40-inch camera and single prism. The lines are reported as being of good quality. The Yerkes spectrograms were taken with one prism and have a scale value of about 30A per mm. at λ4500. The plates are described as having "many lines and poor, which are not well suited for measurement by reason

TABLE I

No.	G. M. T.	Phase	Obs. Vel.	O−C
1	1913, June 3.954	3.280	− 5.5 km.	−0.8 km.
2	1919, July 8.887	2.376	− 3.6	+1.4
3	July 10.708	4.196	−12.3	+0.9
4	1922, June 30.822	2.338	− 5.8	−0.6
5	July 25.829	1.899	−10.5	−1.5
6	Aug. 31.704	0.595	−31.7	−0.3
7	1924, June 24.928	2.128	− 8.1	−1.4
	G. C. T.			
8	1926, Mar. 11.528	2.714	− 2.0 km.	+1.8 km.
9	12.515	3.700	− 8.7	−1.1
10	13.520	4.705	−19.6	+1.4
11	14.517	5.704	−37.2	−0.2
12	15.522	0.347	−34.5	+1.0
13	16.510	1.335	−16.6	+0.6
14	20.500	5.325	−30.4	+1.3
15	21.514	6.339	−39.8	−0.6
16	22.508	0.971	−21.4	+2.7
17	24.503	2.966	− 4.8	−1.0
18	25.502	3.965	−13.3	+3.0
19	27.507	5.970	−37.8	+1.4
20	28.504	0.604	−29.1	+2.1
21	29.487	1.587	−15.3	−2.2
22	Mar. 30.488	2.588	− 3.3	+0.8
23	Apr. 14.502	4.877	−23.1	+0.9
24	20.458	4.471	−18.2	−1.0
25	21.488	5.501	−32.6	+1.8
26	24.490	2.141	− 9.2	−2.6
27	25.493	3.144	− 2.5	+1.7
28	26.494	4.145	−13.4	−0.9
29	28.453	6.104	−42.2	−2.5
30	Apr. 29.463	0.751	−29.7	−1.3
31	May 1.451	2.739	− 2.4	+1.4
32	2.442	3.730	− 7.8	+0.1
33	3.420	4.708	−21.6	−0.5
34	4.441	5.729	−37.8	−0.5
35	May 11.432	6.357	−38.1	+1.0

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[1] *Publ. A. S. P.*, **38**, 222, 1926.

TABLE II

Mount Wilson Observations

No.	G. M. T.	Obs. Vel.	O−C
1	1912, June 27.926	−35.4 km.	+2.8 km.
2	Aug. 2.817	−10.1	−4.0
3	Aug. 26.650	−12.0	−3.3
4	Sept. 1.662	−17.4	−3.9
5	1913, June 24.935	−25.7	+3.8
6	Aug. 16.788	−24.4	+3.7

of diffuseness." On account of the large residuals for the Yerkes observations, Dr. Otto Struve of the Yerkes observatory kindly consented to remeasure the plates. The results are shown in Table III.

TABLE III

Yerkes Observations

No.	G. M. T.	First Meas.		Second Meas.		Quality
		Obs. Vel.	O−C	Obs. Vel.	O−C	
		km.	km.	km.	km.	
1	1913, Apr. 28.888	−18.7	+13.8	−30.0	+ 2.5	g
2	May 23.806	−29.5	− 5.5	−24.5	− 0.5	f
3	June 27.730	− 8.5	+ 4.2	− 5.8	+ 6.9	p
4	July 4.678	−21.1	−14.8	− 9.6	− 3.3	p
5	1920, May 14.858	+ 5.6	+19.2	+ 0.2	+13.8	g
6	1921, Aug. 29.653	−15.6	−11.4	− 2.4	+ 1.8	p

From a study of the 1926 observations the period was noted to be approximately 6.36 days. With the aid of the first seven Lick observations and the six Mount Wilson plates, a final value was obtained. The period resulting from each observation was weighted according to the elapsed number of revolutions. The mean value of the period obtained in this manner is 6.3624 days. The thirty-five Lick observations were grouped into seventeen normal places, each normal value being weighted according to the number of single observations entering into the normal place.

The Lehmann-Filhés graphical method was used to determine the preliminary elements.

Preliminary Elements

Period	$P=6.3624$ days
Periastron Passage	$T=$J.D. 2424589.5
Eccentricity	$e=0.10$
Longitude of Apse	$\omega=185°$
Semi-amplitude	$K=18.00$ km.
Velocity of System	$\gamma=-19.66$ km.

With this set of elements an ephemeris and residuals were computed for the seventeen normal observations. The resulting residuals are shown in Table IV. Considering the period as established and using the equations of condition as outlined by Schlesinger,[2] the following normal equations resulted:

$$\begin{array}{rrrrrr}
+11.669\Gamma & +1.493\kappa & +0.287\pi & -4.739\epsilon & +0.159\tau & -0.007=0 \\
 & +6.392 & +0.637 & +0.024 & +0.102 & +2.060=0 \\
 & & +5.277 & -0.334 & +4.308 & +0.115=0 \\
 & & & +3.064 & -0.207 & +0.686=0 \\
 & & & & +3.556 & -0.029=0
\end{array}$$

The solution of these normal equations gave the following corrections to the preliminary elements:

$\delta T=0.175$ days
$\delta e=0.018$
$\delta\omega=10^\circ.09$
$\delta K=0.002$ km.
$\delta\gamma=0.02$ km.

Final Elements

$P=6.3624$ days
$T=$J.D. 2424589.675 ± 0.608
$e=0.118$ ± 0.034
$\omega=195^\circ.09$ ± $35^\circ.05$
$K=18.00$ km. ± 1.09 km.
$\gamma=-19.64$ km.
$a \sin i=1{,}600{,}000$ km.

The residuals for the normal places resulting from the final elements are tabulated in Table IV. The sum of the squares of the weighted residuals was reduced from 14.6165 to 13.7466. From a comparison between the preliminary and final residuals, it is evident that little was gained from the least squares solution, so far as practical results are concerned.

TABLE IV

Residuals for Normal Places

No.	(O−C)		No.	(O−C)	
	Prel.	Final		Prel.	Final
1	−0.31 km.	+0.12 km.	10	+1.31 km.	+1.60 km.
2	+0.56	+0.88	11	−0.91	−0.59
3	+0.07	+0.12	12	−1.22	−1.11
4	+2.95	+2.64	13	+0.39	+0.09
5	−0.32	−0.95	14	+0.56	+0.09
6	−1.57	−2.02	15	+2.11	+1.51
7	+0.51	+0.29	16	−0.07	−0.41
8	+1.17	+1.20	17	−0.63	−0.58
9	−1.40	−1.08			

In the accompanying figure the separate observations and the orbit curve have been plotted. The probable error of a Lick observation of unit weight is 1.01 km. This is represented in the diagram by the radius of the open and closed circles which represent the Lick observations. The Mount Wilson observations are represented by the barred circles. The velocity of the system is shown by the dotted line.

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[2] *Publ. Allegheny Obs.*, 1, 33, 1910.

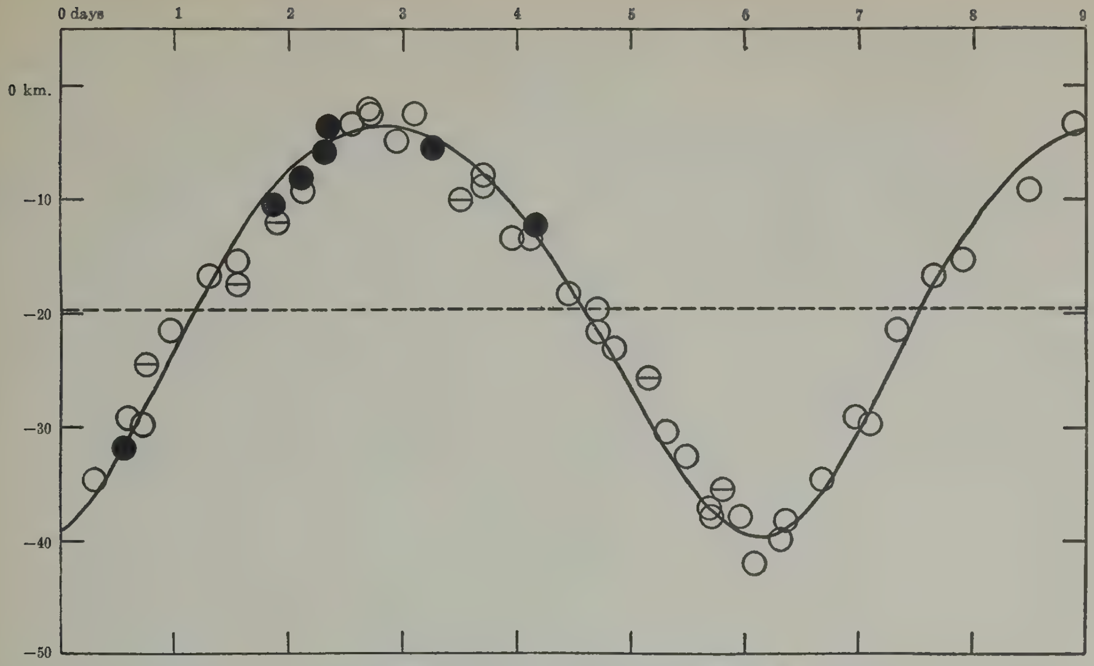

Fig. 1. Velocity Curve of 112 *Herculis*

Open circles are Lick observations, 1926; filled circles Lick observations, 1913–24; barred circles Mount Wilson observations, 1912–13.

July 1, 1926.

Issued September 8, 1926.

UNIVERSITY OF CALIFORNIA PUBLICATIONS
ASTRONOMY

LICK OBSERVATORY BULLETIN
NUMBER 382

ELEMENTS AND EPHEMERIS OF COMET *f* 1926 (COMAS SOLÀ)

BY

CHARLES H. SMILEY and MARGARET K. HOLBROOK

Comet f 1926 was discovered at Barcelona by Comas Solà on November 5. Jeffers describes it as condensed and having a well-defined nucleus. Its estimated magnitude on November 8 was twelve.

From preliminary orbits Crommelin suggested the probable identity of this comet with 1890 VII (Spitaler). This identity was also suspected here by Leuschner on the basis of a preliminary parabolic orbit by Mr. L. A. Brigham and Miss Katherine Prescott. This opinion seems to have been confirmed by others by the application of Tisserand's criterion to periodic orbits computed for this comet.

The orbit given below was obtained by differential correction of a preliminary orbit from short intervals. It is based on the following observations:

	1926, U.T.	α(1926.0)	δ(1926.0)	Observer
(1)	Nov. 8.30589	$2^{\mathrm{h}}53^{\mathrm{m}}26^{\mathrm{s}}.24$	+ 6°42′47″.9	Maxwell, Lick
(2)	Dec. 6.27718	2 27 48.98	+ 9 24 12.9	Maxwell, Lick
(3)	Dec. 28.18902	2 21 58.16	+13 01 45.1	Jeffers, Lick

ELEMENTS

T 1927, Mar. 22.1951 U.T.

ω	38° 27′ 42″.7	1926.0	ω	38° 27′ 44″.7	1927.0
☊	65 36 57.0		☊	65 37 45.5	
i	13 45 17.8		i	13 45 17.9	
log q	0.248538		μ	418″.112	
a	4.16047		P	8.4862 yrs.	
e	0.574015				

CONSTANTS FOR THE EQUATOR 1926.0

$$x=r[9.989569]\sin(193^\circ\ 26'\ 41''.5+v)$$
$$y=r[9.944055]\sin(110\ \ 21\ \ 04.6+v)$$
$$z=r[9.718894]\sin(\ 82\ \ 16\ \ 46.5+v)$$

CONSTANTS FOR THE EQUATOR 1927.0

$$x=r[9.989568]\sin(193^\circ\ 27'\ 32''.8+v)$$
$$y=r[9.944068]\sin(110\ \ 21\ \ 55.7+v)$$
$$z=r[9.718866]\sin(\ 82\ \ 17\ \ 21.6+v)$$

RESIDUALS

O−C	I	III
$\cos\delta\Delta\alpha$	−0″.8	−0″.5
$\Delta\delta$	−0.3	−0.2

Other observations were represented as follows:

	Date	α(1926.0)	δ(1926.0)	O−C $\cos\delta\Delta\alpha$	 $\Delta\delta$
(1)	Nov. 7.0260	$2^{\mathrm{h}}54^{\mathrm{m}}41^{\mathrm{s}}.7$	+6°38′33″	− 5″.1	− 1″.9
(2)	8.1040	53 37	42 02	−21.3	− 4.6
(3)	9.22496	52 31.39	46 00.8	− 1.1	+ 2.4
(4)	10.0056	51 42.8	48 47	−16.5	+ 2.0
(5)	10.08136	51 39.97	49 01.8	+ 1.2	− 0.4
(6)	10.1394	51 35.5	49 16	− 5.5	+ 1.8
(7)	10.82905	50 54.7	51 55	− 0.7	+ 5.4
(8)	10.90320	50 48	52 04	−31.0	− 1.7
(9)	10.95201	50 46.9	52 12	+ 0.7	− 4.0
(10)	10.99900	50 44.09	52 27.7	+ 0.1	− 0.3
(11)	11.20332	50 31.58	+6 53 15.5	− 3.7	+ 0.1
(12)	12.21049	49 45.87	+7 01 52.9	+3′47.0	+4′40.1
(13)	12.92956	48 46.74	00 06.2	− 3.3	− 1.2
(14)	14.29520	47 23.3	05 51.0	− 6.6	−10.2
(15)	14.93920	46 43.6	08 50.2	−14.1	+ 4.4
(16)	14.99734	46 40.67	08 58.9	− 3.5	− 2.5
(17)	21.19813	40 29.29	40 08.0	+ 6.1	− 0.2
(18)	22.01832	39 41.61	+7 44 45.2	+ 2.0	+ 0.3
(19)	25.81861	36 07.11	+8 07 45.5	+ 0.5	+ 3.0
(20)	27.90181	34 14.76	20 53.7	−11.4	−30.2
(21)	28.00104	34 10.2	22 07	− 0.6	+ 4.3
(22)	Nov. 30.13173	2 32 22.60	+8 36 55.4	+ 3.0	+ 0.5

Ephemeris

U.T.	α(1927.0)	δ(1927.0)	log Δ	Br.
Jan. 16.0	$2^h\ 31^m\ 43^s$	+16° 53′.6	0.132	1.05
20.0	35 29	17 46.0	0.140	
24.0	39 47	18 39.1	0.149	
Jan. 28.0	44 38	19 32.6	0.157	
Feb. 1.0	50 00	20 26.3	0.166	0.94
5.0	2 55 52	21 20.1	0.174	
9.0	3 02 14	22 13.6	0.183	
13.0	09 03	23 06.6	0.191	
17.0	16 20	23 59.0	0.199	
21.0	24 03	24 50.4	0.207	
Feb. 25.0	3 32 10	+25 40.5	0.215	0.79

The unit of brightness is that of November 8.

Publication of a periodic orbit was deferred on account of the lack of agreement of the early observations.

Berkeley Astronomical Department,
January 15, 1927.
Issued January 25, 1927.

PRELIMINARY PARABOLIC ELEMENTS OF COMET *f* 1926 (Comas Solà)

BY

Lewis A. Brigham and Katherine Prescott

A preliminary parabolic orbit was based on the following observations:

1926, U.T.	α(1926.0)	δ(1926.0)	Observer
Nov. 7.0260	$2^h 54^m 41^s.7$	+6°38′33″	Krumpholz, Vienna
Nov. 8.30589	2 53 26.24	+6°42 47.9	Maxwell, Lick
Nov. 9.22496	2 52 31.39	+6 46 00.8	Maxwell, Lick

A direct solution by Leuschner's method without differential correction gave:

Elements

T	1927 April 21.33	
ω	58° 35′.6	1926.0
☊	59 10.1	
i	21 25.1	
log q	0.2242	

O−C	I	III
$\cos\delta\Delta\alpha$	+1″.5	−3″.0
$\Delta\delta$	+2.9	−1.7

The elements and an ephemeris were telegraphed to Harvard College Observatory and are published in *H. C. O. Announcement Card No. 12*, November 15, 1926.

Berkeley Astronomical Department,
January 15, 1927.
Issued January 25, 1927.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 383

EIGHTY-FIVE NEW DOUBLE STARS

TWENTY-FIFTH LIST

BY

ROBERT G. AITKEN

The Lick Observatory Double Star Survey was originally planned to extend to −22°0 declination and Professor Hussey, who assumed the zone −10° to −22°, in our division of sky area, actually completed the search to the southern limit in almost the entire zone between 13 hours and 1 hour right ascension. In the remaining 12 hours, which may be called our winter sky, little was done south of −14°, and large gaps remained in the zone −10° to −14°. The gaps in this zone, and the few in the summer sky, I filled in, but planned to leave the area −14° to −22° from 1^h to 13^h, to the observers in the southern hemisphere. For various reasons, these observers preferred not to work so far north and by agreement with Messrs. Innes and van den Bos, I therefore undertook to complete the survey to −18°, it being understood that they would extend their work to that limit. About 400 stars still remain to be examined, but it has seemed advisable to publish now the list of pairs so far discovered because the extension to Burnham's *General Catalogue of Double Stars* which I have in hand, will include only the double star work done prior to January 1, 1927.

In addition to the pairs in the zone −14° to −18°, the list includes pairs found in examining the 9.0 stars in a part of the zone −2° to −10° which had been omitted from the working charts by misunderstanding on the part of an assistant who prepared certain of the charts, and a few pairs found by reëxamining stars marked "possibly double," in my original survey.

The arrangement of the results is similar to that of earlier lists, but I have given the D.M. numbers to identify the stars, instead of A.G. numbers, and have given the star places for the epoch 1950 instead of 1900.

		R.A. 1950.0	Decl. 1950.0	Date	Angle	Dist.	Mag.	B.D. mag.	n	
A3001	+42°294	1^h 21^m 41^s	+42° 55′	1917.97	252°0	0″72	10.2–10.2	9.5	2	
				1921.57	249.8	0.70			2	
A3002	−11.685	3 30 51	−11 21	1921.67	84.3	0.58	9.0–12.5	9.0	2	
A3003	−14.691	3 30 51	−13 49	1918.05	192.3	1.02	9.3–12.5	9.3	2	
				1921.67	190.6	1.02			1	
A3004	−16.674	3 36 00	−15 57	1918.05	22.2	1.92	8.7–12.0	8.7	2	
				1921.67	21.9	1.95			1	
A3005	−15.678	3 48 47	−15 23	1921.69	333.8	0.50	8.5–10.0	8.5	2	
A3006[1]	+ 1.808	4 41 27	+ 1 59	1921.74	314.4	1.01	9.5–13.8	9.5	2	C and D.
				1921.73	155.0	2.14	10.0–10.0	9.3	1	A and B.[1]
				1921.73	248.3	52.44	10.0– 9.5		1	A and C.
A3007	−16.944	4 43 48	−16 36	1925.11	186.1	3.56	8.7–12.8	8.7	2	
A3008	−15.921	4 58 17	−15 49	1925.99	144.8	1.32	9.0–12.5	9.0	2	
A3009	−14.1040	5 03 37	−13 59	1926.00	109.5	0.91	8.0–10.3	8.0	2	

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[1] A3006 is B.D.+1°808; Hall− is B.D.+1°809, and this pair is designated by A and B in the measures given.

		R.A. 1950.0	Decl. 1950.0	Date	Angle	Dist.	Mag.	B.D. mag.	n	
A3010[2]	+18°779	$5^h\ 04^m\ 27^s$	+18° 35′	1912.90	5°9	0″10	6.0– 6.0	5.2	1	
				1914.85	90.3	<0.10			2	
				1915.74	239.3	0.11			1	
				1921.94	267.6	0.09			1	
A3011[3]	−14.1050	5 05 09	−14 37	1926.00	236.4	3.30	8.3–10.8	8.3	2	
A3012	−15.962	5 08 18	−15 46	1926.00	198.3	2.90	8.6–11.0	8.6	2	
A3013	−15.984	5 12 51	−15 09	1926.00	321.9	3.64	9.0–13.5	9.0	2	
A3014	−15.1020	5 19 41	−15 31	1926.01	64.0	4.40	9.0–13.2	9.0	2	
A3015	−15.1023	5 20 49	−15 29	1926.01	323.4	3.20	9.3–12.0	9.3	2	
A3016	−10.1179	5 22 49	− 9 59	1921.82	253.2	4.92	9.7– 9.7	9.0	2	
A3017	−16.1225	5 42 28	−16 51	1926.01	293.7	4.28	8.8–13.0	8.8	2	
A3018	−14.1228	5 43 25	−14 48	1925.11	211.6	0.52	9.5– 9.5	8.7	2	
A3019	−15.1212	5 52 09	−15 44	1926.03	4.8	3.20	11.0–13.2		2	B and C.
				1926.03	41.7	45.62	7.7–11.0	7.7	2	A and B.
A3020	− 3.1310	6 07 06	− 3 23	1926.11	230.9	0.36	9.2–10.5	9.0	2	
A3021	− 4.1380	6 07 33	− 4 19	1926.11	185.1	3.42	9.2–10.7	9.0	2	
A3022	−16.1387	6 09 11	−16 49	1926.03	4.3	0.93	9.0– 9.1	8.5	2	
A3023	−17.1393	6 09 26	−17 26	1926.03	326.0	2.00	8.5–10.0	8.5	2	
A3024	− 4.1401	6 10 17	− 4 16	1926.11	285.5	0.31	10.0–10.0	9.2	2	
A3025	−17.1407	6 12 05	−17 28	1926.03	135.5	0.95	8.5– 9.2	8.2	2	
A3026	−14.1393	6 15 45	−14 12	1926.04	128.1	4.00	9.0–13.0	9.0	2	
A3027	−16.1456	6 20 10	−16 42	1926.82	244.8	1.44	10.5–11.0		2	B and C.
				1926.81	256.1	87.42	8.5–10.5	8.5	1	A and B.
A3028	−14.1436	6 22 54	−14 23	1926.81	16.1	0.44	9.0–10.5	9.0	2	
A3029	−15.1390	6 25 16	−15 16	1926.81	197.2	4.89	7.7–12.7	7.7	2	
A3030	−14.1493	6 31 30	−14 08	1926.06	4.6	2.05	8.9–13.2	8.9	2	
A3031	−17.1564	6 35 45	−17 11	1926.06	315.0	3.08	8.3–13.8	8.3	2	
A3032	−15.1467	6 38 09	−15 48	1926.11	311.9	0.30	8.8– 9.3	8.5	2	
A3033	−17.1587	6 39 35	−17 55	1926.13	201.2	0.40	9.5– 9.7	8.9	2	
A3034	−17.1598	6 41 16	−17 06	1926.13	66.4	1.56	9.5– 9.5	8.7	2	
A3035	−16.1593	6 43 22	−16 50	1926.13	285.2	1.61	9.0–13.0	9.0	2	
A3036	+ 0.1587	6 43 37	+ 0 48	1922.35	44.4	0.38	8.3–10.5	8.3	2	
A3037	−16.1597	6 44 06	−16 49	1926.13	166.2	0.86	9.0–11.0	8.8	2	
A3038	−15.1519	6 46 06	−15 16	1926.11	310.5	0.28	9.5– 9.5		2	B and C.
				1926.07	45.4	7.36	8.0– 9.0	8.3	1	A and BC=Σ972 rej.
A3039	+40.1745	6 50 05	+39 54	1922.79	35.2	0.24	9.2– 9.8	9.0	2	
A3040	−16.1653	6 52 50	−16 26	1926.21	297.3	2.34	9.2–10.3	9.0	2	
A3041	−14.1661	6 55 24	−14 14	1926.24	43.0	0.79	9.0–10.8	9.0	2	
A3042	− 9.1796	6 59 08	− 9 38	1922.84	85.8	0.22	8.3– 8.8	8.0	2	A and B.
				1922.79	76.6	11.00	–15.0		1	A and C.
				1922.79	329.6	12.61	–14.0		1	A and D.
A3043	−15.1657	7 05 37	−15 37	1926.82	338.5	0.25	7.5– 8.5	7.5	2	
A3044	−14.1743	7 07 53	−14 26	1926.82	324.9	0.37	9.4– 9.6	8.9	2	
A3045	−17.1827	7 09 05	−17 13	1926.83	259.4	0.25	9.4– 9.6	9.0	1	
A3046	−17.1898	7 15 17	−18 07	1926.83	67.8	1.24	9.0–10.5	8.9	1	
A3047	− 6.2083	7 20 34	− 7 00	1922.93	39.3	2.39	9.0–11.0	9.0	2	
A3048	− 9.2059	7 25 24	− 9 34	1926.21	21.0	4.12	9.0–13.5	9.0	2	
A3049	+11.1618	7 32 00	+11 45	1921.19	7.8	0.62	9.0–11.5	9.0	2	A and B.
				1921.26	122.0	38.38	9.0–10.0		1	A and C.
A3050	− 6.2206	7 35 18	− 7 08	1918.14	255.6	0.72	9.6– 9.6	9.0	2	
				1922.88	253.8	0.71			1	
A3051	− 6.2210	7 35 29	− 6 29	1918.14	162.4	0.32	9.4– 9.7	9.0	2	
				1922.95	162.6	0.38			2	
A3052	− 9.2174	7 39 15	− 9 18	1916.97	1.6	0.72	10.0–12.0	9.8	3	
				1922.88	358.9	0.80			1	
A3053	−12.2095	7 39 48	−12 46	1917.11	106.5	1.26	9.0–12.5	9.0	2	
				1921.83	105.9	1.26			3	

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[2] A3010 is 104 *Tauri*, for which the Yale Parallax Catalogue gives $\pi=0''057$, $\mu=0''542$ in 88°8. The measures of angle are all very uncertain because of the small angular separation, but the image is certainly that of a very close double star.

[3] A3011. Color contrast, yellow and blue; the spectral class of A is K0.

		R.A. 1950.0	Decl. 1950.0	Date	Angle	Dist.	Mag.	B.D. mag.	n	
A3054	− 8°.2185	$7^h\ 59^m\ 09^s$	− 9° 04′	1926.25	45°.3	1″.64	9.0–11.3	9.0	3	A and B.
				1926.23	354.1	10.59	−12.2		2	A and C.
A3055	−16.2274	8 02 21	−16 42	1926.83	3.0	4.00	9.2–10.7	9.0	1	
A3056	−16.2291	8 04 12	−17 03	1926.83	0.9	4.12	8.5–10.5	8.5	1	
A3057	−17.2466	8 19 51	−17 49	1925.21	250.8	0.33	9.0– 9.2	8.6	2	
A3058	− 9.2497	8 21 15	− 9 18	1918.24	41.4	0.58	9.2–10.7	9.0	2	
				1922.98	41.8	0.62			1	
A3059	− 4.2325	8 21 51	− 4 22	1918.27	139.9	3.26	9.2–10.7	9.0	2	
				1922.98	138.6	3.19			1	
A3060	−15.2456	8 26 50	−15 36	1925.21	103.8	5.14	8.5–13.0	8.5	2	
A3061	+10.1814	8 28 20	+10 11	1923.05	315.5	4.46	9.0–10.5	9.0	2	A and B.
				1923.13	352.5	20.70	−13.0		1	A and C.
A3062	− 9.2547	8 28 29	−10 00	1924.32	222.8	3.96	9.0–11.8	9.0	2	
A3063	−15.2538	8 36 52	−15 51	1926.27	296.3	0.31	8.0– 8.5	7.8	3	A and B.
				1926.26	234.4	12.69	−13.0		2	AB and C.
A3064	−14.2610	8 37 59	−15 07	1926.26	13.7	0.74	9.2– 9.2	8.5	2	
A3065	−16.2550	8 38 36	−17 10	1926.27	250.6	0.20	8.3– 8.6	8.0	3	
A3066	−17.2641	8 43 23	−17 39	1926.11	57.2	0.46	8.8– 9.1	8.5	2	
A3067[4]	−17.2653	8 46 13	−17 51	1926.19	41.7	1.76	8.2–10.5	8.2	2	
A3068	− 8.2498	8 46 46	− 8 38	1918.28	21.1	0.98	9.1–10.6	9.0	2	
				1923.04	20.8	1.03			2	
A3069	−17.2663	8 47 27	−17 57	1926.19	299.2	1.18	9.5– 9.7	9.0	2	
A3070	−14.2715	8 56 27	−14 47	1926.27	55.3	0.57	9.0–11.0	9.0	3	
A3071	− 7.2699	8 59 13	− 8 21	1918.28	149.0	0.92	9.7– 9.7	9.0	2	
				1922.98	150.4	0.97			1	
A3072	−14.2751	9 05 05	−15 18	1926.22	244.3	3.67	9.1–11.5	9.1	2	
A3073	−14.2766	9 07 45	−14 54	1926.22	343.0	1.77	8.5–10.0	8.5	2	
A3074	−17.2792	9 11 17	−18 19	1926.28	218.6	0.82	9.0–11.0	9.0	2	
A3075	−18.2613	9 12 04	−18 28	1926.29	142.4	3.26	8.5–14.0	8.5	2	
A3076	−16.2744	9 16 19	−17 00	1926.26	278.2	1.11	9.0–10.0	8.7	2	
A3077	−15.2763	9 17 11	−15 37	1925.30	189.7	4.14	6.0–11.0	6.0	3	
A3078	−15.2774	9 20 31	−16 20	1925.30	46.0	1.38	9.3– 9.8	9.0	3	
A3079	−15.2898	9 43 54	−16 14	1925.21	124.0	2.53	8.7–10.0	8.5	2	
A3080	−15.2907	9 46 24	−16 13	1925.30	67.0	1.14	8.5–10.2	8.5	3	
A3081	+ 4.2256	9 48 15	+ 3 55	1915.20	272.8	0.26	9.6– 9.6	9.0	2	
				1921.18	272.5	0.23			2	
A3082	− 4.2942	10 43 42	− 4 43	1918.11	72.8	1.43	9.5–10.0	9.0	2	
				1921.26	75.9	1.47			1	
A3083	+11.2344	11 16 20	+10 30	1921.08	241.2	1.42	9.3–11.5	9.3	2	
A3084	− 4.3294	12 28 34	− 5 03	1918.13	226.9	0.15	9.6– 9.6	9.0	1	
				1926.41	239.9	0.18			1	
A3085	−21.5251	19 03 32	−21 46	1926.60	130.9	0.31	9.7– 9.7	9.1	2	A and B.
				1926.61	298.8	17.08	−14.5		1	AB and C.

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[4] A3067. Color contrast: orange-yellow and blue. The brighter star is class G5.

MEASURES OF DOUBLE STARS IN THE YEARS 1923-1926

BY

ROBERT G. AITKEN

Very little time was spent in the years 1923–1926 in measures of double stars other than those of my own discovery, but a few known binaries were kept under observation, as far as seemed necessary, and a few miscellaneous pairs were measured for special reasons. The measures are arranged in the same manner as those published in *Lick Observatory Bulletin* Number **348**, except that the coördinates of position are given for the epoch 1950.0, which has been adopted for the extension to Burnham's *General Catalogue*.

21 Σ 2

R.A. $0^h\ 06^m\ 15^s$ Decl. +79° 26′

6.8 7.1 A3

1923.887	100°9	0″20	22^h8	2
1924.744	94.2	0.23	2.2	3
1924.32	97.6	0.22	2n	

Comparison with Russell's orbit (*Pub. A. A. S.*, **3**, 328, 1917) gives the residuals (O−C) +17°6 +0″03.

104 OΣ 4

R.A. $0^h\ 14^m\ 07^s$ Decl. +36° 13′

8.1 8.8 G0

1923.024	255°6	0″27	3^h0	2
.835	254.2	0.29	22.6	4
.846	255.8	0.31	22.2	3
1923.57	255.2	0.29	3n	

Comparison with Russell's preliminary orbit (*loc. cit.*) gives the residuals (O−C) −11°3 +0″06.

—— Milburn 244

R.A. $0^h\ 18^m\ 00^s$ Decl. +64° 09′

12.0 13.0 ——

1923.818	288°8	2″53	22^h3	2
.846	290.3	2.34	22.9	3
1923.83	289.6	2.44	2n	

153 β 1093

R.A. $0^h\ 18^m\ 20^s$ Decl. +10° 42′

7.0 7.8 A0

1926.731	90°2	0″50	0^h0	3
.740	86.4	0.51	23.3	3
1926.74	88.3	0.50	2n	

236 β 1094

R.A. $0^h\ 27^m\ 32^s$ Decl. +59° 42′

6.0 9.5 B9

1925.803	240°6	0″58	22^h1	3

314 Ho 212 13 *Ceti*

R.A. $0^h\ 32^m\ 40^s$ Decl. −3° 52′

5.6 6.4 G0

1923.607	283°4	0″30	0^h7	4

My orbit (*Publ. Lick Obs.* **12**, 6, 1914) gives for this date: 294°3 0″23.

489 β 1099

R.A. $0^h\ 53^m\ 45^s$ Decl. +60° 06′

6.0 6.7 A0

1925.803	6°3	0″28	22^h6	3
.820	6.0	0.30	22.4	3
1925.81	6.2	0.29	2n	

The angular motion since discovery in 1889 amounts to +96°. It is possible that the point of maximum separation of the components has now been passed and that the motion, which has averaged only 2° a year in the past 11 years, may become more rapid.

508 β 302

R.A. $0^h\ 55^m\ 39^s$ Decl. +21° 08′

6.5 8.0 A0

1922.646	109°2	0″63	22^h5	2

Very slow angular motion, with little change in distance. The discovery position was 1876.27 92°5 0″75.

541 OΣ 21

R.A. $1^h\ 00^m\ 08^s$ Decl. +47° 07′

6.4 8.4 F0

1923.835	188°7	<0″15	22^h9	3

Elongation was certain, but the distance so small as to make the measures subject to considerable uncertainty.

—— Milburn 152

R.A. $1^h\ 08^m\ 21^s$ Decl. +57° 20′

9.8 12.2 ——

1923.818	76°5	1″32	22^h9	2
.846	74.8	1.30	23.4	3
1923.83	75.6	1.31	2n	

The principal star is B.D. +56°213, 9.2 mag.

743 β 1163

R.A. $1^h\ 21^m\ 49^s$ Decl. −7° 16′

6.6 6.8 F0

1923.684	200°5	0″13	2^h8	3

Measures were very difficult. On two later nights I failed to get satisfactory measures because of the small angular separation.

878 H 2069

R.A. $1^h\ 40^m\ 21^s$ Decl. +53° 02′

9.0 9.5 —

1924.801	240°9	18″23	21^h9	2

Measured to identify the pair. The components are respectively +52°418 (9.0) and +52°417 (9.5).

967 Hu 422

R.A. $1^h\ 47^m\ 14^s$ Decl. −14° 29′

8.9 9.2 F5

1922.786	3°7	0″29	2^h0	4
1923.005	3.1	0.26	2.5	2
1922.90	3.4	0.28	2n	

Hussey found: 1901.90 25°7 0″29 3n.

1036 β 513 48 *Cassiopeiae*

R.A. $1^h\ 57^m\ 49^s$ Decl. +70° 40′

4.7 7.2 A3

1923.887	206°9	0″86	22^h9	2

Rabe's elements (letter) give for this date 208°1 0″85.

1074 Σ 208

R.A. $2^h\ 00^m\ 48^s$ Decl. +25° 42′

5.7 7.9 F5

1925.803	174°6	0″39	23^h8	3

It is not yet certain that the point of minimum angular distance has been passed. The change in angle since Struve's first measures amounts to 150°.

1164 Σ 234

R.A. $2^h\ 13^m\ 41^s$ Decl. +61° 07′

8.5 9.4 G0

1923.835	273°6	0″24	23^h2	3
.846	274.4	0.28	23.7	3
1923.84	274.0	0.26	2n	

1209 Joy 1 *o Ceti*

R.A. $2^h\ 16^m\ 49^s$ Decl. −3° 12′

Var. 10. Md

1923.799	132°3	1″01	1^h5	2
.818	130.6	0.93	1.3	3
.835	128.2	0.81	0.3	2
.846	130.2	0.84	0.6	2
.908	130.1	0.94	0.4	3
1924.04	Companion invisible. 3n			
1924.678	131.5	0.80	2.0	2
.681	130.5	0.80	0.8	2
.694	131.3	0.92	1.0	2
.700	133.0	0.83	1.0	3
1925.565	128.9	0.78	2.0	3
1925.80	Companion invisible. 2n			
1926.688	188.7(?)	0.31(?)	1.8	3

o Ceti is listed as **1209** in Burnham's *General Catalogue*, but that refers to the wide pair H VI, 1, and not to the close faint companion whose existence was determined by Joy from the peculiarities of the spectrum of the variable star. The circumstances relating to this discovery and to its verification by me have been recorded elsewhere (*Publ. A. S. P.*, **35**, 323, 1923). The invisibility of the companion in January, 1924, and in October, 1925, was due simply to the brilliance of the primary. The measure in 1926 is not very certain; a companion was suspected in the position given and nothing was seen in the position of the earlier measures, and the seeing was good enough to make it probable that a star would have been seen if it were in that position and as bright as in 1923–1925.

1235 Σ 257

R.A. $2^h\ 21^m\ 54^s$ Decl. +61° 20′

7.8 8.1 B8

1923.835	282°0	0″22	23^h4	3
.846	285.6	0.25	23.9	3
1923.84	283.8	0.24	2n	

1299 OΣ 42

R.A. $2^h\ 29^m\ 53^s$ Decl +52° 05′

7.0 7.5 A2

1923.835	208°2	0″12	23^h7	3

It is possible that the angle should be 28°2; the components were not separated, and the magnitude estimates are therefore uncertain.

1471 β 524

R.A. $2^h\ 50^m\ 33^s$ Decl. +38° 08′

5.6 6.7 F0

1923.835	268°6	0″14	23^h9	3
.846	268.6	0.17	0.2	3
1923.84	268.6	0.16	2n	

The position predicted by my orbit (*Publ. Lick Obs.* **12**, 20, 1914) is 286°5 0″14.

1649 Hu 432

R.A. $3^h\ 15^m\ 24^s$ Decl. −14° 18′

9.9 9.9 G5

1922.786	12°7	0″16	2^h2	3

Hussey's discovery position was 1901.87 46°2 0″16 2n.

1650 A.C. 2 95 *Ceti*

R.A. $3^h\ 15^m\ 48^s$ Decl. −1° 07′

5.9 9.0 G5

1926.688	216°3	0″65	3^h1	3

1657 β 1177

R.A. $3^h\ 16^m\ 18^s$ Decl. −1° 13′

10.6 10.6 G5

1926.688	318°1	0″28	3^h2	3

The total angular motion since discovery in 1890 has been 66°6.

1747 Σ 400

R.A. $3^h\ 30^m\ 55^s$ Decl. +59° 52′

7.0 8.0 F5

1925.820	51°7	0″43	1^h6	3

Both components appeared perfectly round.

1761 Σ 412 7 *Tauri*

R.A. $3^h\ 31^m\ 29^s$ Decl. +24° 18′

6.6 6.7 A2

1923.835	79°3	0″22	0^h9	2
.846	80.2	0.24	1.2	2
1923.84	79.8	0.23		

My orbit (*L. O. B.* **11**, 66, 1923) gives for this date: 79°2 0″23.

1856 β 536

R.A. $3^h\ 43^m\ 17^s$ Decl. +24° 02′

8.4 9.4 K2

1923.835	198°4	0″29	1^h1	2

1900 OΣ 65

R.A. $3^h\ 47^m\ 18^s$ Decl. +25° 26′

6.0 6.3 A3

1923.846	209°2	0″55	1^h5	2

2109 Σ 518 o^2 *Eridani* BC

R.A. $4^h\ 13^m\ 12^s$ Decl. −7° 41′

9.4 10.8 B9, Mdp

1924.056	8°9	3″76	4^h8	3
.081	8.0	3.89	5.2	3
1924.07	8.4	3.82	2n	

The residuals computed from Doolittle's elements (*Proc. Am. Phil. Soc.* **42**, 170, 1903) are: +5°2 +0″72.

2172 Hu 304 66 *Tauri*

R.A. $4^h\ 21^m\ 08^s$ Decl. +9° 21′

5.8 5.8 A2

1924.133	54°5	0″24	5^h4	3
.188	57.8	0.22	7.1	2
1924.16	56.2	0.23	2n	

In 1901.39 Hussey found: 23°9 0″25 3n.

2187 β 1185

R.A. $4^h\ 22^m\ 56^s$ Decl. +18° 45′

8.2 8.8 G0

1924.133	10°7	0″22	5^h6	3
.188	15.5	0.23	7.3	2
1924.16	13.1	0.22	2n	

My orbit (*L. O. B.* **11**, 70, 1923) gives for this date: 10°9 0″19.

12992 Hu 1080

R.A. $4^h\ 26^m\ 07^s$ Decl. +16° 03′

7.7 8.2 G0

1924.133	78°6	0″30	6^h0	3
.188	71.3	0.27	7.5	2
1924.16	75.0	0.28	2n	

The quadrant was marked "fol." on both nights. Earlier measures make it *preceding*, and my observing note may possibly be in error.

2230 Σ 554

R.A. $4^h\ 27^m\ 17^s$ Decl. +15° 32′

5.7 9.0 F0

1924.133	25°5	1″16	6^h1	3
.188	25.0	1.04	7.7	2
1924.16	25.2	1.10	2n	

The distance is steadily increasing with very slow increase in angle. The orbit by van den Bos (*M. N.* **81**, 474, 1921) gives for this date: 24°7 1″16.

2279 β 1295 2 *Camelopardi*

R.A. $4^h\ 36^m\ 00^s$ Decl. +53° 23′

5.4 7.4 F0

1924.744	152°7	0″25	2^h5	3

AB and C = Σ 566 7.4

1924.744	273°1	1″32	2^h7	3

2335 Hu 612

R.A. $4^h\ 43^m\ 51^s$ Decl. +53° 13′

7.0 9.0 F2

1924.744	254°5	0″23	2^h9	3

The position angle has increased about 56° since discovery in 1902, with little change in distance.

2381 β 883

R.A. $4^h\ 48^m\ 25^s$ Decl. +10° 59′

7.7 7.7 F5

1924.133	265°8	0″17	6^h5	2
1925.050	310.2	0.15	3.7	2

The residuals computed from my elements (*Pub. Lick Obs.* **12**, 35, 1914) are respectively: +11°4 +0″06 and +11°9 +0″01. Nearly three revolutions have been completed since Burnham's first measure in 1879.

2383 β 552

R.A. $4^h\ 49^m\ 00^s$ Decl. +13° 34′

6.7 10.0 F5

1924.133	273°4	0″85	6^h7	2
1925.009	272.0	0.90	3.0	2
.031	272.1	0.83	4.0	2
.050	270.8	0.85	3.9	2
1925.03	271.6	0.86	3n	

The residuals for 1924.13 and 1925.03 computed from my elements (*L. O. B.* **11**, 72, 1923) are: −0°6 +0″14 and −4°0 +0″15.

2460 β 314

R.A. $4^h\ 56^m\ 48^s$ Decl. −16° 27′

6.1 6.4 F2

1925.995	152°	0″13±	3^h7	3

The star was carefully identified. On December 26, 1925, no elongation could be detected. On December 30, with better conditions, it appeared to be elongated in the position angle given. Earlier measures give distances ranging from 0″43 (1876) to 1″10 (1907).

J.C. 782 Hu 1222

R.A. $5^h\ 04^m\ 49^s$ Decl. +14° 09′

9.4 9.6 A2

1922.764	148°9	0″20	3^h0	3
.786	157.0	0.22	3.1	3
1922.78	153.0	0.21	2n	

Hussey, in 1904.70, found: 169°4 0″22 1n.

13051 Hu 614

R.A. $5^h\ 18^m\ 33^s$ Decl. +32° 28′

1924.188	2°9	3″22	8^h1	2

The principal star is red, the companion blue. The bright star is UV *Aurigae*, a variable star with peculiar spectrum, possibly class R. (H.D.C.). At the time of measure, the difference in brightness of the two components was estimated at 1.5 magnitudes.

2780 Σ 728

R.A. $5^h\ 28^m\ 06^s$ Decl. +5° 55′

4.4 7.0 B3

1925.05	77°5	0″27	4^h3	2

2857 β 1240 26 *Aurigae*

R.A. $5^h\ 35^m\ 26^s$ Decl. +30° 28′

6.0 6.4 A2

1924.133	118°7	0″14	7^h4	3

—— TU *Tauri*

R.A. $5^h\ 42^m\ 10^s$ Decl. +24° 24′

1924.87. This variable star, marked class Nb in the Henry Draper Catalogue, has been found by Shane to have a composite spectrum—A and N. Careful examination failed to show any evidence of a companion. If one exists, it must be faint and the angular separation of the pair small.

2956 Hu 39

R.A. $5^h\ 44^m\ 10^s$ Decl. +21° 52′

9.0 9.1 G0

1921.932	67°3	0″14	3^h2	2
1922.786	59.5	0.17	3.4	3
1923.005	66.7	0.22	3.6	2
1922.57	64.5	0.18	3n	

Hussey, in 1900.06, found: 43°2 0″30 3n.

3167 Σ 850

R.A. $6^h\ 05^m\ 42^s$ Decl. −3° 56′

9.0 10.5 F2

1926.066	17°8	2″04	4^h4	3

3291 β 895

R.A. $6^h\ 16^m\ 49^s$ Decl. +28° 27′

7.8 7.8 A3

1923.879	111°1	0″19	7^h8	3
1924.119	115.6	0.17	10.3	3
1924.00	113.4	0.18	2n	

AB and C=Σ 888 9.2

1923.879	253°4	2″92	8^h0	3

Van den Bos's ephemeris (*B. A. N.* **2**, 25, 1923) for the close pair gives: 1924.00 119°4 0″18.

3497 OΣ 150

R.A. $6^h\ 35^m\ 47^s$ Decl. +42° 03′

8.5 9.0 A0

1922.843	116°3	0″14	3^h2	2
1924.744	Too close to measure.			3

3596 A.G.C. 1 *Sirius*

R.A. $6^h\ 42^m\ 57^s$ Decl. −16° 39′

−1.6 8.5 A0, F

1923.021	61°9	11″13	8^h1	3
.178	62.5	11.28	6.8	2
.202	61.6	11.28	7.1	2
1924.045	59.2	11.32	6.4	2
.133	59.2	11.23	6.8	3
1925.050	57.8	11.38	6.6	2
.195	59.3	10.84	7.8	2
.220	59.4	10.96	7.7	2
1926.066	55.7	10.98	6.8	4
.106	56.6	11.01	7.5	3
.241	55.5	10.97	7.9	3

The following residuals (O−C) are from places computed from my orbit (*L. O. B.* **9**, 184, 1918):

	θ_0		$\Delta\theta$	$\Delta\rho$
1923.13	62°0	11″23	+0°4	−0″12
1924.09	59.2	11.28	−0.6	−0.03
1925.16	58.8	11.06	+1.0	−0.18
1926.14	55.9	10.99	+0.1	−0.14

4355 OΣ 185

R.A. $7^h\ 54^m\ 43^s$ Decl. +1° 16′

7.1 7.3 F8

1922.764	60°5	0″23	6^h5	3
1924.133	58.1	0.23	7.7	3
.267	57.4	0.28	9.4	2
1924.20	57.8	0.26		

4414 β 581

R.A. 8h 01m 36s Decl. +12° 26′

8.6 8.6 G5

1924.119	190°.7	0″.35	10h.3	3
.133	190.1	0.37	9.1	3
.267	190.8	0.29	9.8	2
1924.17	190.5	0.34	3n	

The position for this date computed from my elements (*Pub. Lick Obs.* **12**, 53, 1914) is: 171°.7 0″.40.

4507 β 904

R.A. 8h 11m 19s Decl. −5° 36′

8.8 10.5

1923.232	80°.4	3″.16	8h.2	2

No change since discovery in 1880.

4537 β 905

R.A. 8h 14m 11s Decl. −16° 10′

7.6 10.5 K0

1923.232	11°.1	3″.47	8h.4	2

No change since discovery in 1879.

4570 Σ 1216

R.A. 8h 18m 48s Decl. −1° 26′

7.0 8.0 A0

1923.231	227°.7	0″.48	8h.6	2

4668 β 205

R.A. 8h 30m 55s Decl. −24° 16′

6.9 7.0 A5

1924.133	189°.4	0″.47	8h.8	2
.267	188.1	0.42	8.9	2
1924.20	188.8	0.44	2n	

4714 β 208

R.A. 8h 36m 58s Decl. −22° 30′

5.2 8.5 G5

1924.133	200°.1	1″.31	8h.6	3
.267	198.0	1.37	9.2	2
1924.20	199.0	1.34	2n	

There is no evidence as yet that the point of apparent maximum separation has been reached; until this point has been passed, any orbit that may be computed will be uncertain.

J.C. 1849 A.G. ——

R.A. 8h 51m 55s Decl. +16° 48′

9.2 9.3 ——

1922.811	231°.1	1″.39	6h.9	3

Noted as double in A.G. Berlin A. The only other measure is my own in 1912.

5235 A.C. 5 8 *Sextantis*

R.A. 9h 50m 02s Decl. −7° 52′

5.8 6.1 A2

1923.350	57°.5	0″.57	11h.4	2
.372	58.9	0.57	11.5	2
1923.36	58.2	0.57	2n	

The computed position from Schroeter's elements (*A. N.* **178**, 189, 1908) is: 48°.7 0″.60.

5652 β 1077 *α Ursae Majoris*

R.A. 11h 00m 41s Decl. +62° 01′

2.0 11.0 K0

1925.349. No certain evidence of presence of the companion, though at moments one was suspected at 0″.3 in 355°. Seeing was good.

5707 Σ 1517

R.A. 11h 11m 06s Decl. +20° 25′

7.5 7.5 G0

1923.320	246°.7	0″.24	13h.4	3
.350	248.3	0.26	13.6	2
.416	248.7	0.23	13.3	2
1923.36	247.9	0.24	3n	

5841 Σ 1555

R.A. 11h 33m 40s Decl. +28° 03′

6.4 6.8 A3

1923.320	22°.1	0″.15	13h.7	3
.350	22.3	0.20	13.9	2
.416	18.7	0.20	13.6	2
1923.36	21.0	0.18		

5848 β 456

R.A. 11h 34m 18s Decl. −12° 04′

9.9 10.2 G5

1924.119	53°.2	0″.51	11h.7	2
.327	52.9	0.52	11.1	2
1925.335	55.0	0.54	11.8	3
.349	56.0	0.53	12.0	3
1924.22	53.0	0.52	2n	
1925.34	55.5	0.54	2n	

The angular motion since the first measures in 1877 has been 167°; the distance, which was 0″.65 in 1877, reached a minimum value of 0″.27 in the years 1902–1906, and has since been slowly increasing.

5951 β 794

R.A. 11h 51m 03s Decl. +74° 02′

7.1 8.1 F8

1923.492	30°.1	0″.38	15h.2	3
.509	26.2	0 43	15.9	2
.520	29.8	0.49	15.9	3
.525	25.5	0.47	15.7	3
1924.382	27.7	0.45	14.2	2
1925.349	32.7	0.42	14.2	3

5951—(*Continued*)

Comparison with my orbit (*L. O. B.* **11**, 77, 1923) follows:

	θ_0	ρ_0	$\Delta\theta$	$\Delta\rho$ (O−C)
1923.51	27°9	0″44	−3°6	+0″14
1924.38	27.7	0.45	−7.6	+0.14
1925.35	32.7	0.42	−7.5	+0.10

6406 Σ 1728 42 *Comae Berenices*

R.A. 13^{h} 07^{m} 36^{s} Decl. +17° 47′

5.2 5.2 F5

1923.530 Image round with 1500-power.

1924.500	13°6	0″16	$15^{h}7$	2
1925.335	11.2	0.25	13.1	3

The distances for the three dates, according to Russell's orbit (*Pub. A. A. S.* **3**, 328, 1917) were: 0″05, 0″14, and 0″21; the position angles for the last two dates: 13°5 and 13°2.

6578 β 612

R.A. 13^{h} 37^{m} 07^{s} Decl. +11° 00′

6.3 6.3 F2

1923.350	236°4	0″24	$14^{h}4$	2
.369	236.0	0.25	16.0	3
1925.335	253.9	0.20	13.4	3
.398	255.6	0.24	13.0	2

The following comparison is with places from my orbit (*Pub. Lick Obs.* **12**, 86, 1914):

	θ_0	ρ_0	$\Delta\theta$	$\Delta\rho$ (O−C)
1923.36	236°2	0″24	+4°4	−0″04
1925.37	254.8	0.22	+9.0	0.00

6663 β 614

R.A. 13^{h} 51^{m} 29^{s} Decl. +10° 23′

8.2 11.2 F0

1926.411	227°8	0″30	$14^{h}7$	3

6711 β 1270

R.A. 14^{h} 01^{m} 15^{s} Decl. +8° 44′

8.5 8.6 F5

1923.350	291°4	0″26	$14^{h}6$	2
.369	292.3	0.26	16.2	3
1925.335	307.3	0.26	13.7	3
.398	305.1	0.25	13.2	2
1926.411	309.3	0.28	14.8	3
.476	314.2	0.25	15.0	3
.496	312.4	0.23	15.6	3

Comparison with places computed from my orbit (*L. O. B.* **11**, 79, 1923) shows residuals (O−C) as follows:

	θ_0	ρ_0	$\Delta\theta$	$\Delta\rho$
1923.36	291°8	0″26	−5°3	0″00
1925.37	306.2	0.26	−4.1	−0.02
1926.46	312.0	0.25	−0.3	−0.04

6832 Σ 1834

R.A. 14^{h} 18^{m} 28^{s} Decl. +48° 44′

8.0 8.1 F8

1923.492	91°0	0″48	$16^{h}7$	3
.525	89.8	0.43	17.9	3
1924.503	92.5	0.45	17.0	3
.525	89.4	0.46	17.5	2

Van den Bos published an orbit for this pair in 1921 (*Proc. Acad. Sci. Amsterdam*, **24**, 72, 1921). Comparison with his ephemeris gives:

	θ_0	ρ_0	$\Delta\theta$	$\Delta\rho$ (O−C)
1923.51	90°4	0″46	−7°8	+0″09
1924.51	91.0	0.46	−7.7	+0.06

6842 β 1111

R.A. 14^{h} 20^{m} 55^{s} Decl. +8° 40′

7.4 7.4 A0

1923.350	249°0	0″22	$14^{h}8$	2
.369	247.3	0.18	16.4	3
1925.335	272.8	0.17	14.1	3
.398	273.3	0.21	13.5	2
1926.411	279.7	0.21	15.7	2
.476	287.5	0.19	15.9	3
.496	281.0	0.22	15.8	3

My orbit, computed in 1919 (*Publ. A. S. P.* **31**, 285, 1919), represents these measures within the error of observation:

	θ_0	ρ_0	$\Delta\theta$	$\Delta\rho$ (O−C)
1923.36	248°6	0″20	−0°7	+0″01
1925.37	273.0	0.19	+3.3	0.00
1926.46	282.7	0.21	+2.1	+0.02

6955 Σ 1865 ζ *Bootis*

R.A. 14^{h} 38^{m} 45^{s} Decl. +13° 57′

4.5 4.9 A2

1923.416	314°4	0″94	$13^{h}9$	2
.427	315.5	0.88	13.7	3
1923.42	315.0	0.91	2n	

My orbit (*Publ. A. S. P.* **28**, 201, 1916) gives for this date: 318°4, 1″04.

7013 Σ 1883

R.A. 14^{h} 46^{m} 24^{s} Decl. +6° 10′

7.5 7.5 F8

1923.350	208°6	0″29	$15^{h}0$	2
.369	208.3	0.26	16.6	3
1925.335	199.8	0.25	14.5	3
.398	198.3	0.23	13.8	2
1926.476	194.8	0.26	16.5	3
.496	195.7	0.24	16.4	3

1923.36	208°4	0″28	2n
1925.37	199.0	0.24	2n
1926.49	195.2	0.25	2n

7332 OΣ 298

R.A. 15^{h} 34^{m} 19^{s} Decl. +39° 58′

7.3 7.6 K0

1923.342	211°4	0″68	$13^{h}1$	2
.372	210.4	0.74	12.2	2
.394	211.1	0.75	12.5	2
1923.37	211.0	0.72	3n	

Celoria's orbit (*A. N.* **119**, 163, 1888) gives for this date: 207°8 0″87.

7360 Hu 580 *ι Serpentis*

R.A. $15^h 39^m 19^s$ Decl. +19° 50′

5.2 5.2 A2

1923.369	245°.9	0″.18	$17^h.6$	3
.372	248.3	0.20	12.5	2
.394	246.2	0.21	12.8	2
1923.38	246.8	0.20	3n	
1924.557	247.2	0.15	17.8	3

7487 Σ 1998 *ξ Scorpii*

R.A. $16^h 01^m 37^s$ Decl. −11° 14′

4.8 5.1 F8

1923.369	184°.1	0″.99	$16^h.8$	3
.424	183.7	0.98	17.4	4
1923.40	183.9	0.98	2n	

My orbit (*L. O. B.* **3**, 147, 1905) gives for this date: 185°.8 1″.20.

7506 β 949

R.A. $16^h 05^m 43^s$ Decl. −9° 58′

7.3 7.4 F8

1923.369	188°.9	0″.34	$17^h.0$	3
.424	190.1	0.37	17.6	4
1923.40	189.5	0.36	2n	
1925.532	191.2	0.40	16.3	3

7591 Hu 481

R.A. $16^h 19^m 07^s$ Decl. +23° 06′

7.7 9.5 F8

1922.622	201°.1	0″.62	$18^h.1$	2

Hussey, in 1902.49, found: 227°.5 0″.51 3n.

7642 Σ 2052

R.A. $16^h 26^m 43^s$ Decl. +18° 31′

7.8 7.8 K0

1923.369	275°.9	0″.59	$17^h.8$	3
.520	277.2	0.59	18.1	2
1923.44	276.6	0.59		

Jackson's orbit (*Greenwich Catalogue of Double Stars*) gives for this date: 279°.9, 0″.66.

7694 A.G. 204

R.A. $16^h 35^m 32^s$ Decl. +1° 45′

9.0 10.0 ——

1926.529	12°.2	3″.95	$17^h.5$	3

The quadrant was definitely noted as *north;* in Burnham's G. C. it is given as *south.*

7726 β 953

R.A. $16^h 36^m 56^s$ Decl. +69° 53′

8.6 9.1 F5

1923.698	148°.7	0″.37	$19^h.2$	3

7717 Σ 2084 *ζ Herculis*

R.A. $16^h 39^m 27^s$ Decl. +31° 41′

3.0 6.0 G0

1923.561	69°.9	1″.33	$18^h.2$	3
.616	70.4	1.37	19.2	4
1923.59	70.2	1.35	2n	

Comstock's orbit (*A. J.* **30**, 139, 1917) gives for this date: 72°.8, 1″.57.

7748 Δ 15

R.A. $16^h 42^m 23^s$ Decl. +43° 34′

8.9 9.1 K5

1925.398	208°.9	0″.62	$14^h.4$	2

Jackson's orbit (*M. N.* **80**, 556, 1920) gives: 1925.40 216°.9, 0″.58.

7778 Σ 2106

R.A. $16^h 48^m 44^s$ Decl. +9° 30′

7.0 8.7 F8

1923.369	261°.4	0″.25	$18^h.2$	2
.561	255.4	0.25	17.9	3
.616	263.0	0.20	18.8	3
1923.52	259.9	0.23	3n	

7783 Σ 2107

R.A. $16^h 49^m 51^s$ Decl. +28° 45′

6.8 8.3 F5

1923.561	36°.8	0″.72	$18^h.7$	3
.616	38.5	0.70	19.0	3
1923.59	37.6	0.71		

Jackson's orbit (*M. N.* **80**, 557, 1920) gives: 1923.59 50°.8, 0″.63.

13364 Hu 1176 c *Herculis*

R.A. $17^h 06^m 16^s$ Decl. +36° 00′

6.1 6.1 A5

1923.692	54°.7	0″.09	18°.7	3
1924.557	22.7	0.11	19.2	3

Measures were very difficult on both nights because of the small angular separation. My orbit (*L. O. B.* **11**, 83, 1923) gives for the two dates, respectively: 54°.6, 0″.13 and 32°.5, 0″.11.

8162 A.C. 7 *μ^1 Herculis*

R.A. $17^h 44^m 32^s$ Decl. +27° 46′

10.0 10.5 Mb

1923.692	245°.2	1″.04	$19^h.0$	3
.698	243.2	1.00	20.8	3
1924.653	246.3	1.09	19.8	3
.672	248.9	0.96	19.6	3
.689	248.5	1.13	19.7	2
.708	248.9	1.01	20.6	3
1925.313	255.0	1.01	16.4	3
1926.329	261.4	0.74	14.5	3

8162—(*Continued*)

The following comparison is with places computed from my orbit (*Pub. Lick Obs.* **12**, 119, 1914)

	θ_0	ρ_0	$\Delta\theta$	$\Delta\rho$ (O−C)
1923.70	244°.2	1″.02	+0°.9	−0″.02
1924.68	248.2	1.05	−0.9	+0.04
1925.31	255.0	1.01	+1.9	+0.04
1926.33	261.4	0.74	+1.1	−0.16

8340 Σ 2272 70 *Ophiuchi*

R.A. $18^h\ 02^m\ 54^s$ Decl. +2° 32′

4.1 6.1 K0, K4

1924.650	128°.9	6″.18	$19^h.9$	2

The position for this date computed from Pavel's elements (*A. N.* **212**, 347, 1921) is: 128°.3, 6″.09.

8353 OΣ 341

R.A. $18^h\ 03^m\ 43^s$ Decl. +21° 26′

7.3 8.2 G0

1923.604	88°.3	0″.43	$20^h.6$	3
.678	89.0	0.50	19.6	3
.692	89.7	0.48	19.4	3
1924.653	90.0	0.55	20.2	3
.672	91.6	0.53	20.0	3
.689	88.3	0.54	20.2	2

Comparison with places computed from my orbit (*L. O. B.* **11**, 85, 1923) gives:

	θ_0	ρ_0	$\Delta\theta$	$\Delta\rho$ (O−C)
1923.66	89°.0	0″.47	+0°.1	+0″.05
1924.67	90.0	0.54	+0.2	+0.06

8380 Σ 2281 73 *Ophiuchi*

R.A. $18^h\ 07^m\ 05^s$ Decl. +3° 59′

5.9 7.4 F2

1923.369	68°.7	0″.65	$18^h.5$	2
.424	69.8	0.73	19.1	3
1923.40	69.2	0.69	2n	

Russell's orbit (*Publ. A. A. S.* **3**, 329, 1917) gives for this date: 68°.9, 0″.51; Jackson's orbit (*Green. Cat. Double Stars*): 67°.7, 0″.54.

8430 Hu 196

R.A. $18^h\ 12^m\ 30^s$ Decl. +8° 58′

9.2 9.4 K0

1922.660	339°.2	0″.26	$18^h.9$	2

Hussey has: 1900.59 345°.1 0″.25 3n.

8489 Hu 197

R.A. $18^h\ 17^m\ 19^s$ Decl. +10° 15′

8.3 9.3 G0

1922.660	285°.4	0″.40	$19^h.2$	2

Hussey gives: 1900.58 28°.0 0″.36 3n.

8535 A.C. 11

R.A. $18^h\ 22^m\ 22^s$ Decl. −1° 36′

6.8 6.8 F5

1923.369	360°.0	0″.34	$18^h.3$	3
1923.424	358.6	0.33	18.8	3
1924.650	357.8	0.39	19.3	2
1923.81	358.8	0.35	3n	

8662 OΣ 359

R.A. $18^h\ 33^m\ 25^s$ Decl. +23° 34′

6.4 6.7 K0

1923.678	84°.3	0″.12	$19^h.9$	3
.692	83.0	0.10	20.2	3
1923.68	83.6	0.11	2n	

The quadrant is indeterminate, and is recorded here to agree with my earlier measures.

8877 Hu 199

R.A. $18^h\ 49^m\ 48^s$ Decl. +11° 44′

8.7 9.1 F5

1922.660	351°.3	0″.39	$19^h.8$	2

For comparison we have: 1900.60 3°.5 0″.23 3n Hussey.

8933 β 648

R.A. $18^h\ 55^m\ 09^s$ Decl. +32° 50′

5.3 8.8 G0

1924.653	33°.2	1″.09	$20^h.5$	3
1925.313	34.6	1.38	17.0	3
1924.98	33.9	1.24	2n	

The orbit by van den Bos (*B. A. N.* **1**, 57, 1922) gives for this date: 29°.4 0″.99.

8965 Winlock —— ζ *Sagittarii*

R.A. $18^h\ 59^m\ 26^s$ Decl. −29° 57′

3.4 3.6 A2

1926.698	288°.6	0″.25	$18^h.9$	3
.720	288.1	0.28	19.2	3
1926.71	288.4	0.26	2n	

My orbit (*Pop. Astron.* **9**, 57, 1901) gives for this date: 299°.2 0″.28.

9011 Hu 757

R.A. $19^h\ 00^m\ 56^s$ Decl. +52° 11′

10.0 10.2 ——

1923.604	72°.6	0″.29	$21^h.2$	3
.659	70.8	0.29	21.2	4
1923.63	71.7	0.29		

This is the fainter component of Σ 2450. Hussey gives: 1904.53 93°.0 0″.23 2n.

8993 H.N. 126

R.A. 19^h 01^m 21^s Decl. −21° 36′

7.7 7.8 G0

1926.592	252°.4	0″.92	19^h.2	2
.614	250.6	0.91	19.2	2
1926.60	251.5	0.92		

—— B 427

R.A. 19^h 06^m 51^s Decl. −19° 53′

7.0 7.0 K0

1926.698	296°.8	0″.12	19^h.4	3
.720	297.7	0.14	19.6	3
1926.71	297.2	0.13	2n	

This pair and *ψ Sagittarii*, below, were discovered by van den Bos with the 26½-inch refractor of the Union Observatory. They are both very difficult objects to measure with the 36-inch refractor here.

9114 Secchi 2

R.A. 19^h 09^m 28^s Decl. +38° 37′

8.7 8.7 G5

1925.313	87°.6	0″.54	17^h.5	3

The position for this date predicted by Russell's orbit (*Publ. Lick Obs.* **12**, 138, 1914) is: 89°.7, 0″.60.

—— B 430 *ψ Sagittarii*

R.A. 19^h 12^m 28^s Decl. −25° 21′

5.6 5.6 F5

1926.698	109°.5	0″.16	19^h.2	3
.720	111.9	0.20	19.4	3
1926.71	110.7	0.18	2n	

9643 A.G.C. 11 *ζ Sagittae*

R.A. 19^h 46^m 45^s Decl. +19° 01′

5.3 6.3 A2

1924.689	156°.9	0″.27	20^h.8	2
.724	158.4	0.26	21.2	2
1924.71	157.6	0.26	2n	

Van Biesbroeck's orbit (*A. J.* **29**, 163, 1916) gives for this date: 164°.4, 0″.27.

10643 Σ 2737 *ε Equulei*

R.A. 20^h 56^m 34^s Decl. +4° 06′

5.8 6.3 F5

1924.546
1926.698 } Elongated on both nights in a north preceding-south following direction. Angular separation very small and good measures could not be made.

10829 OΣ 535 *δ Equulei*

R.A. 21^h 12^m 03^s Decl. +9° 48′

5.3 5.4 F5

1924.546	184°.4	0″.14	22^h.2	3
1926.740	20.0	0.36	20.3	3
.756	21.2	0.35	22.7	2
1926.75	20.6	0.36	2n	

My orbit (*Publ. Lick Obs.* **12**, 158, 1914) gives for the two dates, respectively: 193°.1 0″.14 and 24°.2 0″.35.

10846 A.G.C. 13 *τ Cygni*

R.A. 21^h 12^m 48^s Decl. +37° 50′

3.8 8.0 F0

1924.870	167°.0	1″.01	0^h.0	3

According to my orbit (*Publ. Lick Obs.* **12**, 160, 1914) the position should be: 156°.3 0″.94.

10880 β 163

R.A. 21^h 16^m 10^s Decl. +11° 22′

7.0 9.5 G0

1924.546	261°.7	0″.72	22^h.4	3
1926.740	259.4	0.71	20.8	4
.756	257.3	0.65	22.8	2
1926.75	258.4	0.68	2n	

11125 β 1212

R.A. 21^h 36^m 56^s Decl. −0° 17′

7.3 7.8 F8

1924.546	55°	0″.12	21^h.6	3
1926.693	204.2	0.19	22.7	2

The measure in 1924 is quite uncertain, but in 1926 the two components were clearly separated at the best moments. Additional measures are required to verify the quadrants assigned here.

11210 Ho 166

R.A. 21^h 41^m 40^s Decl. +27° 37′

8.6 8.8 F5

1923.818	189°.8	0″.23	23^h.9	3
1924.870	181.6	0.19	23.8	3
1924.915	182.9	....	0.7	2
1925.551	183.6	0.19	19.5	3
1925.11	182.7	0.19	3,2n	

The angular motion is increasing, with diminishing distance. The total motion since Hough's discovery in 1886 exceeds 120°. The quadrant is here given as noted in my observing books, but there is less difference in brightness than is indicated by the magnitudes assigned to the components. Earlier observers reverse the quadrant.

11222 β 989 *κ Pegasi*

R.A. $21^h\ 42^m\ 23^s$ Decl. +25° 25′

5.0 5.1 F5

1923.818	$82^{\circ}.3$	$0''.15$	$0^h.1$	2
1924.870	37.9	0.10	23.6	3
1925.551	171.5	0.13	19.3	3

Comparison with places computed from Lewis's orbit (*Mem. R. A. S.* **56**, 653, 1906) gives the following residuals (O−C):

1923.82	$+19^{\circ}.5$	$+0''.03$
1924.87	+33.2	0.00
1925.51	+18.4	0.00

The large residuals in angle are probably due more to the difficulty of measure than to inexactness in the orbit elements.

—— Milburn 270

R.A. $21^h\ 46^m.9$ Decl. +66° 11′

10.4 10.6

1923.799	$266^{\circ}.9$	$1''.89$	$23^h.7$	2
.818	267.9	....	23.6	2
.876	267.2	1.96	23.8	2
1923.83	267.3	1.92	3,2n	

11761 Krueger 60

R.A. $22^h\ 26^m\ 17^s$ Decl. +57° 27′

9.3 10.8 Mb

1923.799	$344^{\circ}.5$	$1''.55$	$0^h.2$	2
.818	338.6	1.56	0.5	2
1924.870	314.5	....	0.2	3
1924.915	312.8	1.32	1.0	2
1925.009	316.4	1.26	2.7	2
1925.313	306.5	1.49	18.2	3

A and C −9.2

1925.313	$59^{\circ}.0$	$57''.57$	18.5	3

The measures of the close pair, except the final one, were used in computing the orbit given in *Lick Obs. Bulletin* 365 (*L. O. B.* **11**, 45, 1925). The residuals (O−C) derived from comparison with this orbit, according to which the components were at minimum apparent distance in 1925, are:

	$\Delta\theta$	$\Delta\rho$
1923.81	$+4^{\circ}.1$	$+0''.03$
1925.03	−3.3	−0.15

11763 Σ 2912 37 *Pegasi*

R.A. $22^h\ 27^m\ 26^s$ Decl. +4° 11′

5.7 7.1 F5

1926.693	$121^{\circ}.7$	$0''.48$	$23^h.5$	3
.712	117.1	0.55	22.2	3
1926.70	119.4	0.52	2n	

These measures make it certain that the companion has again passed into the second quadrant and that the angular separation will steadily increase for about half a century.

11873 Ho 295

R.A. $22^h\ 36^m\ 34^s$ Decl. +44° 03′

7.5 7.5 G0

1923.799	$332^{\circ}.8$	$0''.29$	$0^h.7$	2
.818	333.3	0.28	0.7	3
1923.81	333.0	0.28	2n	

12008 Ho 482

R.A. $22^h\ 49^m\ 03^s$ Decl. +26° 08′

7.3 7.5 A3

1923.799	$179^{\circ}.1$	$0''.23$	$1^h.1$	2
.818	181.1	0.23	1.0	3
1923.81	180.1	0.23		

12090 OΣ 536

R.A. $22^h\ 56^m\ 01^s$ Decl. +9° 06′

7.2 7.3 G0

1923.684	$166^{\circ}.0$	$0''.28$	$0^h.6$	3

12276 β 79

R.A. $23^h\ 15^m\ 02^s$ Decl. −1° 48′

8.6 9.6 G5

1925.795	$58^{\circ}.9$	$1''.04$	$23^h.0$	3

12290 β 80

R.A. $23^h\ 16^m\ 18^s$ Decl. +5° 08′

8.6 9.6 K0

1925.795	$258^{\circ}.9$	$0''.95$	$0^h.0$	3

Jackson's ephemeris (*M. N.* **80**, 561, 1920) gives for this date: $259^{\circ}.7$, $0''.83$.

12450 β 721

R.A. $23^h\ 33^m\ 43^s$ Decl. −7° 24′

8.6 8.6 A3

1925.784	$131^{\circ}.9$	$0''.23$	$23^h.7$	3
.795	133.5	0.21	23.3	3
1925.79	132.7	0.22		

The distance has been slowly decreasing since Burnham's first measure in 1878, but the change in angle has not exceeded 10°.

January, 1927.

Issued March 4, 1927.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 384

A PHOTOMETRIC STUDY OF THE FLASH SPECTRUM

BY

E. F. Carpenter

Introduction

It is of considerable interest, both as a further check upon a good deal of theoretical work which has been done in recent years and to present a new aspect of observational material at hand bearing upon the physical state of the solar atmosphere, to study the variation of intensity of solar spectral lines as a function of their height in the chromosphere. The best opportunity for this study is offered by the flash spectrum, where the emission lines may be spread out against the dark sky to show in detail their structure as it depends upon the distance from the limb.

The work which has so far been done with the flash spectrum by Campbell,[1] Dyson,[2] Anderson,[3] Mitchell,[4],[5] Hale and Adams,[6] and others, has been mainly a study of wave-lengths with eye estimates of intensities. Mitchell's second list[5] is unique in its study of the heights of the lines, but this in general leaves something to be desired, because even the relative measured heights are, to some extent, a function of the exposure, and hence of the color sensitivity curve of the plate and apparatus used at the eclipse. If, for example, a line whose intensity falls rapidly with increasing height is compared with one for which the intensity law is more gradual, the latter will be measured as of a lesser relative height in regions where the effective exposure is less. On account of the great variation in intensity in the lines of the flash, there seems no accurate way to correct for this effect unless the law of intensity variation is known beforehand. The success of the method lies in the uniformity of intensity distribution laws of most of the prominent lines, which will be discussed later. A difficulty of this sort has already been pointed out by Evershed.[7]

The Spectrograms Available for Study and the Method of Obtaining Them

The spectrograms that were used in this study were obtained at the several Lick Observatory eclipse expeditions with an apparatus devised by Director Campbell and used originally in India in 1898.[8] At later eclipses the form was somewhat modified, and it may be desirable to re-describe the apparatus here.

The ordinary arcs of the flash spectrum are formed in the usual manner by an objective prism with a lens, usually of 60-inch focus, each arc being a monochromatic image of the chromospheric crescent formed by the Moon just before totality. But not the whole arc is photographed. Instead, a narrow focal-plane diaphragm, $^{1}/_{32}$ of an inch wide and the length of the spectrum, with the long dimension in the direction of dispersion, restricts the photography to a small strip of the arc near the middle. Just before the second contact the exposure is started, and the plate is moved, by a glycerin clepsydra, uniformly in its own plane during the exposure in a direction normal to the dispersion. The speed at which the plate is moved depends upon the required exposure; for most cases it is of the order of $^{1}/_{16}$ inch per second, so that the exposure on any part of the plate amounts to about half a second. A similar procedure is carried out while the Moon is uncovering the chromosphere at third contact.

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[1] Campbell: Several papers in *Publ. A. S. P.*, 1900, 1906, 1908, etc.

[2] Dyson: *Phil. Trans.*, *A* **206**, 403, 1906.

[3] Anderson: *Publ. U. S. Naval Obs.*, Second Series, Vol. X. Appendix. *The Total Solar Eclipses of 1905, 1918, 1923*, B 157.

[4] Mitchell: *Ap. J.*, **15**, 97, 1902.

[5] Mitchell: *Ap. J.*, **38**, 407, 1913.

[6] Hale and Adams: *Ap. J.*, **30**, 222, 1909.

[7] Evershed: *Observatory*, **48**, 45, 1925.

[8] Campbell: *Publ. A. S. P.*, **10**, 127, 1898; *Ap. J.*, **11**, 226, 1900; *Publ. A. S. P.*, **18**, 27, 1906.

Plate IX shows a portion of a plate, the Hγ region, obtained in this way. For the purposes of reproduction in Plates IX and X it has of course been necessary to shade the region of the absorption spectrum, so that in these plates the brightness of the absorption lines seems less than that of the emission, while the curves plotted from measures of the original negative of course indicate the reverse. The details of reversal are well shown, and the intensities of the lines at any level are readily measured. As Moore has pointed out,[9] Mitchell's criticism[10] of using this method of observation for measuring the heights of the lines does not hold. There is one objection to this method, however, which might be very serious, namely, that the portion of the chromospheric arc photographed, might be a disturbed region, even a prominence. No method has, to the writer's knowledge, been employed to avoid this situation, but evidence will be presented later to show that the plates used for this paper were exposed upon fairly representative regions of the limb.

Table 1 lists the five plates of this sort which were obtained by four of the Lick Observatory eclipse expeditions. The accurate focus of the apparatus is very difficult to determine, yet there is only one plate with a focus which cannot be classified as at least *good.* The column headed *Linear Dispersion* refers to the spectral interval of the preceding column. In 1908 a film was used instead of a plate. For the first two eclipses the driving mechanism for the plate was a clock, for the last two a glycerin clepsydra.

TABLE 1

Plate No.	Date	Place	Observer	Reference	Focus of telescope in inches	Dispersing apparatus	Spectral region λλ	Linear dispersion mm.	Definition
1	1898	India	Campbell	*Ap. Jour.*, **11**, 226, 1900	20.7	Rowland grating, third order	5150–5500	50	Good
2	1900	Georgia	Campbell	*Publ. A.S.P.*, **12**, 181, 1900	60	Two 60° prisms	3930–5180	153	Good
3	1905	Spain	Campbell	*Publ. A.S.P.*, **18**, 13, 1906	79	Two 60° prisms, (U.V. Glass)	3200–5200	130	Poor
4	1905	Spain	Arrhenius		60	Two 63° prisms	3800–5200	229	Excellent
5	1908	Flint Island	Campbell	*Publ. A.S.P.*, **20**, 79, 1908	60	Three 60° prisms	3800–5100	318	Good

None of these photographs was taken with the possibility of photometry in mind, and most of them are not well suited for it. The film of 1908 is overexposed in the lower chromosphere and all of the plates, except that by Arrhenius, show a rather too small range of density to yield good photometric results. The plate by Arrhenius, however, is in unusually good focus, has a good grain, and a range of density which insures confidence in photometric results derived from it. Only the exposure at contact III is satisfactory. The results described in this paper are based directly upon this plate. The other plates have found use as checks.

The oversight of the possibility of photometry unfortunately resulted in the omission of standard exposures upon the plate, from which the photometric constant might be obtained. The problem thus introduced has been solved with some difficulty, but probably satisfactorily. Rather more serious is the omission of a permanent record of either the kind of plate or developer used. A microscopic comparison of the grains of several plates indicate that the Arrhenius plate of 1905 may be a Seed 27, which was the fastest of the ordinary Seed plates at that time. However, the effect of an unknown developer upon the grain of a plate renders this method of indentification rather unreliable. It is evident from an inspection of the plate that it cannot be panchromatic or isochromatic, and the similarity of the color sensitivity curves of the ordinary Seed plates,[11] justifies the hope that the lack of fully recorded data at the time of the eclipse may not be fatal to a further photometric study of the plate. In the present paper, however, we shall concern ourselves exclusively with the behavior of the individual lines as a function of their height in the chromosphere, so that we need not be materially troubled by ignorance of the kind of plate used.

Method of Measurement

The Photometric Constant.—The measurements were made by means of a Hartmann equalizing wedge microphotometer. By means of screws in the x and y coördinates, any part of any line could be readily brought into the field of view for measurement. In all cases a line was completely measured at equal intervals, usually of one millimeter except where variation was rapid, beginning at the highest chromospheric level of measurable brightness and continuing to the level on the photosphere where the exposure was stopped, or, in the middle of the spectrum, to the region where the photographic density became too great, which was always after the measurements had reached the photospheric spectrum. In general, two settings were made at each position, and were completely repeated if they did not agree within 0.8 of a division of the wedge scale (corresponding to about 15%). Usually the agreement was within eight or ten per cent. Such an agreement was not expected and was not obtained in the very lightly exposed parts of the plate, where the density gradient of the wedge is slight. The photometer readings were interpreted upon the assumption that the photographic wedge and the spectrogram have exposure-density curves of similar shape (but not necessarily of equal contrast); hence the photometric scale was assumed logarithmic, equal differences of scale reading at any part of the scale representing equal percentage differences of intensity. The justification of this assumption is presented later. The curves representing the measures could then be drawn upon an arithmetic scale where a given photometer reading is taken to represent unit brightness. Since we are ignorant of the color sensitivity curve of the entire spectrograph, these units of brightness in the several parts of the spectrum must be quite arbitrary and unrelated, and the slopes of the curves obtained in this manner cannot in general be compared.

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[9] Moore: *Publ. A. S. P.*, **36**, 19, 1924.

[10] Mitchell: *Eclipses of the Sun.* Second ed., 217.

[11] Luckiesh, Holladay, and Taylor: *Journ. Franklin Inst.*, Sept. and Oct., 1923.

In the absence of standard exposure ratios imposed upon the spectrogram at the time of the eclipse, four ways of obtaining the photometric constant of the plate suggest themselves:

1. To utilize the strip of Fraunhofer spectrum by comparing the photometer measures of its intensity variation with wave-length with the measures by H. H. Plaskett on the spectral intensity curve on the Sun.[12]

2. To compare the photometer measures of the strip of Fraunhofer spectrum as a function of distance from the limb with Abbot and Fowle's measures[13] of the variation of intensity along the radius of the solar disk.

3. To photograph without an eclipse and under high dispersion some of the stronger lines of the flash spectrum upon plates having standard exposure ratios, and to use these as intensity standards upon the eclipse spectrogram.

4. To measure the ratios of intensity of background to intensity of line in the Fraunhofer spectrum near the limb and to compare them with photometer measures of the same quantities on the eclipse plate.

It is at once obvious that methods (1) and (2) cannot be used, for the strip of Fraunhofer spectrum upon the eclipse plate includes only the region of the Sun about one per cent of the radius from the limb, if Mitchell's measures of heights of the lines are used as standards of length. So little is known of spectral conditions in this part of the solar disk that these methods would necessitate a very untrustworthy and even highly perilous extrapolation of either Plaskett's or Abbot and Fowle's curves.

Accordingly resort was had to the last two methods. The third seemed the more promising. There would be, in addition to complications arising from ignorance of the color sensitivity curve, some difficulty in establishing on the two plates equivalent heights in the chromosphere at which intensities were to be compared. But if curves are drawn from measures of the eclipse plate showing intensity in units of the photometer scale as a function of chromospheric height, the curves are roughly parallel for several of the prominent lines, and if the photometer scale is logarithmic then the intensity ratios of these lines should not be greatly dependent upon the chromospheric level, and uncertainties in the latter should not greatly affect the photometric constant derived in this way.

Appropriate apparatus for this work being lacking at Mount Hamilton, Dr. W. S. Adams very kindly arranged to let me carry out the work at Mount Wilson. The Snow telescope was used with the 18-foot vertical Littrow grating spectrograph[14] during the latter part of January, 1925, but the conditions of seeing were so highly unsatisfactory that it was exceedingly difficult to photograph even Hγ in emission, though it could be seen readily enough with an eye-piece.

Hence it was decided to use method (4) for obtaining the photometric constant, and, with the Snow telescope, spectrograms were taken of the Fraunhofer spectrum close to the limb on plates having standard exposure ratios imposed by means of a sectored disk kindly loaned by Mr. Ellerman. In order to secure reasonable accuracy, it was necessary to select rather strong Fraunhofer lines for which the variation in the relative intensities of the line and its continuous spectrum background near the limb was as small as possible. Furthermore, the lines should be in a region of the spectrum where the photographic density in the eclipse spectrogram is not too greatly different from the average density of the flash lines, where the definition is at its best, and where the linear dispersion of the eclipse and Snow telescope spectra are nearly equal. Only one plate taken with the Snow telescope proved to fulfill these conditions satisfactorily, and on this plate the lines λ4003.95, λ4005.41, and λ4045.98 were selected as the best, though since λ4003.95 is much less intense than the others the result from it is given half weight. The ratio of brightness of the background to that of the line for each of these three lines is, from nine measures of each line:

λ4003.95	1.72±0.05 (p.e.)
4005.41	2.92±0.09
4045.98	4.04±0.13

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[12] H. H. Plaskett: *Publ. Dom. Astroph. Obs.*, **2**, 12, 252.

[13] Abbot and Fowle: *Annals of Astroph. Obs. of Smithsonian Inst.*, **3**, 153.

[14] St. John and Nicholson: *Cont. M. W. Obs.* No. 208; *Ap. J.*, **53**, 380, 1921.

The accuracy here is greater than that in determining the same quantities in the eclipse plate.

On account of "boiling" at the limb the exact setting of the Sun's image by the Snow telescope upon the slit of the spectrograph is somewhat uncertain, and much of the accuracy of the results depends upon the degree of constancy of the intensity ratio $\frac{\text{background}}{\text{line}}$ near the limb. On the eclipse spectrogram the brightness on both the emission lines and the continuous spectrum background increases away from the limb, but since the Hartmann photometer scale is logarithmic a constant difference between the scale readings of line and background indicates such a constant light ratio. The degree to which such a constant ratio holds is shown by fig. 1 in which the difference of scale reading, background−line, is plotted against distance from the point of reversal away from the limb. The irregularities are for the most part in the original plate: the dark "bands" in the Fraunhofer region parallel to the spectral dispersion indicate that the running of the clepsydra at the eclipse was not smooth.

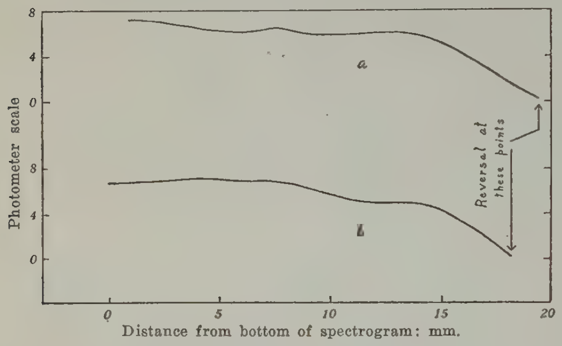

Fig. 1. The Fraunhofer lines: (*a*) λ4005 (Fe), (*b*) λ4046 (Fe). Ordinates are photometer readings: background minus line. Abscissae are distances in mm. from the lower edge of the spectrum (Plate III).

The agreement of the observations with the smooth curve suggests extreme values of the difference in photometer readings beyond which the true value is unlikely to lie, and from them we get the following maximum and minimum values for the photometric constant of the eclipse plate:

Line:	λ4003.95	λ4005.41	λ4045.98
Maximum value:	1.178	1.196	1.240
Minimum value:	1.145	1.165	1.221
Mean:	1.162	1.180	1.230

The weighted mean is 1.195, with a probable extreme range of about 0.013. A difference, then, of n in the photometer readings of two points of the eclipse spectrogram corresponds to a difference of $(1.195)^n$ units in the light represented by these two points on the plate. As has already been pointed out, the unit is quite arbitrary for each line.

THE SOURCES OF ERRORS

We give here a summary of all our assumptions, some of which have already been mentioned, regarding the behavior of the eclipse spectrograph and of the photographic plate, and underlying the interpretation of the spectrogram; and we include a brief discussion to justify these assumptions. The assumptions may be said to concern the errors of observation of a more or less systematic nature, which may be divided into two classes: (1) those incident to the actual taking of the photograph at the time of the eclipse and (2) those associated with the behavior of the photographic plate.

1) Of those of the former class, the most obvious relates to the uniformity of motion imparted to the photographic plate by the glycerin clepsydra. The clepsydra has usually proved satisfactory as a source of uniform motion, and has recently, for example, been found useful at the Norman Lockyer Observatory,[15] but in the eclipse apparatus there was considerable opportunity for uniform action of the weight and piston to be seriously disturbed by variable friction in the wooden ways guiding the plate holder. There is indeed evidence of irregularities of this sort, in the form of the "bands" of lesser exposure in the Fraunhofer region parallel to the focal plane slot (i.e., to the direction of spectral dispersion). All of these are visible in Plate IX and some in Plate X.

It is probable, at first sight, that these irregularities persist throughout the region of the flash spectrum, and in fact there is evidence of such a slight irregularity just above the limb, the effect of which will be discussed later. While there is no continuous background for this effect to show itself on, yet the eye is quite sensitive to irregularities of this sort in regions of rapidly and uniformly varying intensity, and with the exception of this one possibility no such irregularity is evident, nor do the photometer measures suggest any but this one.

Rather more difficult to detect, and far more detrimental to the results, is the existence of a more or less uniform acceleration in the motion of the plate-holder, brought about, let us say, by the variable friction of possibly wedge-shaped guiding ways. It is in testing for this effect that the similar photographs taken at other eclipses have found their usefulness. Photometric measures of flash lines on the Arrhenius plate and on whatever lines of the other plates are photometrically suitable give curves of the same general shape. This check is uncertain to the extent that the photometric constant of only the Arrhenius plate is known, and it is evident that, in so far as the behavior of each line of itself is concerned, a uniform positive or negative acceleration in the motion of the plate-holder can be compensated for by adjustment of the logarithmic photometric constant. Hence, a check of this sort must be associated with a consistent behavior in the relative intensities of lines close together in the spectrum on the check plates, a relation which is in fact found.

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[15] Lockyer: *Mon. Not., R. A. S.*, **85**, 47, 1924.

But a much more cogent indication of the freedom from acceleration comes from the character of the intensity variation, with the chromospheric level, of the low temperature lines of the abundant heavy metals, such as iron. Evidence which will be given in the discussion of the results, indicates that for these substances the density of radiating gas along the line of sight is so great that the length of the column does not affect the total intensity of the light emitted. The emission is saturated, hence, each slice of the chromosphere, of uniform thickness and of width determined by the width of the focal-plane slot, appears of equal brightness, independent of its height in the chromosphere. Therefore, taking the phenomenon at third contact as an example and assuming that the instrument operates perfectly, as the Moon gradually uncovers the lower layers of the chromosphere the total intensity of the light falling upon the moving plate increases *uniformly* and this integrated intensity, if plotted against height in the chromosphere as indicated by vertical distances on the spectrogram, should give a linear curve. The satisfactory manner in which this occurs is shown by fig. 2, which represents the behavior of the low temperature iron line λ4046, typical of many other lines of the same sort.

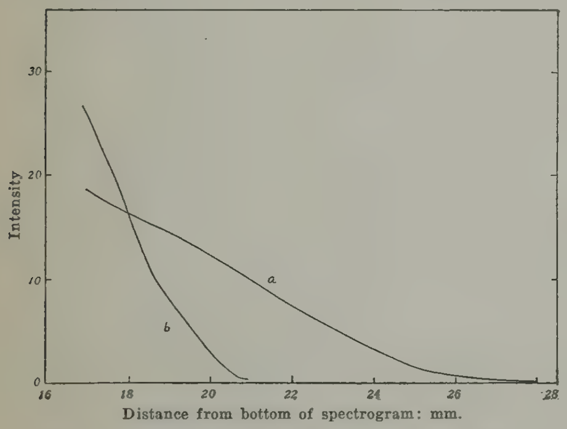

Fig. 2. λ4046 (Fe). Ordinates represent intensities on an arithmetic scale. Abscissae represent distances in mm. from lower edge of the spectrum. (*a*) Curve for the emission line. (*b*) Curve for the photospheric background.

2) Closely associated with this source of error is that arising from cloudiness at the eclipse. At the Spanish eclipse of 1905 the Lick observers were troubled by clouds,—so much so that the second contact was for the most part lost, and the third contact was observed through very thin clouds.[16] But, lacking an absolute scale of the heights on these spectrograms, the effects of variable cloudiness are indistinguishable from those of variable action of the clepsydra, and everything that has been said in that regard applies equally well to the question of cloudiness. Actually of course it would be entirely possible for the errors arising from these two sources to offset each other, though it would be highly improbable, since each variable is independent and continuous. But, even if such were the case, the only error remaining would be in the scale, in kilometers, of the heights of the lines,—a quantity only roughly known in any event. It should be pointed out, also, that the abrupt irregular "bands" in the Fraunhofer spectrum must result from a variation so rapid that the cause cannot readily be attributed to variable cloudiness.

3) There remains only one source of disturbance to our conclusions which could have been introduced at the time of the eclipse, namely, the possibility of photographing an unrepresentative region of the Sun. The chromospheric spectrum is not the same at all eclipses,[17] and it may well be difficult to define accurately what is meant by a representative region of the chromosphere. For the purposes of this paper it will be sufficient to define it as a region free from prominences of any magnitude.

The spectrogram with which this study is mainly concerned was made at third contact: therefore at the west limb. An examination of the series of direct photographs at the Lick Observatory, taken at the same eclipse with the forty-foot camera, shows that, while the eastern limb was very rich in prominences, there was but one of any size at all at the west limb. This is an eruptive prominence of some height, but so narrow that it could occupy but one-sixth or less of the width of the focal-plane slot of the spectrograph. Hence, if its image did fall upon the plate, its contribution to the whole brightness photographed would be small, probably comparable to the errors inherent in the plate and its measurement, for the brightness of the H and K lines in the prominence appears to be appreciably less than their brightness in the chromosphere, according to another set of Lick spectrograms which show the whole chromospheric crescents. The position of the prominence, however, is so far from the center of the crescent that its interference is quite unlikely.

Further evidence of freedom from trouble on this point comes from the agreement in the general shape of the curves from the other similar spectrograms, which has already been mentioned; and finally there is one more indication from the unpublished measures of wave-length made upon the Arrhenius plate by H. C. Plummer. On comparison of these measurements with some (also unpublished) made by Fath and Blair upon a set of flash spectra of the same eclipse, having similar dispersion but showing the entire chromospheric crescents, so that the effects of prominences may be readily eliminated, the agreement as to lines measured and as to their intensity is all that could be expected. Curiously enough, the measures by Fath and Blair are noticeably more consistent in these respects with Plummer's measures than with Mitchell's, though all the spectra were obtained at the same eclipse.

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[16] Campbell: *Publ. A. S. P.*, **18**, 27, 1906.

[17] Mitchell: *Eclipses of the Sun.* Ch. XIII.

4) Of the assumptions regarding the behavior of the photographic plate, that of which the uncertainty is greatest concerns itself with the similarity of the exposure-density curves of the photometer wedge and the eclipse plate. As has been described, the photometric constant of the eclipse plate was determined from the ratio of brightness of background to brightness of line in a restricted region of the spectrum where it is impossible to obtain a knowledge of the dependence of the constant upon photographic density. But there is evidence which indicates that this is inappreciable except possibly in the regions of very light exposure, for, if the variation were sufficient to affect the measures, then the "bands" of lighter exposure in the Fraunhofer region should give rise to irregularities in the differences of photometer readings, background minus line. The absence of these irregularities, in particular the very prominent one which might be expected at 15 mm., is indicated in fig. 1 for the iron line λ4046, which is typical of all lines examined in all parts of the spectrum represented on the plate. The range of density represented in this test includes all but the lightest exposures.

Favorable evidence comes also from the curves which show the relation between the photometric reading (with the same wedge) and the known logarithmic exposure for a number of plates covering a considerable range of contrast. These relations are linear except in the regions of light exposure, corresponding to those not subject to the intensity ratio test on the eclipse plate; at these densities the wedge is presumably of lesser relative contrast than the ordinary plate. For all important parts of the plate, however, the assumption of a uniform photometric calibration seems well justified, although a knowledge of the color sensitivity curve of the entire eclipse spectrograph, including the plate, would furnish an additional check.

5) Two of the most important obstacles to precision photographic spectrophotometry are the influence of the Eberhard effect and of photographic irradiation upon the density of absorption lines. Since the determination of the photometric constant was made to depend upon measures of absorption lines, there might be opportunity for these effects to become serious. Fortunately they work oppositely. There are, moreover, two indications that consideration of them may be safely neglected: some experiments suggested in conversation with Dr. J. H. Oort show that the Eberhard effect becomes appreciable only when associated with much greater background densities than were used in the evaluation of the photometric constant; and, since the linear dispersion of the Snow spectrograms is very nearly equal to that of the eclipse plate in the region used for calibration, the effects of irradiation can depend only upon the difference in the turbidities[18] of the two plates and there can be no measurable influence of distance upon the Eberhard effect.

6) No correction can be made for the photographic Purkinje effect,[19] for this would necessitate a knowledge of the photometric constants in several parts of the spectrum, and any attempt to obtain these by the method already used would be hampered very seriously by irradiation and Eberhard effects due to great densities in some parts of the eclipse spectrogram. Unfortunately, the work of Ross shows that there is a considerable variation in the photometric calibration between the wave-lengths λλ4000 and 4400, and were it not for some of the uncertainties already described this would probably be one of the most serious obstacles to accuracy in this study. It limits considerably the accuracy to which our photometric constant may be trusted, except in the vicinity of the lines used for calibration.

Summarizing, then, it may be pointed out that the manner in which the eclipse spectrogram shows linear relations between integrated intensity of the flash lines and chromospheric level where such relations would be expected argues much for its satisfactory photometric behavior, yet in view of the foregoing discussion too much emphasis cannot be placed upon the danger of a quantitative acceptance of the measures presented in this paper. The conclusions presented later are of a qualitative nature, and as such are believed to be reliable. The discussion emphasizes the inconvenience, and even the danger, of using observations for purposes other than those for which they were obtained. There is every need for repeating spectrograms of this sort at future eclipses, taking special precautions for the photometric purpose of the plate and taking very particular care to make provision for the determination of a photometric scale in all parts of the spectrum used. It is one of the objects of this paper to show the importance of such an eclipse program.

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[18] Ross: *Ap. J.*, **53**, 349, 1921.

[19] Ross: *Ap. J.*, **52**, 86, 1920; **56**, 34, 1922.

DISCUSSION OF RESULTS

Some idea of the unusual excellence of definition of the Arrhenius plate may be obtained from an inspection of Plates IX and X. The former is enlarged some four or five times from the original negative and the latter about ten times. Plate IX shows the region of Hγ and includes many of the abundant lines of ionized titanium in that region. Plate X shows the region of the iron line λ4046, including the He line λ4026 and the ionized strontium line at λ4077.

The most striking feature of the plates is the completely different behavior of the enhanced and normal lines. An inspection of the following lines in Plate X will bring out the main points:

TABLE 2

Line	Element		Line	Element
4012.4	Ti$^+$, Cr$^+$		4040.8	Ce$^{(+)}$, Nd$^{(+)}$
4014.2	Sc, Fe		4041.5	Mn, Fe, Zr
4015.1	Ce$^{(+)}$		4045.8	Fe
4018.4	Fe		4048.8	Fe$^+$
4021.1	Nd$^{(+)}$		4053.9	Ti$^+$ (blend with Fe)
4021.6	Nd$^{(+)}$		4061.4	Nd$^{(+)}$
4025.4	Ti$^+$, Ce$^{(+)}$		4063.7	Fe
4026.2	He		4067.2	Ni$^+$
4028.5	Ce$^{(+)}$, Ti$^+$		4068.1	Fe
4030.8	Mn	ultimate triplet	4071.9	Fe
4033.0	Mn		4077.7	Sr$^+$
4034.4	Mn			

The pair at 4040.8 and 4041.5 is of particular interest.

Aside from the reversal of the enhanced lines relatively very low on the photosphere, which we shall discuss more fully later, one of the most conspicuous differences of behavior is in the manner of reversal. This takes place, in the case of the prominent lines of the Fraunhofer spectrum, with a distinct wedge of absorption. These lines are nearly all unenhanced lines. The enhanced lines, except the very strong ones, such as λ4078, reverse by fading as a whole very gradually with respect to the background. The strong enhanced lines show the wedge as a sharp core which widens as the edge of the solar disk is approached, and since the strong enhanced lines all reach to a high level this absorption is undoubtedly a high level phenomenon.

At the edge of the photosphere all lines are distinctly sharper on the less refrangible edge (the edge defined by the Moon) than on the other, and there is a distinct asymmetry in the sense that the emission lines seem bulged on the short wave-length side. In the lines which reverse near the limb this asymmetry is continued at reversal, the weakened absorption and emission lines at this point appearing to lie side by side with the former towards the red. In the weaker enhanced lines this effect is difficult to detect, but in the strong enhanced lines it shows up again in a large displacement of the absorption core towards the red, as is visible in λ4077 of ionized strontium. It is particularly prominent in the case of the H and K lines, and in these as well as in λ4077 the violet side of the core is much the more brilliant.

All the asymmetry finds explanation in the geometry of the method of photographing, for on the original spectrogram the diameter of the Sun's image is about 18 mm., corresponding to a purely geometrical width of about 0.1 or 0.2 mm. for the lower chromosphere in Plate X. It is at once evident that the asymmetry is of this order of magnitude, and is what would be expected if the cool high-level solar atmosphere increasingly absorbed the emission from the lower levels as the proportion of cool material increased towards the limb. This effect may then be expected quite completely to mask any real asymmetry of brightness which might exist.

The behavior of the wide diffuse wings of the H and K lines is of considerable interest. In the Fraunhofer spectrum these wings attain a total width of some seven angstroms and have usually been attributed to material of very low level. They are conspicuously present in emission in the flash spectrum on the two plates (4 and 5, Table 1) in which the exposure in this region is sufficient to bring them out. But the matter of importance is that they are not confined to the low reversing layer, but rise to a height of several hundred kilometers. The width of the wings is the full amount at the limb, and diminishes gradually to zero at the maximum height. It is not possible to estimate the height with much precision. Some lines in the vicinity which are of similar height (e.g., λλ3938 Cr and 3883 Cy) extend, according to Mitchell, to nine hundred kilometers, but the exposure in this part of the spectrum is in no case sufficient to bring out the upper parts of these lines, so that without the assumption that the law of fading of the H and K wings is similar to that of the lines nearby this is likely to be an overestimate. If the scale is taken from a more strongly exposed part of the plate the height of the wings seems to be about four to six hundred kilometers, but unless it is assumed that the wings are photographed to their full extent in this none too heavily exposed region, this must be an underestimate. Probably these figures represent extreme possibilities, and the true height may be taken to be of the order of six or seven hundred kilometers. These wings exert a strong absorption in the spectrum of the limb on the eclipse plates, the reversal taking place appreciably closer to the limb than in the case of the lines of un-ionized metals. It may be mentioned that against the possibility that these wings may be due appreciably to photographic irradiation there is the evidence that no other lines show them in any easily perceptible degree, either in emission or absorption, although the Balmer lines are of comparable brightness.

The results of the photometric measures are most conveniently presented by curves representing typical lines, as shown in figs. 2 to 14. The curves represent the brightness of lines as functions of position near the limb. Abscissae are distances in millimeters from the bottom of the Fraunhofer spectrum, as it appears on the eclipse spectrogram, away from the center of the Sun, i.e., from the bottom of the spectrum in Plate IX towards the top. One millimeter on the eclipse negative corresponds very roughly to about 225 kilometers in the chromosphere. Ordinates represent intensity[20] upon an arithmetic or logarithmic scale, the logarithmic curves being included in order to show better the behavior of the brighter parts of the lines in the region of the photosphere. The base of the logarithmic scale is 1.195. As previously explained, the unit of intensity in the arithmetic curves is quite arbitrary for each line, so that the arithmetic intensities cannot in general be compared except where lines from the same part of the spectrum are represented in the same figure.

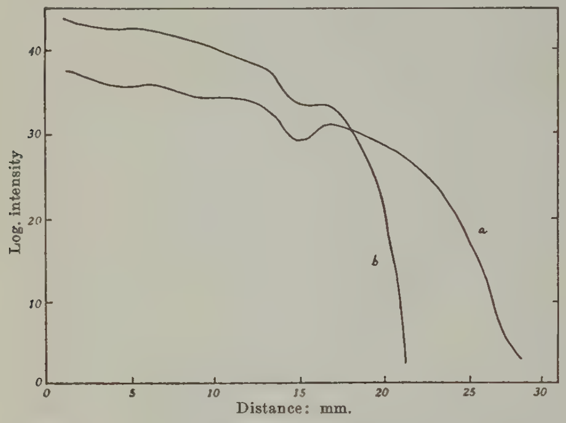

Fig. 3. (*a*) Logarithmic intensity curve for λ4046 (Fe).
(*b*) Curve for the photospheric background.

As in all the intensity curves, abscissae represent distances in mm. from the lower edge of the spectrum, as shown in Plate III.

Figure 3 shows the logarithmic intensity curve for the iron line at λ4046, figure 4 that for the two strongest lines of the manganese ultimate triplet at λ4030 and λ4033, and figure 5 that for the g line of calcium at λ4227. In all the diagrams the very steep curve at the left represents the behavior of the photospheric background. The behavior of these four lines is similar, and is quite typical of the strong lines of the normal metals. The drop in the intensity of line and

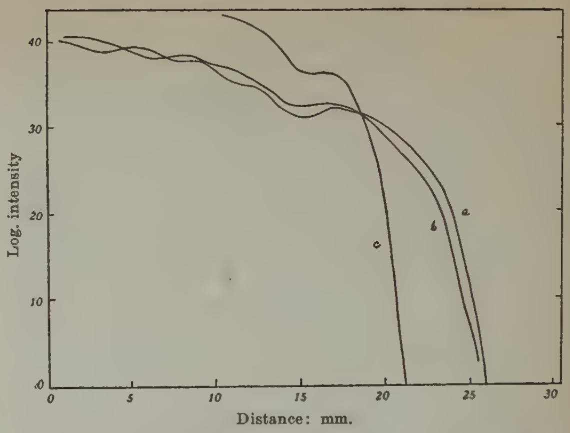

Fig. 4. (*a*) Logarithmic intensity curve for λ4031 (Mn).
(*b*) Logarithmic intensity curve for λ4033 (Mn).
(*c*) Curve for the photospheric background.

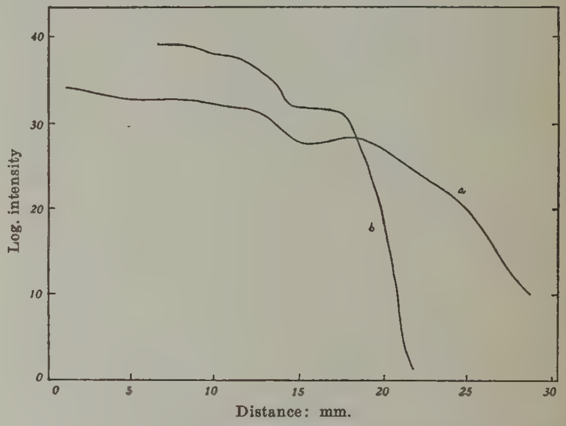

Fig. 5. (*a*) Logarithmic intensity curve for the g line, λ4227 (Ca).
(*b*) Curve for the photospheric background.

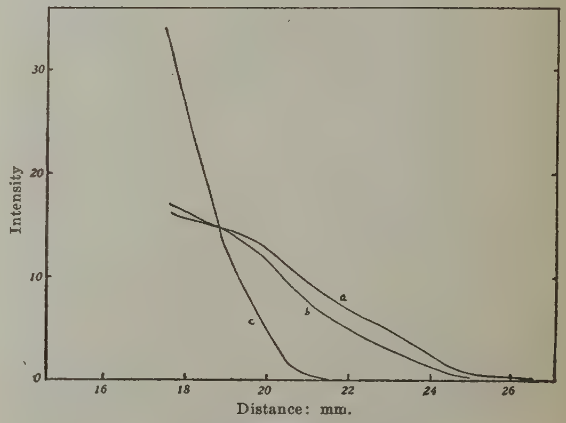

Fig. 6. (*a*) Intensity curve for λ4031 (Mn).
(*b*) Intensity curve for λ4033 (Mn).
(*c*) Curve for the photospheric background.

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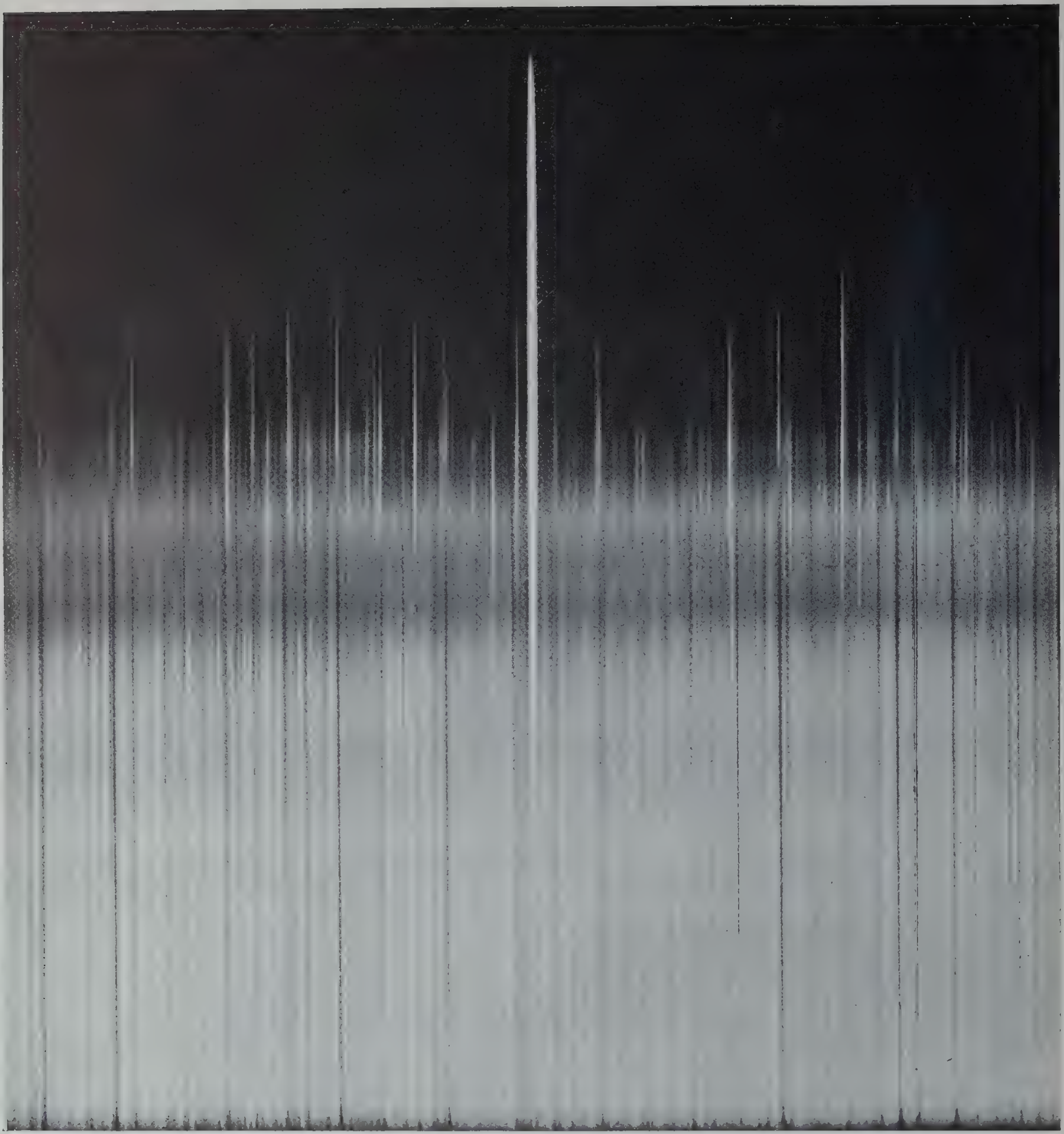
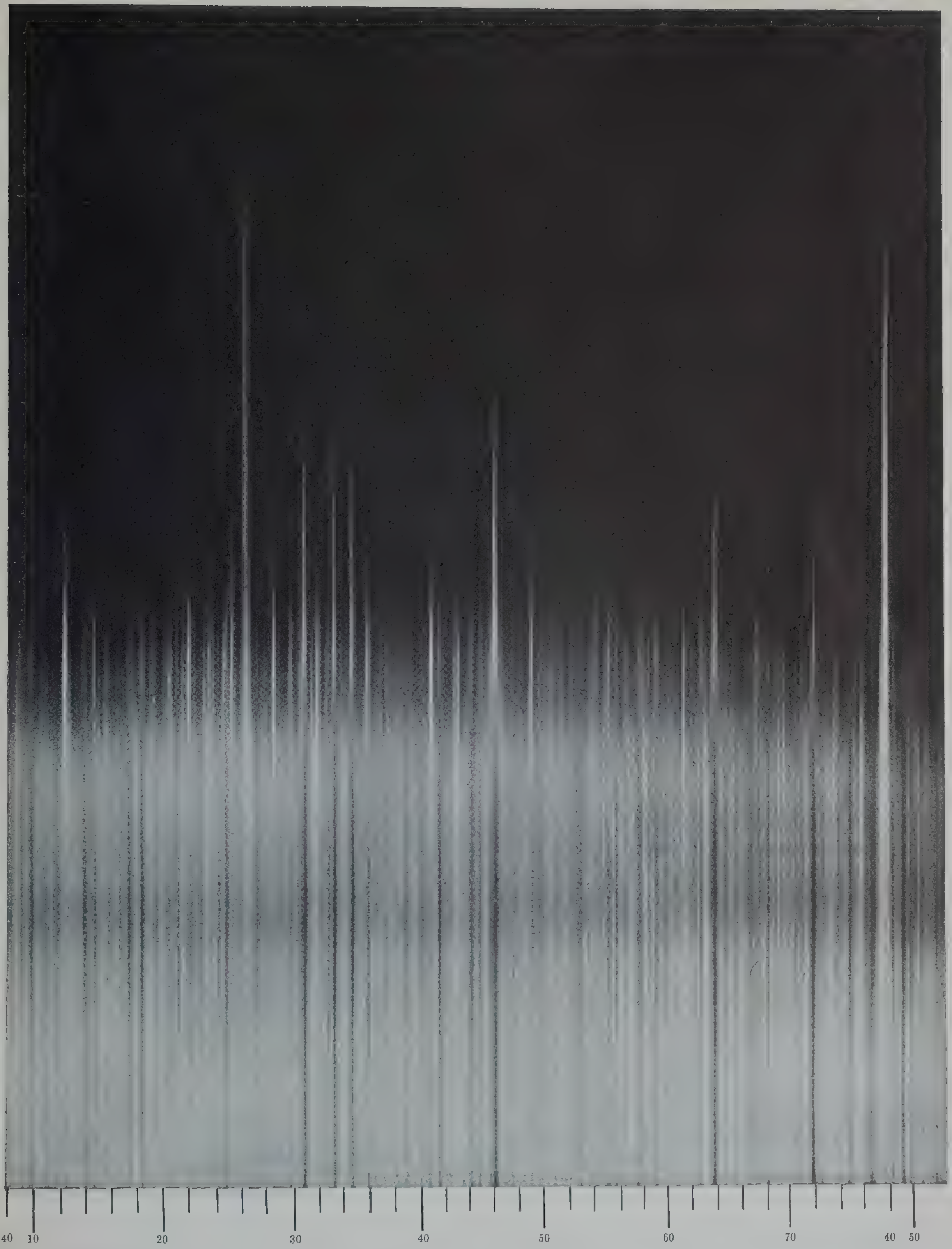
[20] It should be pointed out that the term *intensity* as used in this paper is to be taken as equivalent to brightness or illumination, and not, as in the usual sense, to contrast, or difference in brightness between any absorption line and its background.



Enlarged section of the flash spectrum from the plate of 1905 (Spain). The most intense line is Hγ.
Scale: 1 mm. = 1 A. (Approx.).



Enlarged section of the flash spectrum in the region from He 4026 to Sr^+ 4077, from the plate of 1905 (Spain).
The scale indicates approximate wave lengths.

background together, beyond the reversal, comes from the very strong horizontal band of light exposure visible in Plate IX. Figures 2 and 6 show the chromospheric regions of the iron (λ4046) and manganese (λλ4031, 4033) curves on an arithmetic scale, together with the curves for the photospheric background.

Figure 7 shows the behavior of λ4077, a member of the first pair of the ultimate lines of ionized strontium, on a logarithmic scale; it is represented on an arithmetic scale in fig. 8. The curve is typical of those for all the strong enhanced lines, and resembles closely in shape the curves for all the Balmer lines, which are, therefore, omitted. The less prominent enhanced lines have nearly linear logarithmic intensity curves, which on an arithmetic scale are represented by sharply rising curves. As typical examples, fig. 9 represents the intensity curves for the ionized titanium line λ4468 and for a member (λ4554) of the ultimate pair of ionized barium.

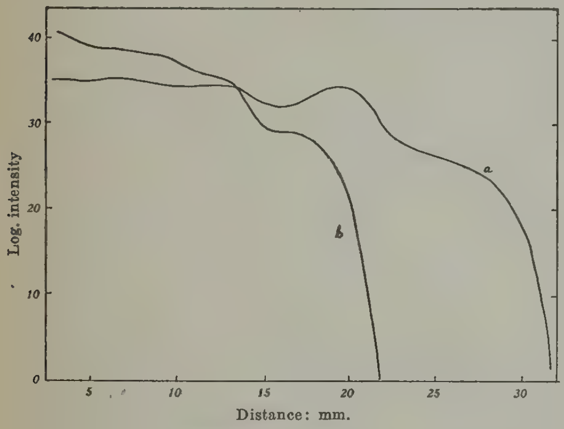


Fig. 7. (*a*) Logarithmic intensity curve for λ4077 (Sr^+).
(*b*) Curve for the photospheric background.

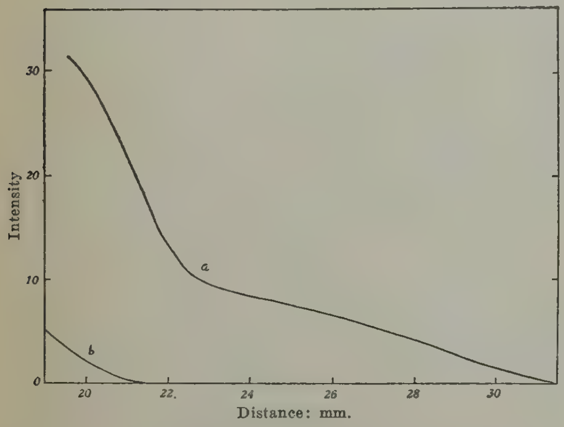


Fig. 8. (*a*) Intensity curve for λ4077 (Sr^+).
(*b*) Curve for the photospheric background.

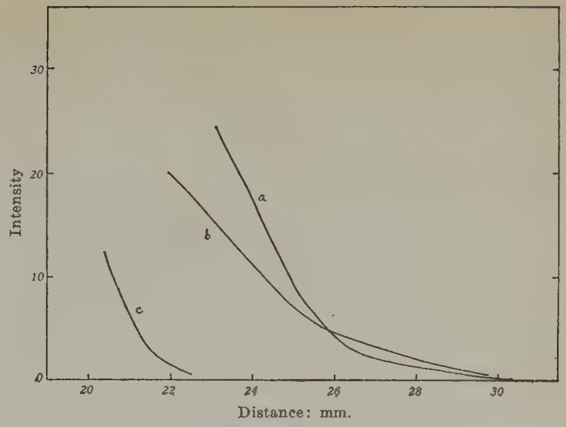


Fig. 9. (*a*) Intensity curve for λ4468 (Ti^+).
(*b*) Intensity curve for λ4554 (Ba^+).
(*c*) Curve for the photospheric background.

The existence of strong wings on all of the most prominent lines of the Fraunhofer spectrum suggests that these lines are saturated in absorption even at the center of the Sun's disk. This view is well corroborated in the case of the strong enhanced lines by their very slight or even negligible diminution in the sun-spots,[21] where ionization is very much less than in the photosphere. If saturation exists at the center of the disk, then it certainly must exist at the limb, and in the chromosphere the emission of these lines must also be saturated. Hence for some distance from the limb the chromosphere in these lines must be of uniform brightness, and the integrated intensity curves on an arithmetic scale, such as those of figs. 2 and 6, must be straight lines.

The intensity curves of the normal metals, then, present no difficulty of interpretation, but those for the Balmer lines and for the intense lines of the ionized metals indicate a distinct discontinuity, as typified in fig. 8. It corresponds to a uniform bright chromosphere surrounded by a uniform fainter ring, and to account for such a phenomenon is a matter of great difficulty, so great, in fact, that its reality may well be considered highly unlikely.

Indeed, there is evidence, as will be seen from Plate IX, that there was irregularity of motion in the plate-holder just at the limb, and perhaps just above it, and since there is as much likelihood of the friction in the guiding ways of the plate-holder increasing as decreasing—of the exposure being too long as too short—it may be presumed that the second abrupt rise in the logarithmic curve which contributes so much to the discontinuous rise in the arithmetic curve for the typical enhanced line is quite spurious, and that we may again have here the linear relation which we expect. The curves for the manganese lines and for the g line of calcium just suggest such a possible irregularity, but of an order of magnitude entirely too small to produce any such effect as appears in the curves of the enhanced lines.

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[21] Russell: *Ap. J.*, **55**, 119, 1922.

On the other hand the reality of the abrupt rise is suggested by fig. 10, which represents the logarithmic curve for λ4077 as given by the film of 1908, taken with a different apparatus. It will be noticed that the upper part of the chromosphere is seriously troubled by photographic fog, but apart from this the agreement is quite satisfactory. If this fogginess in the background is allowed for, there is also a fair agreement of the curves for the normal metals as taken from the film of 1908 and from the Arrhenius plate of 1905. The plate holder carrying the film was operated more slowly than that carrying the plate of 1905, giving greater exposure and a shorter vertical scale. The motion was also far more uniform, so that the possibility of spurious irregularities in the curves from the film is relatively smaller. Hence there is fairly consistent evidence from these spectrograms of a phenomenon which seems highly improbable. In view of this improbability it seems hardly safe to assume the reality of the effect, but this much may be stated with some assurance: that close to the limb the cumulative intensity of the strong enhanced lines is very greatly increased as compared with that of the unenhanced lines. Whether the difference is due to the unexpected behavior of the former or of the latter is an important matter which can be decided only by similar observations at future eclipses.

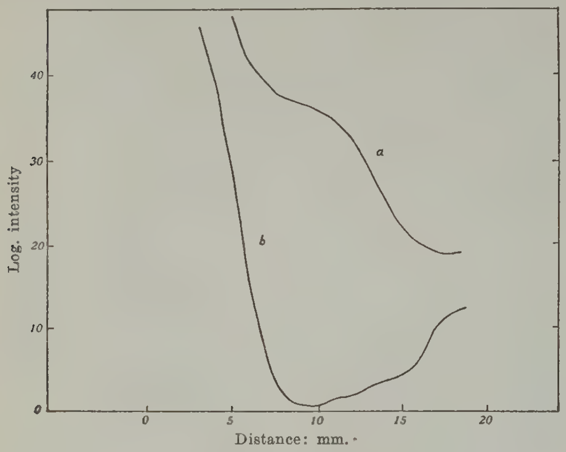

Fig. 10. (*a*) Logarithmic intensity curve for λ4077 (Sr^+) from the film of 1908.
(*b*) Curve for the photospheric background.

On account of the saturation of the strong lines in the lower chromosphere, it is impossible to obtain from observations of this sort any quantitative information concerning the density distribution law in this region, but above the level of saturation the curves for the normal lines indicate a region of diminishing intensity. The accuracy of the curves does not by any means permit the testing of any quantitative laws, but future observations carried out with this problem in mind should yield reliable data.

The tailing off of the enhanced lines takes place too high in the chromosphere to be observed on the Arrhenius plate, for which the exposure was started slightly too late at third contact, but the film of 1908 shows the effect well, though it is too disturbed by photographic fog to yield photometric results. An inspection of the film shows that on the average the diminution in brightness is much more gradual than in the case of the normal lines. This is quite as may be expected, for since the proportion of ionization in the chromosphere increases with height the law of density distribution for elements in the ionized state must be more gradual than for those in the normal state, the proportion of the former increasing in the upper chromosphere at the expense of the latter.

Milne has shown[22] on theoretical grounds that, for certain elements which have essentially only two relevant states and whose lines are so situated that radiation pressure is powerful, the law of density distribution in the upper chromosphere approximates to an inverse square form: a very gradual law as compared with the usual exponential law which results when ordinary gas pressure is important. Ionized calcium and ionized strontium are two elements which fulfill these requirements. Mitchell has already measured their ultimate lines up to very great heights,[5] and while even the film of 1908 does not reach the upper ionized calcium chromosphere, some of that of ionized strontium is shown, and the gradual tailing off is qualitative evidence for some such gradual density law as Milne finds.

Milne[22] also suggests that ionized barium may have such a gradual density law. It does not rise to nearly such a height as ionized strontium, but in so far as the data of fig. 9 may be trusted a more gradual density law is suggested than for ionized titanium, for which radiation pressure is not likely to be so effective. In this case a photometric comparison of the two lines is legitimate since they are in the same region of the spectrum.

On the whole the intensity law for the enhanced lines of normal strength is not greatly different from that for the photosphere, though it is perhaps appreciably less abrupt. Freedom from saturation is clearly indicated. On account of the apparent irregularities in the spectrogram very close to the limb it is not safe to derive a law of darkening at the limb from these plates, though the method of observing is unusually well adapated for such a study very close to the limb,—a region where observation would be of particular value in checking a considerable amount of theoretical work.

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[22] Milne: *Mon. Not., R. A. S.*, **85**, 111, 1924.
[5] Mitchell: *Ap. J.*, **38**, 407, 1913.

The case of un-ionized helium is of unusual interest. The absence of its lines in the Fraunhofer spectrum and their great extent in the flash spectrum are well known. An inspection of the eclipse spectrograms indicates a behavior very similar to that of enhanced lines, and measurements indicate that this behavior is indeed much accentuated, except in the matter of intrinsic brightness. Perhaps here, however, the effect is, partially at least, one of actual "leaking" of the photospheric light through the line. The diffuse series line at λ4026 is shown in Plate X, and curves for it and for the other two prominent helium lines on the spectrogram are shown on an arithmetic scale in figs. 11, 12, and 13.

The absence of these lines in the Fraunhofer spectrum and the abrupt increase in intensity at the limb indicates freedom from saturation. A curve of upward concavity should then be expected to represent the chromospheric intensity, which argues again for a slight increase in the intensity just outside the limb, as previously suggested. In any case the rate of decrease of intensity from the limb is not nearly so

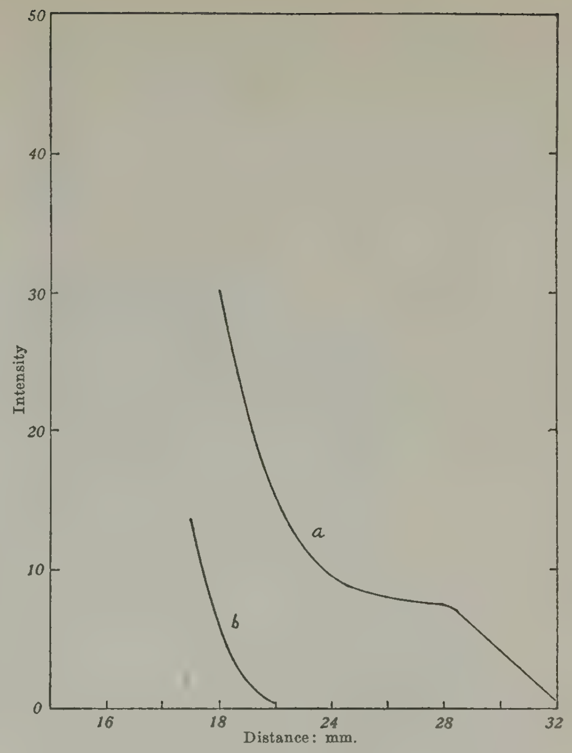

Fig. 12. (*a*) Intensity curve for λ4471 (He).
(*b*) Curve for the photospheric background.

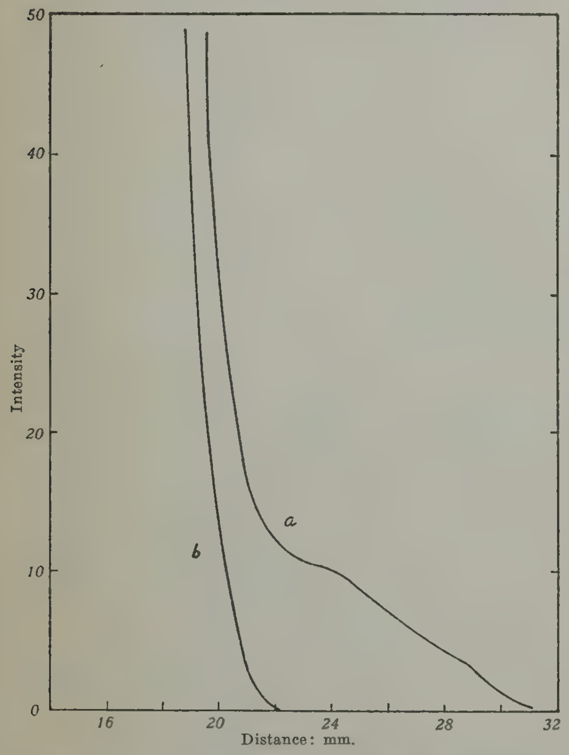

Fig. 11. (*a*) Intensity curve for λ4026 (He).
(*b*) Curve for the photospheric background.

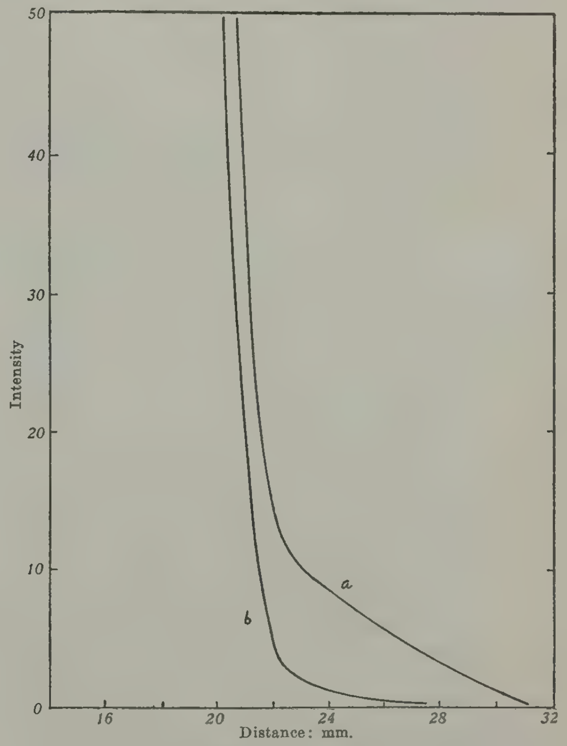

Fig. 13. (*a*) Intensity curve for λ4713 (He).
(*b*) Curve for the photospheric background.

great as for such average bright enhanced lines as those represented in fig. 9, and an extremely gradual density distribution law is indicated, which according to the film of 1908 holds as well far into the outer chromosphere. Fig. 14 shows the relative behavior, to scale, of λ4026 (He) and λ4046 (Fe). The line λ4686 of ionized helium is of course present in the eclipse spectrograms, but it is too faint for measurement in the Hartmann photometer.

It is of some importance to note that all the lines of the rare earths which have been examined behave as characteristic enhanced lines, suggesting a low ionization potential, in accordance with physical evidence, as for example, the relations between atomic number and ionization potential.[23] In Table 2 these lines are suggested as enhanced.

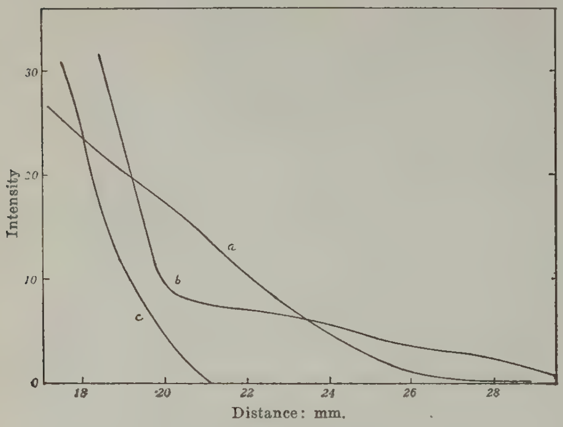

Fig. 14. (*a*) Intensity curve for λ4046 (Fe).
(*b*) Intensity curve for λ4026 (He).
(*c*) Curve for the photospheric background.

The explanation of the low reversals of the enhanced lines is a matter of much difficulty. We cannot interpret the results as a transmission of the photospheric light, for in quantum line absorption the process is not true absorption and we cannot use the usual equation for the intensity of a bright source observed through a column of luminous gas:

$$I=i(1-e^{-kM})+Ie,^{-kM}$$

where M = the mass of the gas in the line of sight
k = the mass absorption coefficient
i = the emission per unit mass of gas
I = the intensity of the bright source.

Instead, a column of gas for which the line "absorption" is saturated must be opaque in the saturated lines to a bright object behind it. Moreover, in the case of the strong enhanced lines the flash line outside the limb is far brighter than the photosphere.

Schuster[24] has shown that if scattering is important in a radiating atmosphere, bright lines will result if the radiation gradient is small. These conditions are met in the case of the enhanced lines, as has been suggested, on account of the conditions of ionization, and also probably in the case of hydrogen and helium on account of low atomic weight, and at the limb the radiation gradient in the line of sight is greatly diminished. But Schuster has also pointed out[25] that his treatment may not hold without further consideration if scattering is largely selective, and it is doubtful if, after all, the phenomenon we wish to explain is so simple.

The very great intensity of the enhanced lines suggests that they radiate with a higher spectroscopic temperature than do the normal lines, and if an explanation of this can be found the problem of the position of reversal may possibly not be so difficult of solution. In view of the extremely low pressures known to prevail in the chromosphere, particularly at a little distance from the reversing layer,[26] and in view of the intense stream of photospheric radiation traversing the chromosphere, with its attendant powerful radiation pressure and photoelectric excitation, both of these processes being highly selective, it is not surprising that widely varying effective radiation temperatures should exist among different kinds of atoms, but in the present state of our knowledge we shall probably have to be satisfied with pointing out the difference without attempting a satisfactory explanation.

Summary

1. A photometric study has been made of flash spectrograms obtained at four eclipses by the Lick Observatory in such a way as to show the behavior of the emission lines in the chromosphere and at reversal. The spectrograms are wholly unsuited for quantitative interpretation, but there is evidence to indicate that certain qualitative conclusions may be drawn with reasonable safety. The advantages of this method of observing the flash spectrum for data of quantitative value make its inclusion in programs of future eclipses a matter of prime importance.

2. The behavior of the normal and enhanced lines at reversal is very different, the normal lines reversing abruptly approximately to their relative Fraunhofer intensities and the enhanced lines extending in emission relatively far onto the solar disk and showing in general a greater brightness.

3. The lines of hydrogen and of un-ionized helium behave closely like enhanced lines, and in the case of

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[23] C. H. Payne: *Proc. Nat. Acad. Sci.*, **10**, 322, 1924.

[24] Schuster: *Ap. J.*, **21**, 1, 1905.

[25] *Ibid.*, p. 17.

[26] Russell and Stewart: *Ap. J.*, **59**, 197, 1924.

helium the characteristics are exaggerated except in the matter of increased brightness.

4. There is evidence for a uniform saturated chromosphere in the intense lines, but close to the limb the strong enhanced lines appear to be relatively much brighter than the strong normal lines. The weaker lines appear unsaturated.

5. The greater surface brightness of the saturated enhanced lines indicates a higher effective radiation temperature for these lines as compared with that for the un-ionized lines.

6. In the strong lines, above the region of saturation, there is a region of diminishing brightness. In accordance with other evidence, this extends much higher and is much more gradual in the case of the enhanced lines than in the case of the normal lines, and in this respect, too, the un-ionized helium lines strongly resemble the enhanced lines.

7. Contrary to usual supposition, the wide diffuse wings of the H and K lines are not confined to the low reversing layer, but show in emission to a height of six or seven hundred kilometers.

8. All the prominent lines of the rare earths behave like enhanced lines.

The writer is greatly indebted to President Campbell for his generous permission to work with the eclipse spectrograms, to Director Adams for the opportunity to work at the Mount Wilson Observatory, and to the members of the staff of the Lick Observatory for their encouragement and assistance.

December, 1926.
Issued, March 4, 1927.

END OF VOLUME XII.

INDEX TO VOLUME XII

PAGE

Absolute Luminosities of Class A Stars, A Proposed Method for the Determination of the. C. S. Yü 155

Aitken, R. G. Eighty-five New Double Stars. Twenty-Fifth List 170
  Measures of Double Stars in the Years 1923–1926 173
  The Orbit of Krueger 60 45
  The Orbit of β 581 47

Alexeievsky, A. P. Elements and Ephemeris of Comet *c* 1925 (Orkisz) [with J. D. Shea] 35
  Preliminary Elements and Ephemeris of Minor Planet Y. O. 23 [with A. D. Maxwell and J. D. Shea] 3

Allen, Leah B. The Radial Velocities of Twenty Southern Variable Stars of Class Me 71
  A Study of the Changes in the Spectrum of T *Centauri*. 73

Applegate, Dorothy. The Spectrum of α *Cygni* between Wave-lengths 3020A and 3300A [with W. H. Wright] 81

Azimuth Corrections for Sunset and Sunrise Transits. R. H. Tucker 65

Baker, R. H. A Photoelectric Study of *u Herculis* 130

Boss Proper Motions in Declination, On the Determination of the Systematic Error in the. P. van de Kamp 38

Brigham, L. A. Preliminary Parabolic Elements of Comet *f* 1926 (Coma Solá) [with Katherine Prescott] 169

Carpenter, E. F. A Photometric Study of the Flash Spectrum 183

Cepheid Variable Stars, A Re-determination of the Radial Velocity Curves of Certain. T. S. Jacobsen 138

Class A Stars, A Proposed Method for Determining the Absolute Magnitude of the. C. S. Yü 155
  On the Continuous Hydrogen Absorption in Spectra of. C. S. Yü 104
  See also, The Spectrum of α *Cygni*.

Cluster Messier 11, The. R. J. Trumpler 10

Comet *c* 1919 (Metcalf-Borrelly), Second Elements of. W. F. Meyer 37

Comet *b* 1924 (Encke), Measures of. H. M. Jeffers 9

Comet *c* 1924 (Finsler), Measures of. H. M. Jeffers 8

Comet *d* 1924 (Wolf), Measures of. H. M. Jeffers 62

Comet *a* 1925 (Shajn–Comas Solá) Elements and Ephemeris of. H. B. Kaster and Katherine Prescott 31
  Measures of. H. M. Jeffers 63, 124

Comet *b* 1925 (Reid), Elements and Ephemeris of. A. D. Maxwell 33
  Measures of. H. M. Jeffers 63, 125

Comet *c* 1925 (Orkisz), Elements and Ephemeris of. J. D. Shea and A. P. Alexeievsky 35
  Measures of. H. M. Jeffers 64, 125

Comet *d* 1925 (Tempel II), Measures of. H. M. Jeffers 64, 125

Comet *e* 1925 (Wolf), Measures of. H. M. Jeffers 126

Comet *f* 1925 (Borrelly), Measures of. H. M. Jeffers 127

Comet *g* 1925 (Brooks), Measures of. H. M. Jeffers 127
  Preliminary Elements and Ephemeris of. N. W. Storer and H. S. Mendenhall 77

Comet *h* 1925 (Faye), Measures of. H. M. Jeffers 127

Comet *i* 1925 (Peltier–Wilk), Measures of, H. M. Jeffers. 128

Comet *j* 1925 (Van Biesbroeck), Elements and Ephemeris of. A. D. Maxwell and L. C. Damsgard 76, 79, 122
  Measures of. H. M. Jeffers 128

PAGE

Comet *b* 1926 (Blathwayt), Measures of. H. M. Jeffers. 129
  Preliminary Elements and Ephemeris of. A. D. Maxwell and N. W. Storer 87

Comet *f* 1926 (Comas Solá), Elements and Ephemeris of. C. H. Smiley and Margaret K. Holbrook 168
  Preliminary Parabolic Elements of. L. A. Brigham and Katherine Prescott 169

Damsgard, L. C. Elements and Ephemeris of Comet *j* 1925 (Van Biesbroeck) [with A. D. Maxwell] 76, 79, 122

Double Stars, Eighty-five New, Twenty-fifth List. R. G. Aitken 170
  Measures of, in the Years 1923–1926. R. G. Aitken 173
  The Orbit of β 581. R. G. Aitken 47
  The Orbit of Krueger 60. R. G. Aitken 45
  See also, Spectroscopic Binary.

Eclipsing Binary Stars: See Variable Stars.

Flash Spectrum, A Photometric Study of the. E. F. Carpenter 183

Holbrook, Margaret K. Elements and Ephemeris of Comet *f* 1926 (Comas Solá) [with C. H. Smiley] 168

Jacobsen, T. S. A Re-determination of the Radial Velocity Curves of Certain Cepheid Variable Stars 138

Jeffers, H. M. Measures of Comets 124
  Measures of Comets and of Minor Planet 1924 TD (Baade) 8, 62
  Observations of the Satellites of *Mars* 4

Kaster, H. B. Elements and Ephemeris of Comet *a* 1925 (Shajn–Comas Solá) [with Katherine Prescott] 31

Leuschner's Direct Method for Determining the Orbits of Disturbed Bodies, On the Number of Solutions in. R. H. Sciobereti 17

Mars: Observations of the Satellites of. H. M. Jeffers 4
  Photographs of, Made with Light of Different Colors. W. H. Wright 48

Maxwell, A. D. Elements and Ephemeris of Comet *b* 1925 (Reid) 33
  Elements and Ephemeris of Comet *j* 1925 (Van Biesbroeck) [with L. C. Damsgard] 76, 79, 122
  Preliminary Elements and Ephemeris of Comet *b* 1926 (Blathwayt) [with N. W. Storer] 87
  Preliminary Elements and Ephemeris of Minor Planet Y. O. 22 [with T. D. Shea] 2
  Preliminary Elements and Ephemeris of Minor Planet Y. O. 23 [with J. D. Shea and A. P. Alexeievsky]. 3
  Second Elements of Minor Planet Y. O. 22 78

Mendenhall, H. S. Preliminary Elements and Ephemeris of Brooks' Periodic Comet *g* 1925 [with N. W. Storer] 77

Meyer, W. F. The Orbit of the Spectroscopic Binary 112 *Herculis* 165
  Second Elements of Comet *c* 1919 (Metcalf–Borrelly). 37

Minor Planet: 1924 TD (Baade), Corrected Elements and Ephemeris of. H. Thiele 1
    Measures of. H. M. Jeffers 9, 62
  Y. O. 22, Preliminary Elements and Ephemeris of. A. D. Maxwell and J. D. Shea 2
    Second Elements of. A. D. Maxwell 78
  Y. O. 23, Preliminary Elements and Ephemeris of. J. D. Shea, A. D. Maxwell and A. P. Alexeievsky 3

Orbit Methods: See Leuschner's Direct Method.

Photometric Study of the Flash Spectrum, A. E. F. Carpenter 183

PAGE

Prescott, Katherine. Elements and Ephemeris of Comet *α* 1925 (Shajn–Comas Solá) [with H. B. Kaster] 31
  Preliminary Parabolic Elements of Comet *f* 1926 (Comas Solá) [with L. A. Brigham] 169

Proper Motions: See Boss Proper Motions.

Radial Velocities of Twenty Southern Variable Stars of Class Me. Leah B. Allen 71

Sciobereti, R. H. On the Number of Solutions in Leuschner's Direct Method for Determining the Orbits of Disturbed Bodies 17

Shea, J. D. Elements and Ephemeris of Comet *c* 1925 (Orkisz) [with A. P. Alexeievsky] 35
  Preliminary Elements and Ephemeris of Minor Planet Y. O. 22 [with A. D. Maxwell] 2
  Preliminary Elements and Ephemeris of Minor Planet Y. O. 23 [with A. D. Maxwell and A. P. Alexeievsky] 3

Smiley, C. H. Elements and Ephemeris of Comet *f* 1926 (Comas Solá) [with Margaret K. Holbrook] 168

Spectra of Class A Stars, On the Continuous Hydrogen Absorption in. C. S. Yü 104

Spectroscopic Binary 112 *Herculis*, The Orbit of the. W. F. Meyer 165

Spectrum of *α Cygni* between Wave-lengths 3020A and 3300A. W. H. Wright and Dorothy Applegate 81

Storer, N. W. Preliminary Elements and Ephemeris of Brooks' Periodic Comet g 1925 [with H. S. Mendenhall] 77

PAGE

  Preliminary Elements and Ephemeris of Comet *b* 1926 (Blathwayt) [with A. D. Maxwell] 87

Sun's Velocity with Respect to Stars of Magnitude 9 and 10, A Determination of the. P. van de Kamp 88

Thiele, H. Corrected Elements and Ephemeris of Minor Planet 1924 TD (Baade) 1

Trumpler, R. J. The Cluster Messier 11 10

Tucker, R. H. Azimuth Corrections for Sunset and Sunrise Transits 65

Van de Kamp, P. A Determination of the Sun's Velocity with Respect to Stars of Magnitude 9 to 10 88
  On the Determination of the Systematic Error in the Boss Proper Motions in Declination 38

Variable Stars: A Photoelectric Study of *u Herculis*. R. H. Baker 130
  The Radial Velocities of Twenty Southern Variable Stars of Class Me. Leah B. Allen 71
  A Study of the Changes in the Spectrum of T *Centauri*. Leah B. Allen 73
  See also, Cepheid Variable Stars.

Wright, W. H. Photographs of *Mars* Made with Light of Different Colors 48
  The Spectrum of *α Cygni* between Wave-lengths 3020A and 3300A [with Dorothy Applegate] 81

Yü, C. S. On the Continuous Hydrogen Absorption in Spectra of Class A 104
  A Proposed Spectroscopic Method for Determining the Absolute Luminosities of Class A Stars 155

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